CLOSE RANGE PHOTOGRAMMETRY USING GEOMETRIC PRIMITIVES FOR EFFICIENT CAD MODELLING OF INDUSTRIAL PLANT

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ABSTRACT

This paper outlines the straightforward derivation of several simple mathematical models describing the relationship between some given 3D geometric primitives in the world co-ordinate system and the 2D primitives forming the projected views of those 3D objects.
As a starting point, the co-planarity condition is adapted to relate the parameters of a line in the world co-ordinate system, to the parameters of the corresponding 2D line in an image. An extension of the same co-planarity condition is then employed, to link the parameters describing a cylinder to the parameters of the two lines defining the occluding edges in an image of that cylinder. Basic geometry and simple vector algebra are then used to complete the model for the cylinder, with the third set of equations linking the parameters of a cylinder to the ellipses forming the images of it's end-caps.
These mathematical models have been partially integrated into an existing system designed for the remote measurement of industrial plant. Their potential, to increase the ease with which CAD models of existing plant may be either generated or updated, will be demonstrated.

1. INTRODUCTION

Due to increasingly stringent safety regulations imposed by the relevant governing authorities, and a desire to replace or alter existing equipment, as-built CAD models of industrial plants are necessarily becoming more sought after by those companies operating industrial sites.

In some instances CAD models may already exist from the planning stages, but may not reflect a true picture of the site, since necessary and unforeseen changes had to be made during the plant construction. For older sites red lined design drawings may be the only record of the plant layout.

Conventional or digital terrestrial photogrammetry provides a relatively efficient means to generate or update CAD models, see (Chapman et al., 1992). By providing a record of the plant in question the images may then be used immediately or archived for future use. At any point in time they may be used to determine a few critical dimensions, to model a small local section of the site, or to create a model of the entire site.

An industrial plant is often a highly complex environment consisting of great quantities of equipment separated by narrow corridors for access. Such an environment requires the stationing of cameras at frequent intervals with a separation of the order of several metres, in both the horizontal and vertical sense. At each location a panorama of images will have to be captured. This scenario is essential in order to provide full coverage of the plants equipment and enable that equipment to be modelled to the required accuracy.

With the large numbers of images relating to each site, both the localisation of camera positions and subsequent plant modelling can become very labour intensive. Any automation of this process necessarily provides an increase in productivity. It will be shown how an existing CAD model may be used to effect some level of automation. In addition, images within such an archive may often be found which are crucial to linking together the three dimensional grid of camera locations, but which see few points which could be used as part of a conventional bundle adjustment. This paper will indicate how the use of edges and cylinders may help overcome these problems.

The equipment comprising the majority of many industrial plants may be modelled by a number of basic geometric primitives (cylinders, spherical dishes, boxes, tori). The standard collinearity equations can be used to determine points on the surface or edges of objects viewed in any two or more images, after which primitive fitting routines may be used to determine the descriptive parameters of the viewed object.

An alternative approach is to by-pass the collinearity equations, and to use equations relating the descriptive parameters of the objects in the world co-ordinate system, and the parameters describing the actual or occluding edges in the image of that object. A considerable amount of work has already be done in this area for lines, for example (Schwermann, 1994; Tetsa & Patias, 1994; Tommaselli & Tozzi, 1992), and for some other geometric objects, (Li & Zhou, 1994). Some alternative mathematical models describing these relationships for a 3D line and a cylinder in the world co-ordinate system are given in this paper. These models are relatively simple, two based largely upon the co-planarity condition, and do not involve the introduction of any unwanted parameters. They could be used during the localisation stage, to help determine the camera locations and an initial set of primitives for the CAD model. Their use during the subsequent modelling stage using the localised camera positions is demonstrated.

2. 3D LINE PROJECTION

In a typical industrial scene straight lines may be found as the edges of boxes or as part of markings or signs on the equipment. As mentioned in the introduction the use of lines in photogrammetry is nothing new. A few simple derivations are made here for the projection of a line in object space. These are based upon the equation of the plane passing through both the optical centre of the camera and the 3D line in object space, transformed into the camera co-ordinate system. This will be seen to be nothing more than the familiar co-planarity condition. The intersection of this plane with the focal plane of the camera yields the equation of the 2D projected line image in terms of the orientation parameters of the camera and the parameters of the 3D line, without the introduction of any nuisance parameters.

Two alternative arrangements of this equation can also be seen to be useful in different scenarios.

2.1 Projected Line

Consider a line in object space. This line will define a plane which passes through the optical centre of the camera. If we then rotate this plane into the co-ordinate system defined by the camera (camera space), the observed line in the image must also lie in this plane.

Let the relationship between the camera co-ordinate system and the world (object) co-ordinate system be defined by the collinearity equations thus (this convention will be retained throughout the paper),

$$(\mathbf{x}-\mathbf{x}_0)=\lambda\mathbf{R}(\mathbf{X}-\mathbf{X}_0) \tag{1}$$

where

$\mathbf{X}=(X \quad Y \quad Z)^2$ - point in object space
$\mathbf{X}_0=(X_0 \quad Y_0 \quad Z_0)^2$ - camera optical centre
$\mathbf{x}=(x \quad y \quad z)^2$ - point in camera space
$\mathbf{x}_0=(x_0 \quad y_0 \quad 0)^2$ - camera optical centre
λ -scale parameter
$\mathbf{R}$ -rotation matrix

Let the line in object space be defined by the equation,

$$\mathbf{X}=\mathbf{a}+\gamma\mathbf{l} \tag{2}$$

where

$\mathbf{a}$ -point on the line
$\mathbf{l}$ -line vector
γ -scale parameter

Now the normal to the plane in object space defined by this line, and the optical centre of the camera is given by

$$\mathbf{n}=\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0) \tag{3}$$

We can rotate this to give the form of the same vector in camera space

$$\mathbf{Rn}=\mathbf{R}[\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0)] \tag{4}$$

Therefore the equation of the plane in camera space is given by

$$(\mathbf{x}-\mathbf{x}_0)\bullet[\mathbf{R}[\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0)]]=0 \tag{5}$$

The line determined by the intersection of this plane, (5), with the image plane is found simply by letting

$$z=f \tag{6}$$

where f is the focal distance of the camera.

Thereby yielding an equation of the form

$$Ax+By+C=0 \tag{7}$$

2.2 Observations to a Line

It is also possible to derive a relationship between the actual points observed along the line in the image, and the line in object space.

Again consider the co-planarity requirement on the optical centre of the camera, the line in object space and any of the points along the observed line in the image. We can use the equivalent of the co-planarity condition (the triple scalar product) to derive the following relationship between them,

$$[\mathbf{R}^{T}(\mathbf{x}-\mathbf{x}_0)]\bullet[\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0)]=0 \tag{8}$$

The direct relationship between equations (5) and (8) is obvious. The former relating to camera space, the latter to object space.

2.3 Planes Passing Through a Line

An alternative approach to that described above, is to determine the normal to the plane, in object space, which passes through the line and the optical centre of the camera. The conditions that this plane passes through the line in object space can be shown to be,

$$\mathbf{n}\bullet\mathbf{l}=0 \tag{9}$$

$$\mathbf{n}\bullet(\mathbf{a}-\mathbf{X}_0)=0 \tag{10}$$

where vector, $\mathbf{Rn}$, is the normal to the plane fitted through the optical centre of the camera and the observed points along the line in camera space, and all other variables are as defined in §2.1.

Again the relationship between equations (8) and (10) is obvious. If one considers the identities of the triple scalar product, we see that,

$$\mathbf{n} \equiv \left[\mathbf{R}^{T}(\mathbf{x}-\mathbf{x}_0)\right]\otimes\mathbf{l} \tag{11}$$

Note that equation (10) relates the distance from our point on the line to a plane, and equation (9) constrains us to a particular plane.

3. CYLINDER PROJECTION

Cylinders in the form of pipes and vessels are abundant in the typical industrial plant, and dominate the CAD models of that plant. The equations of the two tangent planes to the cylinder constrained to pass through the optical centre of the camera will be derived. As with the straight line the intersection of these two planes with the focal plane of the camera yields the 2D line equations of the two occluding edges of the observed cylinder.

As before, a similar approach yields the equation linking the planes tangent to a cylinder, to the parameters of that cylinder.

The picture may be completed by considering the equation of the cone whose base is the edge of the circular end-cap of a cylinder in object space, and whose apex is the optical centre of the camera. Once again the intersection of this cone with the focal plane of the camera yields the equation of the ellipse representing the observed image of the circular end-cap.

3.1 Projected Occluding Edges of a Cylinder

In §2 we used the triple scalar product to establish the co-planarity of our 3D line and our image line. Since the triple scalar product may also used to determine the distance between two lines, it can be used in an identical manner to that in §2 to give an equation for the tangent planes to a cylinder.

Let us define our cylinder as follows,

$$\mathbf{a} = \begin{pmatrix} a_x & a_y & a_z \end{pmatrix}^{T} \quad \text{- point on the cylinder axis} \tag{12}$$

$$\mathbf{l} = \begin{pmatrix} l & m & n \end{pmatrix}^{T} \quad \text{- cylinder axis vector} \tag{13}$$

$$r \quad \text{-cylinder radius} \tag{14}$$

The distance between a line tangent to the cylinder, and the cylinder axis will be, r. Therefore we can write,

$$(\mathbf{x}-\mathbf{x}_0)\bullet\left[\mathbf{R}\left[\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0)\right]\right] = \pm r|\mathbf{x}-\mathbf{x}_0| \tag{15}$$

The term on the right hand side is scaled since, $\mathbf{x}$-$\mathbf{x}_0$, is not a unit vector.

We can remove the ambiguity of the sign on the right hand side of equation (15), by squaring both sides. Therefore,

$$\left\{(\mathbf{x}-\mathbf{x}_0)\bullet\left[\mathbf{R}\left[\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0)\right]\right]\right\}^2 = \left\{r|\mathbf{x}-\mathbf{x}_0|\right\}^2 \tag{16}$$

Unfortunately the individual equations of the two planes are not readily extracted from equation (16). As an alternative we can derive the unit normal vectors to the two tangent planes as follows.

Let point, $\mathbf{P}$, be that point closest to $\mathbf{X}_0$ which lies on the line defined by the axis of the cylinder.

Then we have,

$$\mathbf{P} = \mathbf{a} + \left[(\mathbf{X}_0-\mathbf{a})\bullet\mathbf{l}\right]\mathbf{l} \tag{17}$$

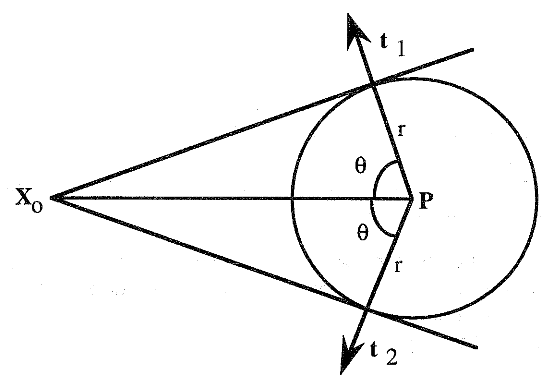

Figure 1 Normal Vectors to a Cylinder

Now by definition, the vector, $(\mathbf{X}_0 - \mathbf{P})$, is perpendicular to the surface of the cylinder. If we consider Figure 1, it can be seen that

$$\mathbf{t} = \mathbf{R}_l \frac{(\mathbf{X}_0-\mathbf{P})}{|\mathbf{X}_0-\mathbf{P}|} \tag{18}$$

where, $\mathbf{R}_l$, is a rotation matrix providing a rotation by angle, $\pm\theta$, about axis, $\mathbf{l}$, see (Bowyer & Woodwark, 1993; Thompson, 1969).

Upon substitution for the terms of the rotation matrix, $\mathbf{R}_l$, we find,

$$\mathbf{t} = \left(\left[\frac{(\mathbf{X}_0-\mathbf{P})}{|\mathbf{X}_0-\mathbf{P}|}\right]\otimes\mathbf{l}\right)\sin(\theta) + \left[\frac{(\mathbf{X}_0-\mathbf{P})}{|\mathbf{X}_0-\mathbf{P}|}\right]\cos(\theta) \tag{19}$$

and therefore upon substitution for the trigonometric functions (refer to Figure 1) we get the equations for the two tangent normal vectors,

$$\mathbf{t} = \frac{(\mathbf{X}_0-\mathbf{P})r \pm \left[(\mathbf{X}_0-\mathbf{a})\otimes\mathbf{l}\right]\sqrt{|\mathbf{X}_0-\mathbf{P}|^2 - r^2}}{|\mathbf{X}_0-\mathbf{P}|^2} \tag{20}$$

These vectors may then be rotated into the camera co-ordinate system, and the equations for the planes in camera space will be given by,

$$(\mathbf{x}-\mathbf{x}_0)\bullet(\mathbf{Rt})=0 \tag{21}$$

with notation as in §2.1.

The intersection of these two planes with the focal plane, give us the equations of the two lines forming the occluding edges of the cylinder in the image, in the form of equation (7).

3.2 Tangent Observations to a Cylinder

By direct analogy with the equations derived in §2 for a 3D line, we obtain the following equation for the observations, in an image, of points along the occluding edge of a cylinder,

$$\left\{\left[\mathbf{R}^{\mathrm{T}}(\mathbf{x}-\mathbf{x}_0)\right]\bullet\left[\mathbf{l}\otimes(\mathbf{a}-\mathbf{X}_0)\right]\right\}^2=\left\{\mathrm{r}\left|\mathbf{x}-\mathbf{x}_0\right|\right\}^2 \tag{22}$$

3.3 Tangent Planes to a Cylinder:

Again by direct analogy, we can derive the equations for planes to be tangent to a cylinder,

$$\mathbf{n}\bullet\mathbf{l}=0 \tag{23}$$

$$\{\mathbf{n}\bullet\mathbf{a}+\mathrm{d}\}^2=\mathrm{r}^2 \tag{24}$$

The relationship between equations (24) and (22) is not so obvious in this case, but vector, **n**, is still given by equation (11), and we note that,

$$\mathbf{n}\bullet\mathbf{X}_0=-\mathrm{d} \tag{25}$$

3.4 Projected Cylinder End-Caps

To determine the equations of the ellipses forming the projection of a cylinder's end-caps into an image, we follow a similar procedure to that in the previous sections. However, in this instance, we determine the equation of the cone whose base is the end-cap of the cylinder, and whose apex is the optical centre of the camera.

Let us retain the definition of a cylinder given in §3.1, but further define the point, **P**, to be the centre of one of the cylinder's end-caps. Thus,

$$\mathbf{P}=\begin{pmatrix}\mathrm{P_X} & \mathrm{P_Y} & \mathrm{P_Z}\end{pmatrix}^{\mathrm{T}} \tag{26}$$

We can therefore define the circular edge of the end-cap to be the intersection of the following two surfaces, a plane and a sphere,

$$(\mathbf{X}-\mathbf{P})\bullet\mathbf{l}=0 \tag{27}$$

$$(\mathbf{X}-\mathbf{P})\bullet(\mathbf{X}-\mathbf{P})-\mathrm{r}^2=0 \tag{28}$$

The cone we are seeking to define is that surface generated by the straight line passing through the point, $\mathbf{X}_0$, which intersects the curve defined by equations (27) and (28). Let us define this straight line as follows,

$$\mathbf{X}=\mathbf{X}_0+\alpha\mathbf{t} \tag{29}$$

where,

$$\mathbf{t}=\begin{pmatrix}\mathrm{t} & \mathrm{u} & \mathrm{v}\end{pmatrix}^{\mathrm{T}}$$

Substitute for, **X**, from equation (29) into both equations (27) and (28), and then eliminate, α, between them. Upon gathering terms we reach the following equation,

$$\begin{aligned}&(\mathbf{t}\bullet\mathbf{t})\left[(\mathbf{X}_0-\mathbf{P})\bullet\mathbf{l}\right]^2\\ &-2\left[\mathbf{t}\bullet(\mathbf{X}_0-\mathbf{P})\right]\left[(\mathbf{X}_0-\mathbf{P})\bullet\mathbf{l}\right](\mathbf{t}\bullet\mathbf{l})\\ &+\left[(\mathbf{X}_0-\mathbf{P})\bullet(\mathbf{X}_0-\mathbf{P})-\mathrm{r}^2\right](\mathbf{t}\bullet\mathbf{l})^2=0\end{aligned} \tag{30}$$

Now, equation (30) is a homogeneous equation which the direction-cosines, **t**, must satisfy for the line to pass through the optical centre of the camera and the edge of the circular end-cap. From (Bell, 1950) we can therefore state that the equation of the cone we are seeking is given by the same homogeneous equation as below,

$$\begin{aligned}&\left[(\mathbf{X}-\mathbf{X}_0)\bullet(\mathbf{X}-\mathbf{X}_0)\right]\left[(\mathbf{X}_0-\mathbf{P})\bullet\mathbf{l}\right]^2\\ &-2\left[(\mathbf{X}-\mathbf{X}_0)\bullet(\mathbf{X}_0-\mathbf{P})\right]\left[(\mathbf{X}_0-\mathbf{P})\bullet\mathbf{l}\right]\left[(\mathbf{X}-\mathbf{X}_0)\bullet\mathbf{l}\right]\\ &+\left[(\mathbf{X}_0-\mathbf{P})\bullet(\mathbf{X}_0-\mathbf{P})-\mathrm{r}^2\right]\left[(\mathbf{X}-\mathbf{X}_0)\bullet\mathbf{l}\right]^2=0\end{aligned} \tag{31}$$

To derive the equation of the ellipse forming the projected view of the cylinder end-cap, is now numerically a straightforward two stage process. To start we transform this cone into our camera space, and then we determine the intersection of this transformed cone with the focal plane of the camera. The algebra of this process is not detailed in this paper, but results in an equation of the form,

$$\mathrm{Ax}^2+\mathrm{By}^2+2\mathrm{Hxy}+2\mathrm{Gx}+2\mathrm{Fy}+\mathrm{C}=0 \tag{32}$$

4. APPLICATION EXAMPLES

The equations derived in the previous sections have all been developed for inclusion in software incorporated into a digital photogrammetric measurement system (HAZMAP), see (Chapman et al., 1992), with the aim of increasing the automation of the CAD modelling process. The work to date has concentrated upon the modelling of pipework and cylindrical vessels, using the equations derived.

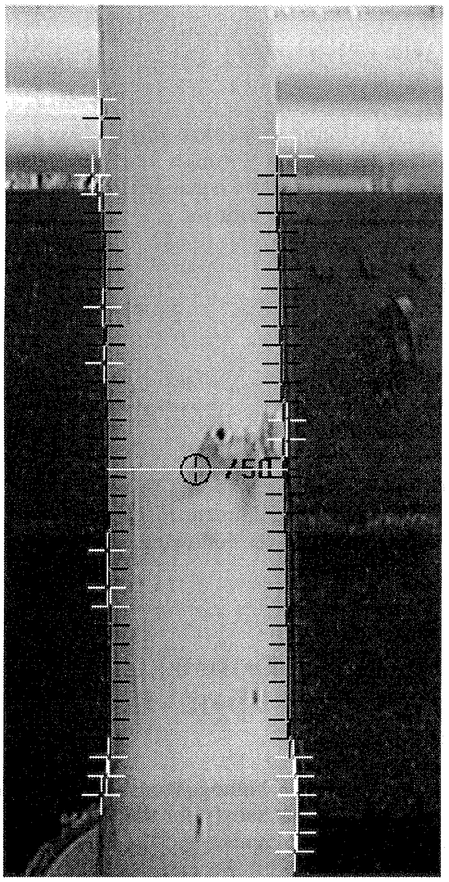

Figure 2. Result of edge detection to locate the occluding edges of a pipe.

The first step in this process was to increase the functionality of a primitive fitting program (geofit), to fit not just 2D and 3D geometric primitives to swarms of points, but cylinders to a selection of tangent planes. Data is supplied to this routine from HAZMAP, where operator directed edge detection filters are applied to the digital images in order to derive points along the occluding edges of a cylinder.

These points are passed to geofit in two stages: the first determines the equations of the tangent planes -Figure 2 shows some results with the light crosses being image points which have been rejected as outliers; the second determines the best fit cylinder to the observed tangent planes. The resultant cylinder is then injected into the HAZMAP images, where it may be manually extended to fit the observed pipe and exported to the CAD model.

The modelling process has now been further automated by using the cylinder projection equations. This is done to either update the parameters of a cylinder from an existing CAD model, or to generate a new cylinder having first roughly positioned an injected solid using several convergent HAZMAP images. The parameters of this cylinder are used, together with the spatial relationships held in the HAZMAP database, to select a group of images in which the pipe in question is likely to appear. The equations of the occluding edges of the cylinder, as viewed in each of these images (Figure 3), is then determined. These equations are used to direct the edge detection routines, and the results are processed as before, to generate the parameters of the cylinder for export to the CAD model.

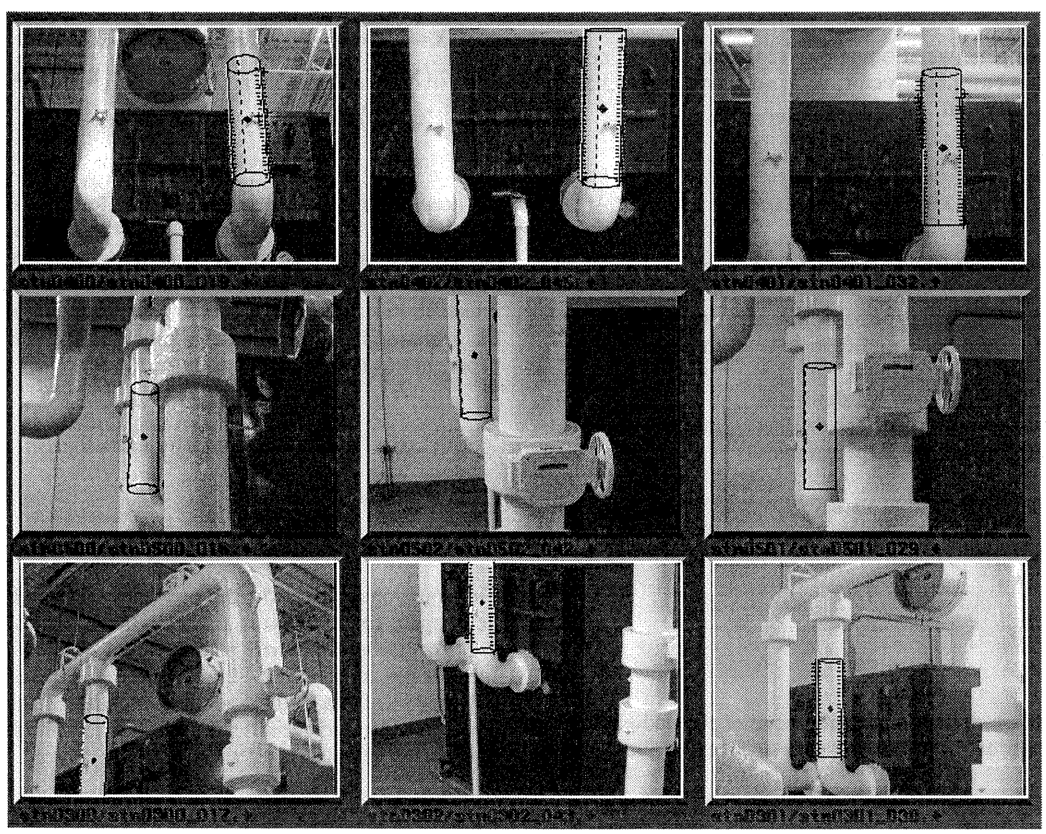

Figure 3. Cylinder projected into nine images of a pipe, five of which are used to update the pipe location.

5. CONCLUSIONS

A derivation of a number of mathematical models has been outlined that will provide useful tools for the modelling of industrial plant. The models defined are not encumbered by the introduction of large numbers of nuisance parameters. The basis of these models, on the coincidence of planes, and cones, in both object and camera space has the second advantage of by-passing the unknown scale parameter, λ, of the collinearity equations, (1).

A typical industrial plant can largely be modelled by using a small number of geometric primitives. The cluttered nature of many industrial sites complicates the generation of detailed CAD models, requires the use of many images, and can therefore prove to be very time consuming. As shown, through the use of mathematical models relating actual objects to the images of them, we can increase the productivity in modelling them. Indeed it can become a semi-automatic process.

The HAZMAP system has already begun to address the automation of the modelling process, building upon the information stored in it's image database and using software based on the equations described. The use of objects and their occluding edges as photogrammetric data would appear to provide great potential. Work is currently underway to extend a similarity transformation program to deal with the parameters of objects, as well as point co-ordinates. A bundle adjustment program, able to deal with both points, and the selection of geometric primitives encountered in a CAD model, is also being contemplated. The two programs could then be used as part of the interior, relative, and absolute orientation processes.

There is currently much talk about "range cameras" replacing close range photogrammetric approaches once their accuracy has been improved. Although photogrammetry will always require two or more images for precise modelling work, the direct extraction of object parameters without recourse to point observations will certainly increase the utility of such systems.

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