FRACTAL_BASED ANALYSIS AND REALISTIC DISPLAY THE THEORY AND APPLICATION TO DTM

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ABSTRACT

3-Dimensional(3-D) surface representation and reconstruction and surface analysis are important problems in variety of disciplines including geographic data processing ,computer vision,computer graphics,computer-aided design,and so on. The current terrain analysis successively derives contours or slopes and aspects to represent the local details of terrain features.However,it is still the most difficult problems to extract the global features of terrain surface such as the roughness and terrain types which are more and more necessary and important for creating precise construction cost's economic model, land classification and planning,assisting image interpretation of remote sensing etc. After introducing the fractal Brownian model (fBm) as a mathematical model for describing real natural surface,the author presents two parameters H and σ which respectively represents the complexity and global slope of surface.This paper also analyzes its physical meanings and introduces the estimating method . The combination of H and σ can completely represent the global and local features of complex surfaces.

3-D reconstruction is very important for comprehension of terrain structure and facilitating its automatic manipulation and analysis.Because of the irregularity and complexity of natural surface detail,the realistic modelling of earth surface is one of the most challenging problems.Since the current methods always omit the details and over smooth the natural surface,this paper presents an adaptive local stationary fractal interpolation technique to model the real terrain details realistically.Compare with other fractal models,the author's proposal is more complete theoretically and can generate more realistic geometric texture.

The research results achieved in this paper have solved some crucial problems in digital terrain analysis and 3-D terrain reconstruction, which have potential applications in GIS,surveying, civil engineering,and land classification and planning etc.

1. INTRODUCTION

3_D surface representation and reconstruction and surface analysis are important problems in variety of disciplines including geographic data processing, computer vision, computer graphics, computer-aided design, and so on. To achieve its goal, to describe efficiently the shape of a complex surface such as irregular natural terrain surface is a key problem. Usually, a set of simple shape primitives such as spheres, cylinders, cones, and cubes are used to define the shape of man-made object. However, such an approach is not suitable for describing natural shapes, since it requires a huge amount of data and computation to describe them in detail, this is too difficult to accomplish. In recent years,fractal geometry is considered as a hopeful model for this purpose(Yokoya et al.,1989).Specially, the fractal Brownian function fBf (Pentland,1984),which is a mathematical generalization of Brownian motion, is very frequently used to describe such kind of complex surfaces, and in most cases, fractal dimension(or parameter H)is used to characterize the complexity or roughness of the shape. In fact, it is impossible to use this single fractal value to distinguish different surfaces, for example, the fractal dimension of runway at the JFK airport may be approximately the same as that of the Himalayas(Musgrave et al.,1989). Fortunately, we can use another feature of fBf, i.e. the standard deviation σ, to describe a surface more completely. In this paper, the author will develop the physical meaning of σ, and presents a practical method for estimating H and σ of a DEM.

3-D Visualization will be used to enhance the user's perception and understanding of the structure and relations of spatial data, so many applications of GIS in present day need some kind of realistic and accurate display of existing landscapes to visualize the spatial query and spatial analysis. However, in most practical activities, the smallest shapes are always omitted during surveying, that means it is not feasible to obtain all the necessary data for the generation of realistic and accurate display. Moreover, because real terrain surface is usually crumpled and crenulated,the realistic modeling of earth surface is then one of the most challenging problems. Since the current methods always omit the details and over smooth the natural surface, this paper introduces an adaptive local stationary fractal interpolation technique to reconstruct the terrain details realistically and accurately.

2. FRACTAL MODEL AND FRACTAL-BASED ANALYSIS

The author also uses the fractal Brownian function to represent a real terrain surface(DEM).In the following,the definition of fBf and its characteristics will be introduced,especially the physical meanings of the two features H and σ,which will be extracted from real DEM and characterize the DEM,will be discussed.
The fBf f(x) is a real-valued random function such that,for all x and Δx,

$$\Pr\left\{\frac{f(x+\Delta x)-f(x)}{\|\Delta x\|^{H}}<Z\right\}=F(z) \tag{1}$$

where, x is a vector quantities in 2-dimensional Euclidean space,and F(z) is a cumulative distribution function (Pentland,1984).The parameter H($\in$[0,1]) is an indicator of the surface complexity(Polidori,1991),and the relationship between H and fractal dimension of the surface is as follows:

$$D=3-H \tag{2}$$

The smaller H, the larger D and the more irregular the surface.On the contrary,the larger H ,the smaller D and the simpler the surface.
If let f(x,y) denote the DEM,we have the following expression:

$$E\left[\left|f(x+\Delta x,y+\Delta y)-f(x,y)\right|\right] = \frac{2}{\sqrt{2\pi}}\sigma\left(\Delta x^2+\Delta y^2\right)^{H/2} \tag{3}$$

$$\because \quad F(z)\sim N\left(0,\sigma^2\right) \tag{4}$$

$$\therefore \quad E\left(|z|\right)=\frac{2\sigma}{\sqrt{2\pi}} \tag{5}$$

Let C=E(|z|), (6)

and

$$\Delta x^2+\Delta y^2=1 \tag{7}$$

then

$$E\left(\left|f(x+\Delta x,y+\Delta y)-f(x,y)\right|\right)=\frac{2}{\sqrt{2\pi}}\sigma \tag{8}$$

From Eqs. (8) ,we can see that C equals the height up or down through a unit distance, it is obviously the slope. Therefore, C represents the average slope of the whole fractal surface with respect to all the Δx and Δy.

The greater the σ,the larger the slope,or vice versa.This is the physical meaning of the fractal feature.
Because σ represents the total character of the surface relief, it can be used to distinguish different kinds of shapes, which may be have the similar fractal dimensions such as broken level terrain and smooth mountainous terrain.
Based on Eqs.(3) we can derive the following equation:

$$\log E\left(\left|f(x+\Delta x,y+\Delta y)-f(x,y)\right|\right) - H\log\sqrt{\Delta x^2+\Delta y^2}=\log C \tag{9}$$

since both H and C are constant,Eqs.(9) implies that a plot of $E\left(\left|f(x+\Delta x,y+\Delta y)-f(x,y)\right|\right)$ as a function of $\sqrt{\Delta x^2+\Delta y^2}$ on a log-log scale lies on a straight line and its slope is H,and the intersection between the line and function axis indicates the logC.
For practical applications,such as extract H and σ from a real DEM,there are a few problems should be dealed with carefully. For example,because of the errors caused by various sources, a real DEM possesses a largest and a smallest limit of scale($\sqrt{\Delta x^2+\Delta y^2}$) between which the surface has a constant fractal features and can be described by a single fBf.Then it is important to determine the range of the scales.Otherwise,an important fact is that fractal features are not constant over all areas of the real DEM but vary smoothly from position to position(Yokaya et al.,1989).So in order to describe the DEM more completely,it is also necessary to extract some subareas'fractal features which is called adaptive fractal analysis. In our experiences, the dimension of the subarea may be two times the largest scale or more larger.
The fractal features H and σ with respect to global area or local area extracted from real DEM can be used for many purposes such as terrain assessment and classification, DEM quality assessment, remote sensing information classification precise engineering calculations and the "shape preserved" terrain interpolation, etc.

3. ADAPTIVE LOCAL STATIONARY FRACTAL SUBDIVISION MODEL

This is a kind of fast recursive subdivision technique, i.e. the midpoint displacement scheme(Fourier et al.,1991). As illustrated in Figure 1,let 01,02...,and 0n denote the initial grid DEM point (o) with the interval as d0. Each subdivision level include two interpolation steps, the first step is to interpolate the height h11 of the center point(x) from the heights h0i(i is from 1 to 4)of four neighboring grid points. We have the relationship between h11 and h0i as follows:

$$h11=\frac{1}{4}\sum_{i=1}^{4}h0i+\Delta 1 \tag{10}$$

$$\text{where } \Delta 1\sim N(0,V\Delta^2) \tag{11}$$

After this step, the initial square grid is changed to a rhombic grid with the interval of d1as shown in figure 1. It is easy to derive the following equation:

$$d1 = \frac{1}{\sqrt{2}} d0 \qquad (12)$$

The second step is to interpolate the height of the center point(.) from the heights of four neighboring rhombic lattices. In the same manner, we can get expression

$$h12 = \frac{1}{4}\sum_{i=1}^{4} h1i + \Delta 2 \qquad (13)$$

After this step, the rhombic grid is changed into a square grid with the d2 interval as shown in figure 1:

$$d2 = \frac{1}{\sqrt{2}} d1 \qquad (14)$$

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o . o . o . o . o
. x . x . x . x .
o . o . o . o . o
. x . x . x . x .
o . o . o . o . o
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Figure 1, midpoint subdivision scheme

A constrain of such kind of subdivision is not to change the previous computing points in the later subdivision level. So in every interpolation step, the heights of grid are known,and the only problem is to determine the random displacement value(Δ).

Many existing fractal model used for modeling a virtual natural surface is based on the statistical criteria opposed by visual acceptability, so the choice of Δ is only considered with respect to the basic requirements for approximating the fBf, such as

$$\Delta = scale \times 2^{-nH/2} \times gauss \qquad (15)$$

where,scale is displacement factor, and gauss~N(0.1).

For virtual terrain generation, the choices of scale and H may depend on the tests or experiences. However, for real data,the parameters scale and H should be coincide with the σ and H extracted from DEM.

With regard as a "shape preserved" fractal interpolation for real data reported by Yokoya et al.(1989),the Δ model was as follows:

$$\Delta i = di^{H} \times \sigma \times \sqrt{1 - 2^{2H-2}} \times gauss \qquad (16)$$

where the i is the iteration level of interpolation.

In fact, this is a approximation to fBm in 1-dimensional case(voss,1985). Apparently it is not stationary for midpoint displacement in more than 1-dimensional space. A local stationary midpoint displacement model should be as follows(Qing Zhu,1995):

$$\Delta i = di^{H} \times \sigma \times \sqrt{1 - 2^{H-3} - 2^{2H-4}} \times gauss \qquad (17)$$

Compare Eqs.(16) with Eqs.(17), it is easy to see that the Yokoya's model is intended to smooth the real relief. In order to control the different details efficiently in different subareas, it is important to use the results of adaptive analysis. On the other hand, because the midpoint displacement subdivision is usually accomplished in object space, it is difficult to relate the depth of recursion to the last screen coordinates. However,it is possible to relate it to the intervals in world coordinate system.

4. RESULTS OF EXPERIMENT

Figure 2 shows the contour map of a studied area which is a square grid DEM with the interval as 10m and size of 70x70 points.

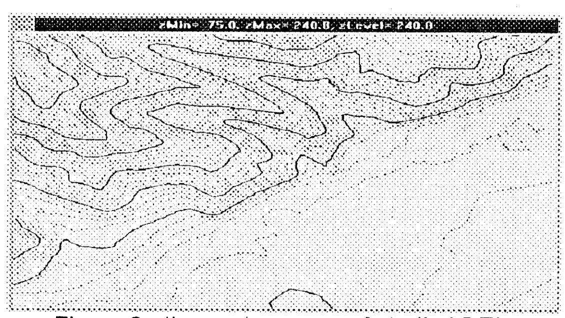

Figure 2: the contour map of studied DEM

Because it is impossible to consider all the Δx and Δy to estimate the H and σ by using Eqs.(9), in order to extract the isotropic fractal features of an area, it is a good choice to compute the average height difference along several different directions. In our experiment, four main directions of east-west, south-north, northeast-southwest and southeast-northwest are chosen to compute the profiles. Meanwhile, the scale limits is ranged from 5m to 60m, and the window size of adaptive analysis is about 110m. The results of adaptive analysis are shown in table 1 and table 2.

local fractal feature: H				
0.992	0.995	0.579	0.629	0.989
0.948	0.991	0.806	0.660	0.999
0.897	0.973	0.986	0.919	0.387
0.888	0.783	0.947	0.999	0.859
0.897	0.987	0.928	0.955	0.935

Table 1: the results(H) of adaptive fractal analysis of studied DEM

local fractal feature: σ				
0.159	0.072	0.146	0.116	0.039
0.387	0.212	0.137	0.098	0.025
0.673	0.456	0.567	0.123	0.180
0.739	0.941	0.569	0.389	0.181
0.674	0.593	0.745	0.428	0.749

Table 2: the results(σ) of adaptive fractal analysis of studied DEM

Figure 3 illustrates the shading display of original DEM, it is easy to see that the surface is very smooth, because there is no details.

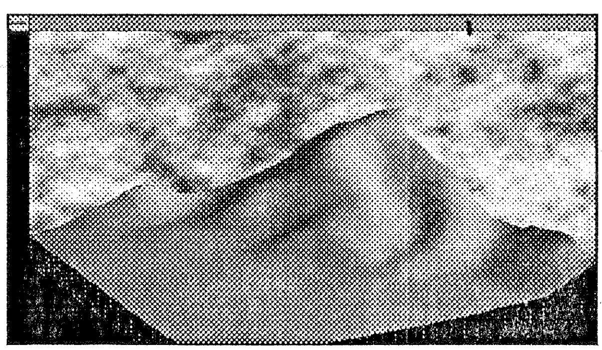

Figure 3. the shading display of original DEM

Based on the adaptive fractal features extracted from DEM, after 1 recursive subdivising by means of the model expressed in Eqs.(17), a new DEM which has 139x139 points with 5m interval is obtained, and its shading display is illustrated in figure 4. It is obvious that such display is more vivid and intuitive, because the micro relief of the real terrain has been reconstructed realistically.

Figure 4: the shading display of the results of adaptive local stationary fractal interpolation of studied DEM

5. CONCLUSION

Generally, the applications of fractal theory to real terrain surfaces (usually the DEMs) are rarely discussed with respect to surface analysis and realistic reconstruction, even though there are some examples in a simple way. In this paper, the author wants to stress that the fractal model fBf is useful for digital terrain analysis and interpolation. For practical applications, both the two features H and σ are important in describing the terrain relief, and an adaptive analysis is also necessary for most cases. For realistic 3D visualization applications, in order to provide a more accurate visual model, the local stationary property and adaptive interpolation is needed in any approximation to fBf model. The adaptive local stationary midpoint displacement tehnique introduced in this paper is a good choice for such purpose.

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