OBJECT RECONSTRUCTION WITHOUT INTERIOR ORIENTATION

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ABSTRACT:

This paper develops an algorithm for linear object reconstruction without interior orientation. First we introduce the Thompson and Longuet-Higgins equation as well as the fundamental matrix. After defining the affine model, we show that some of its components can be linearly withdrawn from the fundamental matrix, which in turn is linearly determined up to a scale factor by minimum eight image correspondences. This decomposition of the fundamental matrix leads to a full use of the information within a stereo. Unlike the well-known DLT algorithm where minimum six known points are required on each image of a stereo, our algorithm requires that only four of them appear on the other one. In addition to the fully compatible accuracy with the DLT algorithm, tests with an aerial stereo show the robustness of this algorithm as well.

1. MOTIVATIONS

For quite a long time it seemed to be a rule that the interior orientation has to be completed before any other photogrammetric computation is done. This was overthrown by the time the well-known DLT (Direct Linear Transformation) algorithm was publisched by Abdel-Aziz and Karara in 1971 (c.f, Slama, 1980, pp.801-803). It directly relates the object point to its *oblique* image coordinates. This problem seems to be fully solved if we neglect the drawbacks of the DLT algorithm. In fact it recovers an object directly from its images rather than from its *photogrammetric model*, therefore, the inherent information behind the stereo is not fully utilized. Moreover, it requires at least *six* known points on *each* of the images to reconstruct the object. Keeping those issues in mind, the question arises that, how could we reconstruct an object without knowledge of interior orientation by fully employing the information behind a stereo ?

Our motivation also has its deep root in computer vision and close-range photogrammetry, where uncalibrated camera is widely adopted and the interior elements are either unknown or different from image to image.

Linear solution always benefits a lot, especially in computer vision and close-range photogrammetry, where finding reasonable initial values is crucial to the success of iterative algorithms.

Regarding to these backgrounds, our purpose is to find a *linear* solution for object reconstruction without interior orientation, which can fully use the information behind a stereo and has less requirement on known points than the DLT algorithm.

2. REVIEW OF THE RELATED WORK AND OUR SCOPE

As our topic falls both in photogrammetry and computer vision, the related work in both areas should be mentioned. Photogrammetrists seem to rely much on the DLT algorithm and hence on sufficient number of known object points. In contrast, besides paying interests in camera calibration, scientists in computer vision area have fully studied the problem " motion or relative displacement estimation from uncalibrated camera". Most recently, quite a few literatures are focused on this issue. They claimed that without interior elements the object can be reconstructed up to either an affine or a perspective transformation (Faugeras, 1992; Hartley, 1992; Hartley, et al, 1992). Obviously this is of fundamental importance for object reconstruction without interior orientation (Faugeras, 1993; Hartley, et al, 1993).

A linear solution for model reconstruction may date back to the well-known contribution of Longuet-Higgins (1981) in computer vision. However, the basic idea behind this solution was essentially originated from the early work of Thompson (1968), where he expressed the coplanarity equation via an unknown 3×3 matrix which is acknowledged today as *essential matrix* (c.f., Longuet-Higgins, 1981; Huang et al, 1989; Faugeras, et al, 1990; Hartley, 1992; Hartley et al,1993). Photogrammetrists did not recall Thompson's idea for quite a time. Recently, Brandstätter (1992) employed this idea for image rectification. Wang's work (1995) threw a light on this idea upon which a linear algorithm was designed to reconstruct the photogrammetric model *based on* the interior orientation. Most recently, the stability of this algorithm was studied by Barakat et al (1994), Deriche et al (1994), Förstner (1995) and Luong et al (1994).

Our wotk is highly inspired by the work of Hartley et al, Faugeras et al and Wang. Section 3 starts from an affine transformation in the image plane, and then generalizes the Thompson and Longuet-Higgins equation to the case of unknown interior elements. Section 4 is focused on the affine model and the recovery of its components. Unlike Faugeras's work (Faugeras, 1992) where traditional projective geometry is utilized, we fully take advantage of the properties of the skew-symmetric matrix and make our development as parallel as possible to photogrammetry. After defining the affine model analogous to the traditional one, we show some of its components can be withdrawn from the so-called *fundamental matrix*. This leads to a complete employment of a stereo. In section 5 we use a 3D affine transformation to fully recover the object. Unlike the well-known DLT algorithm where minimum six known points are required on each image of a stereo, our algorithm allows that one image may have only four of them. Tests with an aerial stereo in section 6 show that our algorithm is robust both to the configuration of known points and to the affine image deformation. Fully compatible (or even slightly better) results with the DLT algorithm are obtained as well.

3. THOMPSON AND LONGUET-HIGGINS EQUATION

In this section we derive the Thompson and Longuet-Higgins equation which plays a fundamental role in our problem. A short comment is thereafter made on the fundamental matrix.

We write the well-known coplanarity equation as (Slama,1980, pp.54-56)

$$\mathbf{x}_1^T[\mathbf{b} * (\mathbf{R}\mathbf{x}_2)] = 0 \tag{1}$$

In Eq.(1)

$$\mathbf{b} = \begin{pmatrix} B_X & B_Y & B_Z \end{pmatrix}^T \tag{2}$$

is the *base component vector*. $\mathbf{R}$ is the orthogonal rotation matrix of the second image relative to the first one which is assumed to be as a reference. And

$$\mathbf{x}_1 = \begin{pmatrix} x_1 & y_1 & -f_1 \end{pmatrix}^T \quad \mathbf{x}_2 = \begin{pmatrix} x_2 & y_2 & -f_2 \end{pmatrix}^T \tag{3}$$

are coordinates of conjugate image points p_1, p_2 in the first and second image spaces respectively. In Eq.(1) $*$ denotes the scalar product of two vectors.

For any 3×1 vector $\mathbf{x}$ we have

$$\mathbf{b} * \mathbf{x} = \mathbf{B}\mathbf{x} \tag{4}$$

where $\mathbf{B}$ is a 3×3 skew-symmetric matrix whose entries are composed of the elements of $\mathbf{b}$, i.e.,

$$\mathbf{B} = \begin{pmatrix} 0 & -B_Z & B_Y \\ B_Z & 0 & -B_X \\ -B_Y & B_X & 0 \end{pmatrix} \tag{5}$$

Applying Eq.(4) to Eq.(1) yields

$$\mathbf{x}_1^T \mathbf{E} \mathbf{x}_2 = 0 \tag{6}$$

where

$$\mathbf{E} = \mathbf{B}\mathbf{R} \tag{7}$$

Eq.(6) is namely the *Thompson and Longuet-Higgins equation* which was initially derived by Thompson (1968) and rediscovered by Longuet-Higgins(1981). Matrix $\mathbf{E}$, which is the product of the *base component matrix* $\mathbf{B}$ and the *orthogonal rotation matrix* $\mathbf{R}$, is named as *essential matrix* by Longuet-Higgins(1981) and thereafter widely accepted and studied in computer vision (Huang,et al,1989; Faugeras et al ,1990).

It is straightforward to generalize Eq.(6) to the case when the interior orientation is not done. Suppose image points are measured in an *oblique* image coordinate system $(\bar{x}, \bar{y})$ which is generally considered as a linear or an affine transformation of (x, y), namely we have

$$\mathbf{x}_1 = \mathbf{A}_1\bar{\mathbf{x}}_1 \qquad \mathbf{x}_2 = \mathbf{A}_2\bar{\mathbf{x}}_2 \tag{8}$$

where

$$\mathbf{A}_1 = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -f_1 \end{pmatrix}_1 \quad \mathbf{A}_2 = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -f_2 \end{pmatrix}_2 \tag{9}$$

$$\bar{\mathbf{x}}_1 = \begin{pmatrix} \bar{x}_1 & \bar{y}_1 & 1 \end{pmatrix}^T \quad \bar{\mathbf{x}}_2 = \begin{pmatrix} \bar{x}_2 & \bar{y}_2 & 1 \end{pmatrix}^T \tag{10}$$

$\bar{\mathbf{x}}_1$ and $\bar{\mathbf{x}}_2$ are essentially *homogeneous coordinates* of image points. Substituting Eq.(8) back to Eq.(6) we obtain

$$\bar{\mathbf{x}}_1^T \bar{\mathbf{E}} \bar{\mathbf{x}}_2 = 0 \tag{11}$$

where

$$\bar{\mathbf{E}} = \mathbf{A}_1^T \mathbf{E} \mathbf{A}_2 = \mathbf{A}_1^T \mathbf{B}\mathbf{R}\mathbf{A}_2 \tag{12}$$

is called *fundamental matrix* (Faugeras et al, 1992). Eq.(11) and (12) are generalization of Eq.(6) and (7).

Note that matrix $\bar{\mathbf{E}}$ could only be determined up to a scale factor. This fact directly follows if we notice that Eq.(11) is *homogeneous* and therefore multiplication of a scale factor does not change its value. Therefore, there are only *eight linearly independent* parameters in matrix $\bar{\mathbf{E}}$. We need minimum eight image correspondences to *linearly* determine the $\bar{\mathbf{E}}$ matrix. However, since $|\bar{\mathbf{E}}| = 0$, only *seven* degrees of freedom, i.e., free parameters, exist.

There are three degrees of rank deficiency in the reconstruction of the affine model. A simple numeric counting may show this fact. Among 2×11 degrees of perspective transformation in a stereo, 7 could be recovered in relative orientation step and 12 in absolute orientation step. This remains $2 \times 11 - (7 + 12) = 3$ degrees of deficiency to be removed by leaving them to the exterior orientation step, that will yield $12+3 = 15$ exterior orientation parameters.

4. AFFINE MODEL AND RECOVERY OF ITS COMPONENTS

In the following derivation we suppose the fundamental matrix $\bar{\mathbf{E}}$ has been determined up to a scale factor by at least eight image correspondences with Eq.(11).

4.1 Affine model and its components

Applying collinear condition to the first and second image respectively, we obtain

$$\mathbf{p} = \lambda_1 \mathbf{x}_1 \qquad \mathbf{p} = \lambda_2 \mathbf{R} \mathbf{x}_2 + \mathbf{b} \tag{13}$$

where

$$\mathbf{p} = \begin{pmatrix} X & Y & Z \end{pmatrix}^T \tag{14}$$

is the coordinates of object point P in the first image space coordinate system. λ_1 and λ_2 are scale factors of vector $\mathbf{x}_1$ and $\mathbf{x}_2$ relative to vector $\mathbf{p}$ and $\mathbf{p} - \mathbf{b}$, which take the first and second perspective center as their origins respectively. Substituting Eq.(8) to Eq.(13) yields

$$\mathbf{p} = \lambda_1 \mathbf{A}_1 \bar{\mathbf{x}}_1 \qquad \mathbf{p} = \lambda_2 \mathbf{R} \mathbf{A}_2 \bar{\mathbf{x}}_2 + \mathbf{b} \tag{15}$$

The next step is essential for solving our problem. We write Eq.(15) in a way similar to Eq.(13)

$$\bar{\mathbf{p}} = \bar{\lambda}_1 \bar{\mathbf{x}}_1 \qquad \bar{\mathbf{p}} = \bar{\lambda}_2 \bar{\mathbf{R}} \bar{\mathbf{x}}_2 + \bar{\mathbf{b}} \tag{16}$$

where

$$\bar{\mathbf{p}} = \mathbf{A}_1^{-1} \mathbf{p} = \begin{pmatrix} \bar{X} & \bar{Y} & \bar{Z} \end{pmatrix}^T \tag{17}$$

$$\bar{\mathbf{b}} = \mathbf{A}_1^{-1} \mathbf{b} = \begin{pmatrix} \bar{B}_X & \bar{B}_Y & \bar{B}_Z \end{pmatrix}^T \tag{18}$$

$$\bar{\mathbf{R}} = \mathbf{A}_1^{-1} \mathbf{R} \mathbf{A}_2 \tag{19}$$

$\bar{\lambda}_1$ and $\bar{\lambda}_2$ are proportional to λ_1 and λ_2 respectively, since they may take into account the multiplication factor inherent in $\bar{\mathbf{E}}$ matrix.

We define $\bar{\mathbf{p}} = \begin{pmatrix} \bar{X} & \bar{Y} & \bar{Z} \end{pmatrix}^T$ as the *affine coordinates* of object point P, since it is a linear, i.e., affine transformation of its Cartesian coordinates. The collection of all affine points forms an *affine model* of the object. Similarly, $\bar{\mathbf{b}}$ is known as *affine base component vector*, and $\bar{\mathbf{R}}$ is specified as *affine rotation matrix.*

Now we are due to rewrite the fundamental matrix $\bar{\mathbf{E}}$ in Eq.(12) as

$$\bar{\mathbf{E}} = (\mathbf{A}_1^T \mathbf{B} \mathbf{A}_1)(\mathbf{A}_1^{-1} \mathbf{R} \mathbf{A}_2) = \bar{\mathbf{T}} \bar{\mathbf{R}} \tag{20}$$

where $\bar{\mathbf{T}} = \mathbf{A}_1^T \mathbf{B} \mathbf{A}_1$ is a skew-symmetric matrix. As $\bar{\mathbf{T}} \bar{\mathbf{b}} = \mathbf{0}$, we are led immediately to

$$\frac{\bar{T}_X}{\bar{B}_X} = \frac{\bar{T}_Y}{\bar{B}_Y} = \frac{\bar{T}_Z}{\bar{B}_Z} = c$$

i.e., $\bar{\mathbf{T}}$ is proportional to $\bar{\mathbf{B}}$. As any multiplication factor in matrix $\bar{\mathbf{T}}$ can be taken into account in the fundamental matrix, we may simply let $c = 1$. Thus, Eq.(20) becomes

$$\bar{\mathbf{E}} = \bar{\mathbf{B}} \bar{\mathbf{R}} \tag{21}$$

where

$$\bar{\mathbf{B}} = \begin{pmatrix} 0 & -\bar{B}_Z & \bar{B}_Y \\ \bar{B}_Z & 0 & -\bar{B}_X \\ -\bar{B}_Y & \bar{B}_X & 0 \end{pmatrix} \tag{22}$$

is defined as the *affine base component matrix* analogues to $\mathbf{B}$ in Eq.(5).

Being parallel to the essential matrix $\mathbf{E}$ of Eq.(7), Eq.(21) reveals the following primary and important fact:

The fundamental matrix $\bar{\mathbf{E}}$ can be decomposed as a product of $\bar{\mathbf{B}}$ and $\bar{\mathbf{R}}$, where $\bar{\mathbf{B}}$ is a skew-symmetric matrix composed of the affine base components, representing the displacement of the second perspective centre in the first affine image space, and $\bar{\mathbf{R}}$ is an affine rotation matrix representing the orientation of the second camera with respect to the first one.

4.2 Recovery of the affine model components

We start from computing the affine base component vector $\bar{\mathbf{b}}$. Since $\mathbf{B}^T \mathbf{b} = \mathbf{0}$, we have

$$\bar{\mathbf{E}}^T \bar{\mathbf{b}} = \mathbf{0} \tag{23}$$

This is a set of homogeneous equations. By setting $\bar{B}_X$ an arbitrary positive constant, we then could obtain the other two affine base components $\bar{B}_Y$ and $\bar{B}_Z$.

Now we may move on to computing matrix $\bar{\mathbf{R}}$. Eq.(21) is now written columnwise as

$$\bar{\mathbf{B}}\bar{\mathbf{r}}_i = \bar{\mathbf{e}}_i \qquad (i = 1, 2, 3) \qquad (24)$$

where $\bar{\mathbf{r}}_i$ and $\bar{\mathbf{e}}_i$ are the column component vectors of $\bar{\mathbf{R}}$ and $\bar{\mathbf{E}}$ respectively. Since $rank(\bar{\mathbf{B}})$=2, for each column of $\bar{\mathbf{R}}$ we can only determine two parameters, namely three indepedent parameters (degrees of rank deficiency) remain unknown altogether. By choosing $\bar{r}_{11}, \bar{r}_{12}, \bar{r}_{13}$ as independent parameters, in such a way a solution will always be assured, we have

$$\begin{array}{ll} \bar{r}_{21} = (\bar{B}_Y\bar{r}_{11} + \bar{e}_{31})/\bar{B}_X & \bar{r}_{31} = (\bar{B}_Z\bar{r}_{11} - \bar{e}_{21})/\bar{B}_X \\ \bar{r}_{22} = (\bar{B}_Y\bar{r}_{12} + \bar{e}_{32})/\bar{B}_X & \bar{r}_{32} = (\bar{B}_Z\bar{r}_{12} - \bar{e}_{22})/\bar{B}_X \\ \bar{r}_{23} = (\bar{B}_Y\bar{r}_{13} + \bar{e}_{33})/\bar{B}_X & \bar{r}_{33} = (\bar{B}_Z\bar{r}_{13} - \bar{e}_{23})/\bar{B}_X \end{array} \qquad (25)$$

Moreover, we can also get the length ratio, i.e., $\bar{\lambda}_2/\bar{\lambda}_1$, of the two conjugate projective rays. Equalizing the two equations in Eq.(16) and multiplying $\bar{\mathbf{B}}$ in both sides yields

$$\frac{\bar{\lambda}_2}{\bar{\lambda}_1}\bar{\mathbf{E}}\bar{\mathbf{x}}_2 = \bar{\mathbf{B}}\bar{\mathbf{x}}_1 \qquad (26)$$

Its least squares solution is

$$\bar{k} = \frac{\bar{\lambda}_2}{\bar{\lambda}_1} = \frac{(\bar{\mathbf{E}}\bar{\mathbf{x}}_2)^T(\bar{\mathbf{B}}\bar{\mathbf{x}}_1)}{(\bar{\mathbf{E}}\bar{\mathbf{x}}_2)^T(\bar{\mathbf{E}}\bar{\mathbf{x}}_2)} \qquad (27)$$

In summary, with pure image correspondences or the fundamental matrix, we could recover the *two ratios* of the three affine base components, as well as *six relationships* among the nine components of the affine rotation matrix. Moreover, for each image correspondence, we could determine its length ratio of the two conjugate projective rays.

Since there are three degrees of rank deficiency, the affine model can not be fully reconstructed without known object points.

5. OBJECT RECONSTRUCTION

In this section we first present the transformation between object space and the affine model. Then a linear algorithm is designed to perform the exterior orientation of the partially recovered affine model.

It is trivial to show that the transformation between object space and the affine model takes the form (*affine transformation*)

$$\bar{\mathbf{p}} = \mathbf{A}\mathbf{u} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \end{pmatrix} \begin{pmatrix} U \\ V \\ W \\ 1 \end{pmatrix} \qquad (28)$$

where $\mathbf{u}^T = (U \; V \; W \; 1)$ is the homogeneous coordinates of a point in object space, $\mathbf{A}$ is the transfomation matrix specified by twelve independent parameters.

Inserting Eq.(16) into Eq.(28) yields

$$\mathbf{A}\mathbf{u} = \bar{\lambda}_1\bar{\mathbf{x}}_1 \qquad \mathbf{A}\mathbf{u} = \bar{\lambda}_2\bar{\mathbf{R}}\bar{\mathbf{x}}_2 + \bar{\mathbf{b}} \qquad (29)$$

which are elementary to perform the exterior orientation. Eq.(29) has 12 (from matrix $\mathbf{A}$)+3 (from matrix $\bar{\mathbf{R}}$)=15 independent orientation parameters.

To design a linear solution, we eliminate $\bar{\lambda}_1$ in Eq.(29) and get a DLT-type equation

$$\begin{aligned} \bar{x}_1 &= \frac{a_1U + a_2V + a_3W + a_4}{c_1U + c_2V + c_3W + c_4} \\ \bar{y}_1 &= \frac{b_1U + b_2V + b_3W + b_4}{c_1U + c_2V + c_3W + c_4} \end{aligned} \qquad (30)$$

The twelve parameters could be linearly determined up to a scale factor with six given object points appearing on the first image, namely the ratio $a_i' = a_i/c_4, b_i' = b_i/c_4, c_i' = c_i/c_4$ ($c_4' = 1$) are obtained.

Immediately after that, the remainder four parameters are determined linearly with the second set of Eq.(29) by minimum *four* given object points, i.e.,

$$\mathbf{A}'\mathbf{u} = \bar{k}\bar{\lambda}_1'\bar{\mathbf{R}}\bar{\mathbf{x}}_2 + \frac{1}{c_4}\bar{\mathbf{b}} \qquad (31)$$

where matrix $\mathbf{A}'$ is composed of a_i', b_i', c_i' similar to matrix $\mathbf{A}$,

$$\bar{\lambda}_1' = c_1'U + c_2'V + c_3'W + 1 \qquad (32)$$

and $\bar{k}$ is computed from Eq.(27).

The object reconstruction is then finally performed by inversing Eq.(28)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}^{-1} \begin{pmatrix} \bar{X} - a_4 \\ \bar{Y} - b_4 \\ \bar{Z} - c_4 \end{pmatrix} \qquad (33)$$

Comparing to the DLT algorithm which relates every image independently to the object space, our algorithm fully employs the information within a stereo. Therefore, only 15 instead of 2×11 parameters are dependent on known object points. To ensure a linear solution, one image of a stereo may have minimum six known object points, while minimum four of them appear on the other one. It is apparent that in this minimum configuration, the DLT algorithm fails, while a linear solution is available in our algorithm.

Furthermore, this algorithm could be feasibly applied to successive images for object reconstruction if only each of them has minimum four conjugate known points.

6. TESTS AND ANALYSES

In this section we first describe the implementation of our algorithm and then report the test results with an aerial stereo.

The primary step in this algorithm is to determine the fundamental martix $\bar{\mathbf{E}}$. As only the ratios among the entries of $\bar{\mathbf{E}}$ could be determined, we may simply let one of its component equal to one. It is proper to set $\bar{e}_{32} = 1$ as it is approximately equal to $\bar{B}_X \bar{r}_{22}$ which may never be zero. All image correspondences are included to determine $\bar{\mathbf{E}}$. Moreover, since there are only seven degrees of freedom in the fundamental matrix, the condition

$$|\bar{\mathbf{E}}| = 0 \tag{34}$$

may also be included in the solution procedure (Barakat et al,1994).

In exterior orientation, the entries of matrix $\mathbf{A}'$ is determined with six known points by Eq.(30). Other four parameters are then obtained by Eq.(31). After that the object could be fully reconstructed.

We use an aerial stereo to evaluate our algorithm. Its primary parameters and the distribution of the six ground control points (GCPs) are shown in Fig.1 and Tab.1 respectively

Tab.1 Photographic parameters

Flight height:	ca. 2250m
Principle length:	88.94mm
Frame size:	230mm*230mm
Camera:	RC-10
Overlap:	ca. 65%

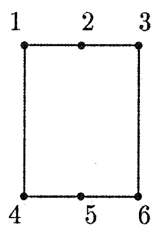

Fig.1 GCPs distribution

Altogether 36 image points as well as their 3D ground coordinates are measured with an analytical plotter. The latter are treated as "best values" to check the validity of our algorithm. Moreover, the DLT algorithm and the traditional collinear algorithm are also implemented. In order to check the efficiency of our algorithm, results under different control configurations and various image deformations are presented respectively in Tab.2 and Tab.3, in which all numerics are relative to the "best values".

In Tab.2, the DLT algorithm and collinear algorithm are implemented with all six conjugate GCPs, while our algorithm is evaluated with different GCPs configuration on the second image and six common GCPs on the first image.

It is no wonder that the collinear algorithm holds the best results (c.f., *item Colli.* in Tab.2). *Item a* and *item DLT* in Tab.2 show our algorithm obtains essentially the same rigorous results as the DLT algorithm when they have the same GCPs configuration. Through *item b to f* of Tab.2 where the DLT algorithm is not applicable, our algorithm behaves completely robust to various GCPs configurations. The small differences, rarely up to maximum decimeters, are within the tolerance of GCPs themselves. Moreover, the most encouraging is in each minimum GCPs configuration we could still reach the same accuracy as the full GCPs configuration - a benefit due to the complete employment of the imformation within a stereo.

Tab.2 Results under different GCPs (in meters)

GCPs config. on the second image	RMSE to best values		
	σ_X	σ_Y	σ_Z
a: 1-2-3-4-5-6	1.936	1.595	1.722
b: 2-3-4-6	1.892	1.455	1.722
c: 1-2-4-5	1.944	1.648	1.723
d: 2-3-5-6	1.887	1.444	1.722
e: 2-4-5-6	1.915	1.539	1.722
f: 1-2-3-5	1.954	1.633	1.722
DLT algorithm	1.954	1.580	1.736
Colli. algorithm	1.376	1.365	1.745

Tab.3 shows the results of our algorithm under different affine image deformations, where s, α and d refer to the scale factor, rotation angle and the disparity of the principal point respectively. In order to testify the validity of our algorithm, simulated affine deformations based on these parameters are added to the original image observations, where the first and second image take different signs of the parameters respectively. The GCPs configuration for this table is *item b* in Tab.2. Since the DLT algorithm presents the same result under different image deformations, it is appended there only in the last row.

Tab.3 Results under image deformations (in meters)

Amount of image deformation parameters	RMSE to best values		
	σ_X	σ_Y	σ_Z
1. no deformation	1.892	1.455	1.722
2. s=1.1,$\alpha = 10^0$,d=10mm	1.926	1.460	1.715
3. s=0.9,$\alpha = 20^0$,d=20mm	1.907	1.466	1.719
4. s=1.3,$\alpha = 30^0$,d=30mm	1.923	1.465	1.748
5. s=0.7,$\alpha = 40^0$,d=40mm	1.911	1.488	1.743
DLT algorithm	1.954	1.580	1.736

It could be clearly seen that our algorithm is practically invariant and robust to different amount of affine image deformations, since only trivial changes (maximum up to centimeters) might occur among them.

7. CONCLUSIONS

Object reconstruction without interior orientation can be linearly accomplished with the aid of the affine model. By making complete employment of a stereo we can determine 2 ratios of the affine base components and 6 relationships among the 9 entries of the affine rotation matrix. The partially reconstructed affine model is oriented to an object frame via determining 15 independent parameters. Unlike the DLT algorithm where minimum 6 known points are required on each image of a stereo, this algorithm allows one image may have only 4 of them. In addition to its completely compatible accuracy with the DLT algorithm, it is robust to control configurations and image deformations.

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