ROBUST MIXED PIXEL CLASSIFICATION IN REMOTE SENSING

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ABSTRACT

In this paper we present a novel method for mixed pixel classification where the classification of groups of mixed pixels is achieved by using robust statistics. The method is demonstrated using simulated data and is also applied to real Landsat TM data for which ground data are available.

1 INTRODUCTION

The problem of mixed pixel classification is a major issue in Remote Sensing and Geography and many approaches have been developed to deal with it [Adams et al., 1986, Foody et al., 1993, Lennington et al., 1984, Li et al., 1985, Marsh et al., 1980, Settle et al, 1993, Smith et al.,1990]. In the past we addressed the problem of mixed pixel classification when whole regions of mixed pixels have to be classified by treating the distribution of pixels in each region as a random distribution [Bosdogianni et al., 1994]. In this work we address the same problem but in a way that is applicable to cases that our previous approach is unreliable, namely when outliers are present.

The motivation of our work is to monitor burned forests for a few years after the fire so that the regeneration processes can be evaluated. In particular, we are interested in assessing the danger of desertification conditions ensuing in the site of a burned forest in the Mediterranean region. If the forest does not show signs of recovery a couple of years after the fire, it probably has to be artificially re-forested to prevent further erosion. Quite often, different types of vegetation grow in a burned region. It is usually the case that this new vegetation presents a deterioration of the quality of the flora of the region. The main type of forests that are common in the Mediterranean region consist of aleppo pine (pinus halepensis). Thus, for the purpose of our work, we are interested in assessing the degree of presence of three classes in a region: aleppo pine, bare soil and other vegetation, using Landsat TM images.

There is a major problem, however, when one deals with real data: The data tend to be very noisy and inaccurate. The statistics computed from them tend to be distorted and it is difficult to obtain consistent results. Thus, a more robust way of solving the problem is needed.

2 THE PROPOSED METHOD

In the linear mixing model adopted here, it is assumed that the pixel value in any spectral band is given by the linear combination of the spectral responses of each component within the pixel, so the model can be expressed as:

$$w = ax + by + cz \tag{1}$$

where w is the known spectral reflectance of a mixed pixel, x, y and z are the known spectral reflectances of the three possible cover components within the mixed pixel and a, b and c are the proportions for each component contained in the mixed pixel that have to be estimated.

If we consider again the linear mixing equation mentioned above, we see that it actually is the equation of a hyper-plane in luminance space where we measure one type of luminance along each axis. What we are interested in identifying are the parameters a, b and c for this plane. The method usually used for this purpose is that of least squares fitting. It is well known, however, that the method of least squares is particularly sensitive to outliers. What we propose in this paper is the use of Hough transform to identify the best values of a, b and c. Hough transform is known to be a robust technique which can tolerate large amounts of outliers and still produce good results. In its most commonly used form it is used to identify straight lines in images, but more generally Hough transform can be thought of as a transformation into the parametric domain where we seek to identify sets of real data that indicate the same values of the parameters for the parametric hyper-surface they define.

In our case this hyper-surface is a plane defined in the 3D domain (x, y, z), which is parameterised by different values of w. Thus, our method consists of the definition of an accumulator 3D array defined in the parametric (a, b, c) domain. For each quadrupole (x, y, z, w) we have a different plane defined in the (a, b, c) domain. The surface of this plane intersects various cells of the accumulator array the occupancy number of which is incremented by 1. When all possible quadruples of the data have been considered, the highest peak in the accumulator array defines the best values of the mixing parameters a, b and c. In reality, of course the problem is even simpler than that, because we know that the values of these parameters have to sum up to 1, so we can eliminate the third one in terms of the other two and the linear model of equation (1) now looks like:

$$w - z = (x - z)a + (y - z)b \tag{2}$$

Then our accumulator array is only 2D and it can be sampled with sufficiently high accuracy. The next step in our optimisation procedure is to estimate the bin size in the accumulator space for the two parameters a and b. In our applications we do not really need to know the values of a and b to better than two significant figures accuracy, so the size of our accumulator array will not be larger than 100×100, but generally it will be a lot smaller.

In the above expression x, y and z are the reflectances of pure classes. Due to intraclass variability, however, each of these variables can be thought of as a random variable distributed according to some distribution, which given enough data, can be modeled parameterically. As the reflectances of the pure classes that enter into equation (2) are drawn from these distributions, it is obvious that w is expected to have its own variability, and that even if we have an exact value for it, parameters a and b cannot possibly be estimated with accuracy higher than the accuracy dictated by the intraclass variability of x, y and z.

We can better explain that by realizing that a given reflectance w of a certain mixed pixel can be created by more than one combinations of values x, y and z, all of which could be legitimate reflectances of the corresponding pure classes. Therefore, the values of a and b we find must reflect this uncertainty. Thus, the bin sizes we have to use in the accumulator array have to reflect the uncertainties in x, y and z.

When we solve (2) for a, or b we get:

$$\begin{aligned} a &= \frac{w - z - b(y-z)}{x-z} \\ b &= \frac{w - z - a(x-z)}{y-z} \end{aligned}$$

The standard error for a and b can then be computed by:

$$\begin{aligned} \Delta a &= \left|\frac{\partial a}{\partial x}\right| \Delta x + \left|\frac{\partial a}{\partial y}\right| \Delta y + \left|\frac{\partial a}{\partial z}\right| \Delta z \\ \Delta b &= \left|\frac{\partial b}{\partial x}\right| \Delta x + \left|\frac{\partial b}{\partial y}\right| \Delta y + \left|\frac{\partial b}{\partial z}\right| \Delta z \end{aligned}$$

Thus the standard error for a and b turns out to be:

$$\begin{aligned} \Delta a &= \left|\frac{a}{x-z}\right| \Delta x + \left|\frac{b}{x-z}\right| \Delta y + \left|\frac{b+a-1}{x-z}\right| \Delta z \\ \Delta b &= \left|\frac{a}{y-z}\right| \Delta x + \left|\frac{b}{y-z}\right| \Delta y + \left|\frac{b+a-1}{y-z}\right| \Delta z \end{aligned}$$

Note that a and b are non-negative numbers, while $a+b \leq 1$. If for simplicity we consider that x, y and z vary within the same range, i.e. if we assume for the moment that $\Delta x = \Delta y = \Delta z = \sigma$, we can see that the uncertainty in a and b is:

$$\begin{aligned} \Delta a &= \frac{\sigma}{|x-z|} \\ \Delta b &= \frac{\sigma}{|y-z|} \end{aligned} \qquad (3)$$

The above assumption is clearly an oversimplification as there is no reason to expect that the reflectances of each pure class due to intraclass variability vary within the same range of values. We may say that for a conservative estimate of the error in the parameters we shall take all these ranges of the reflectances of the pure classes to be equal to the largest one.

For multiband data we shall have different ranges of the reflectances of the pure classes for each band. As we want to combine the data from as many bands as possible in order to estimate the mixing proportions, we must have accumulator arrays that are compatible with each other. For this reason, when we create the accumulator array for a certain band, we do not use equations (3) to decide the size of the accumulator cells but we rather use equations (3) to distribute the vote of each possible set of reflectances in a number of fine cells. That is, we set up an accumulator array which has the finest resolution we desire, say 100 bins along each of the two axes. Let us say that from equations (3) we find that in this particular band $\Delta a \simeq n$ and $\Delta b \simeq m$ where n, m are some integer numbers. As we form the accumulator array then, when we find a particular set of (a, b) values, instead of incrementing by 1 the value of the cell that contains that value, we increment by $\frac{1}{nm}$ the values of $n \times m$ cells that form a rectangle around the (a, b) value. This is equivalent to a Hough transform with distributed voting and a "top hat" kernel. At the end we sum up all the accumulator arrays that have been created from the different bands.

3 TESTING THE PROPOSED METHOD WITH SIMULATED DATA

Our method is at first assessed using simulated data to represent the pure and the mixed classes. Three distributions are artificially created to represent the three "pure" classes e.g. classes X, Y and Z , which are assumed normally distributed in each band. Next, a mixture distribution was created, from the three pure distributions with known mixing proportions. Finally, some outliers were added to this mixed distribution.

We represent each distribution by a set of points and we try to estimate the proportions of the classes in the set of mixed pixels. The sets used to represent the "pure" classes are the same for all the experiments, described below and are plotted in Figure 1. The three pure classes were generated using the same covariance matrix. Each set consists of about 30 samples. The proportions used to create the mixed sets are $a = 30\%$, $b = 60\%$ and $c = 10\%$. Only two bands were used for the proportion estimation. The values of Δa and Δb were found to be 5 and 11 for the first band and 10 and 11 for the second respectively.

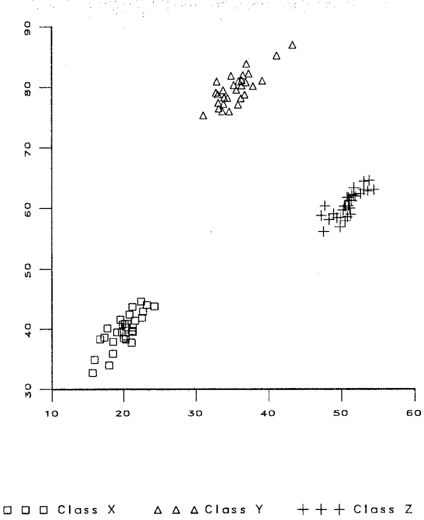

Figure 1: *The sets used to represent the pure classes.*

The parameters that vary in the following experiments are:

the type of outliers, the number of outliers and the position of outliers added to the mixed distribution. We can distinguish two main types of outliers, the ones that follow a certain pattern e.g. a cluster (coherent outliers) and those that are positioned in random places (random outliers). The influence of the outliers depends on their distance from the mixed distribution, so various distances will be examined to demonstrate how they will affect the obtained results. We will examine how many outliers the method can tolerate.

The performance of the Hough transform method will be compared with the solution obtained by the least square error method, where the mean of each distribution is computed and used as a mixed pixel or as an "endmember" in the classical unmixing approach.

3.1 Coherent outliers.

The outliers of this type tend to create clusters in an arbitrary distance from the mixed distribution. Such outliers are shown in Figure 2. Outliers that are placed too close to the distribution do not create much, if any, distortion to the obtained results, while outliers placed too far away are very unlikely to be present in real applications.

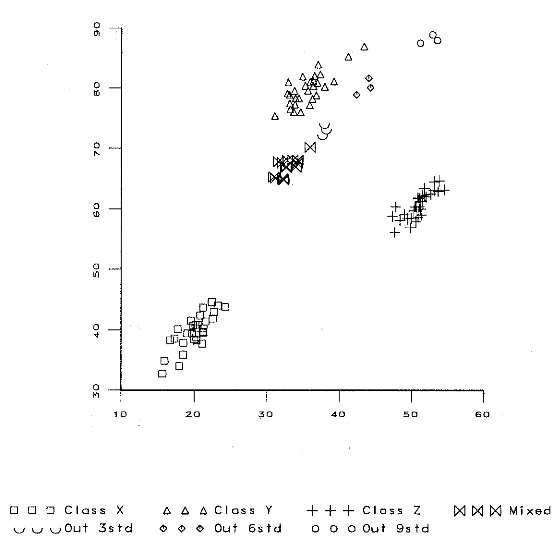

Figure 2: *Outliers placed on a given distance (3, 6 and 9 times the standard deviation) from the mixed set.*

However, distant outliers may be present in the mixture distribution due to the existence of another class in the distribution, that we have no data to describe it. At first we will consider a fixed amount of outliers (e.g. 10% of the given set) and we will place the cluster of those outliers in different distances from the mixed distribution. The unit used to express these distances is the standard deviation of the mixed distribution, which in these experiments is rather tight. The standard deviation of the cluster of outliers will be half the standard deviation of the mixed set. Then we will examine different number of outliers placed at various distances. The results obtained from this experiment are shown in Table 1.

From Table 1 we can see that estimates of the mixing proportions of the Hough method are not affected by the outliers.

Dist	Out	Hough			LSE		
		$a(\%)$	$b(\%)$	$c(\%)$	$a(\%)$	$b(\%)$	$c(\%)$
3	10%	27	60	13	31	59	10
6	-	27	60	13	32	58	10
9	-	27	60	13	34	54	12
3	20%	27	60	13	33	56	11
6	-	27	60	13	23	63	14
9	-	27	60	13	19	64	17
3	30%	27	60	13	24	63	13
6	-	27	60	13	20	64	16
9	-	27	60	13	14	66	20

Table 1: *Effect of number of outliers in the mixed distribution. Mixed set composition $a = 30\%$, $b = 60\%$, $c = 10\%$.*

The LSE method though seems to be affected, as it was expected, and its performance deteriorates as the outliers were placed further away and increase in numbers.

If the cluster of outliers is within the convex hull defined by the "pure" class sets, then these outliers may indicate that the mixed set is not actually homogeneous as assumed so far, but patchy and the outliers in this case represent a patch of another mixed area with different composition. In this case it would be interesting to be able to identify the two mixture compositions. In order to achieve this, we will examine the second significant peak in the Hough space as well.

For the next experiment we created a testing set that is comprised of two mixtures with different compositions. If the mixture of outliers had similar composition to the one of the mixed set, then it would have been very difficult to distinguish between them. That is why for the outliers mixture only, compositions with different dominant class were examined. The original mixed set still had composition $a = 30\%$, $b = 60\%$, $c = 10\%$ as in the previous experiments. The results obtained can be seen in Table 2. For this experiment 1/3 of the test set belongs to the outlier mixture. The second peak in the Hough space is also examined.

Outliers Mix	Hough		LSE
	First Peak	Second Peak	
30-10-60	32-60-8	17-38-45	31-44-25
10-30-60	32-60-8	7-38-55	25-51-24
60-10-30	32-60-8	52-16-32	40-45-15
60-30-10	32-60-8	58-16-26	41-51-8

Table 2: *Test set comprised of two mixtures 2/3 from a mixture with composition $(30\% - 60\% - 10\%)$ and the other 1/3 (outliers) with varied composition.*

As we can see in Table 2 the other peaks in the Hough space may be used to identify sets of coherent outliers. Suppose that we used only one band for the proportion estimation. If the distribution of the mixed set is not very coarse in comparison to the bin size used to discritized the Hough space, then the corresponding points in the accumulator tend to confine in a line. If we use in addition a second band, as is the case here, the points of the mixed set in the second band tend to give a different line that crosses the first line and the crossing point will be the answer. These two lines are not equally important, depending on how separable are the classes in each band and of course on the variance of the mixed set in each band.

When we examine two different mixtures at the same time, then we will generally expect to see four different lines that intersect in four different points. For this case we visualise the Hough space as shown in Figure 3 to check what is going on. If the mixtures have the same number of points then all four intersection points are equally probable to be the right answer and the result will depend on the position of each mixture and the shape of that set. If the two mixtures are not very separable then the corresponding lines will be very close to each other and the corresponding crossing points may be reduced to three, two or even one. In these cases the mixtures are almost identical. If one of the mixtures has fewer points than the other, the lines generated by it will be degraded and the dominant mixture should be responsible for the outcome.

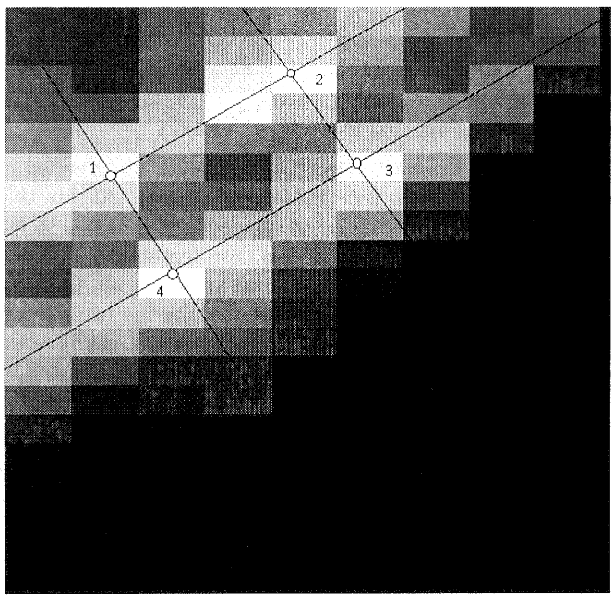

Figure 3: *The mixed set contains points from two mixtures: half of the points belong to a mixture with composition 30-60-10 and the other half to a mixture with composition 30-10-60. The lines were added to show the lines in the Hough space of each mixture in each band. The four numbers indicate the four crossing points and correspond to the following mixtures: 1 → (27-16-47), 2 → (12-49-39), 3 → (27-60-13), 4 → (47-27-26)*

3.2 Random outliers

This type of outliers does not form a coherent set and their distance from the mixed set is randomly chosen in the range between 0 and 12 standard deviations. Such outliers can be seen in Figure 4.

In this experiment, for a certain number of random outliers, a number of mixed sets were generated and tested. The error in proportion estimation was calculated, and finally the average and the standard deviation of the errors (given in brackets) in proportion estimation based on 100 experiments are shown in Table 3. The error in estimation of each proportion (i.e proportion a) was calculated as $error_a = 100 \times \frac{|a-a_T|}{a_T}$, where a is the estimated value for a and a_T is the true value of a.

As we can see in Table 3 the Hough method performs very well and remains remarkably stable throughout the experiment. The LSE method seems to be affected by the outliers and its performance vary depending on the position of the outliers.

Ironically, the more the outliers are and the more uniformly distributed about the mixed distribution they are, the better the LSE method will perform because the mean of the mixture distribution will not be affected by their presence. However, such an improvement in the performance of the classical method is clearly artificial.

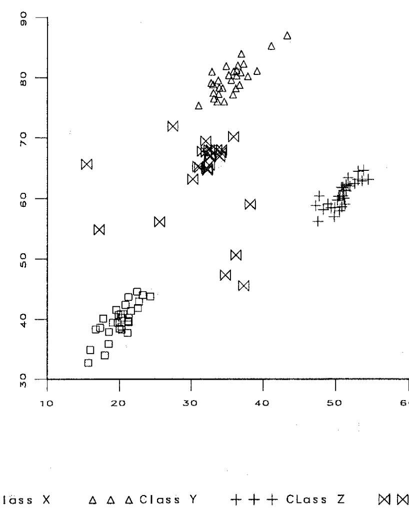

Figure 4: *Outliers placed randomly.*

Out	Hough			LSE		
	a	b	c	a	b	c
10%	10 (0)	0 (0)	30 (0)	5 (4)	3 (3)	21 (16)
20%	10 (0)	0 (0)	30 (0)	8 (4)	4 (4)	30 (20)
30%	10 (0)	0 (0)	30 (0)	7 (5)	6 (4)	41 (24)
40%	10 (0)	0 (0)	30 (0)	9 (7)	5 (4)	33 (27)

Table 3: *Effect of random outliers in the mixed distribution. Mixed set composition $a = 30\%$, $b = 60\%$, $c = 10\%$.*

4 WHAT IF SITES OF PURE CLASSES ARE NOT AVAILABLE?

Ideally, for pure classes we would like to use sets of pixels representative of the pure classes extracted from the remotely sensed image itself. However sometimes, especially if the terrain tends to vary at smaller scales than the size of the test sites, it is difficult to find homogeneous test sites that belong solely to a given pure class.

A solution to this problem is to derive the attributes of the pure classes from test sites for which ground measurements are available [Pech et al., 1986] [Gong et al., 1994]. According to our model we have:

$$w = ax + by + cz \tag{4}$$

We can make use of the Hough transform again to identify the best values for x, y and z, if we consider that equation 4 is an equation of a plane in the 3D space (a, b, c), which is parameterised by different values of w and we are interested in identifying the luminances x, y, z. In this case we have a 3D accumulator array defined in the parametric (x, y, z) domain. We can then use the luminance values w of the training sites, with the estimated (by ground inspection) values of their mixture parameters, to identify values of (x,

y, z) which can be considered to be the means of the corresponding pure classes. Clearly we must perform a different Hough transform for each band. Since x, y and z are luminances, they can take integer values in the range 0 to 255, so we have a 3-D accumulator array $256 \times 256 \times 256$. Instead of searching exhaustively for all possible combinations of x, y and z we can select three samples from three different sites at a time. Thus instead of computing one parametric plane for a given w, we solve a system of three equations similar to equation (4) for the values of x, y and z. Then only the corresponding cell in the accumulator array is incremented.

After the training part of the classification is concluded, it follows the testing part when we are going to use these derived values for x, y and z to classify any mixed set. Therefore, we need to know the intraclass variability in x, y and z in order to calculate the bin size for a and b. The intraclass variability can be estimated by examining the steepness of the peak in the Hough space. Let us assume that for a derived triplet (x_0, y_0, z_0) we have a peak value f_{x_0,y_0,z_0} in the Hough space. Then at the point (σ_x, y_0, z_0) we have:

$$\frac{f_{(x_0,y_0,z_0)}}{f_{(\sigma_x,y_0,z_0)}} = e^{-\frac{1}{2}}$$

Form the above we can derive σ_x and in a similar way σ_y and σ_z. These standard deviations are likely to be different. In such a case we use the biggest one to calculate the bin size for a and b from equations (3).

5 APPLICATION TO REAL DATA

Since the simulation results showed that our model performs well, we then tested it with some real data. The aim was to decide on the type of vegetation in an area located close to Athens, the capital of Greece in the province of Attica. Four test areas (Penteli, Pateras, Varnavas and Lavrio) have been selected because there were forest fires in each of these areas within the last ten years. The training site data used in this work were collected by the Institute of Mediterranean Forest Ecosystem - National Agricultural Research Foundation (NARF) of Greece for evaluating the risk of desertification.

The primary vegetation in this study area is composed of conifers, mainly *Aleppo pine* and a variety of shrub species. So the vegetation cover is categorised in three main classes: *bare soil, aleppo pine* and *other vegetation*. For training we used different sites for which ground data were available. We have no regions solely composed of one class so we derive the spectral characteristics of the real pure classes from sites for which we know their composition, using the Hough transform.

39 training sites were used for this purpose. The algorithm was then tested on 14 sites which had not been used for training and for which the composition was known from ground inspection as well. Two criteria were used to evaluate the obtained results. According to the first criterion a classification result is considered a "hit" if the dominant class is identified correctly, otherwise we have a "miss". The second criterion is more strict, a classification result is considered a "hit" if the dominant class is identified correctly with accuracy $\pm 15\%$. The results of the Hough transform were compared with the results obtained by the Least Square Errors method.

In Tables 4 and 5 S stands for soil, AP for Aleppo Pine, V for Other vegetation. The numbers are percentages of coverage by the corresponding class. Under the heading "LSE" we give the results obtained by using the Least Square Errors. Under the heading "Hough" we give the results obtained by using the Hough transform. All the results presented in the following tables were calculated using only two bands, the ones that give the maximum discrimination for the three "pure" classes (in our case bands 3 and 5).

With the LSE method according to the first criterion 24 sites out of the 39 were classified correctly and according to the second criterion 18 sites out of the 39 were classified correctly. Using the Hough model, according to the first criterion 25 sites were classified correctly, while according to the second criterion we had 19 "hits". The detailed results obtained for these sites are shown in Table 4.

At the second stage of the evaluation of our method, we tested our model using 14 sites that they had not been used for the derivation of the pure classes. According to the first and second criteria the LSE method classified correctly 5 sites. The Hough model had 8 "hits" in accordance to the first criterion and 6 "hits" according to the second criterion. The detailed results obtained for these sites are shown in Table 5.

6 DISCUSSION AND CONCLUSIONS

The simulation results showed that the Hough transformed method can tolerate large amount of outliers and still retain an acceptable performance. So the Hough method seems more attractive in terms of performance, but the price that one has to pay is the increase in computational complexity.

The problem of exponential explosion of the number of quadruples one can use has also to be considered. Indeed, if each one of the distributions that represents a pure class and the mixed distribution consists of 30 points, we have to consider 30^4 possible combinations which is about 10^6 combinations. This is really the limiting factor in our approach: It is not feasible to use it for large data sets or for many "pure" classes. However, the method is not really meant for large data sets as it is only introduced for the case that the datasets are not sufficiently large to allow reliable statistics to be extracted from them.

The problem of multiple peaks in the Hough space when more than the mixtures are present can basicly be tackled by considering pairs of equations and solving for a single (a, b) and incrementing only one cell in the accumulator array at a time. The problem of combinatorial explosion is dealt with in [Kälviäinen et al., 1996]

7 ACKNOWLEDGEMENTS

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Name	Ground			LSE			Hough		
	S	AP	V	S	AP	V	S	AP	V
L-1	20	30	50	17	40	43	17	33	50
L-2	40	25	35	23	33	44	30	30	40
L-3	15	30	55	11	39	50	8	39	53
L-4	35	20	45	35	23	42	30	30	40
L-5	55	15	30	50	16	34	53	3	44
L-6	10	70	20	0	54	46	1	54	45
PD1-1	15	55	30	5	51	44	5	48	47
PD1-2	40	20	40	45	23	32	57	9	34
PD1-3	25	25	50	44	24	32	37	24	39
PD1-4	15	45	40	34	17	49	37	17	46
PD1-5	10	30	60	19	30	51	8	30	62
PD1-6	20	60	20	12	46	42	10	54	36
PD1-7	20	35	45	22	41	37	17	48	35
PD2-1	0	75	25	0	59	41	1	39	60
PD2-2	5	55	40	3	45	52	1	36	63
PD2-3	0	70	30	0	53	47	5	48	47
PD2-4	0	60	40	5	55	40	3	54	43
B-1	15	0	85	17	35	48	5	36	59
B-2	10	0	90	0	53	47	8	56	36
B-3	30	0	70	0	46	54	1	35	64
B-4	5	35	60	0	61	39	1	54	45
B-5	0	0	100	0	66	34	1	9	90
B-6	5	0	95	1	51	48	17	56	27
P-1	30	30	40	31	23	46	28	21	51
P-2	55	10	35	36	23	41	34	13	53
P-3	30	30	40	46	2	52	46	1	53
P-4	20	35	45	24	18	58	24	17	59
P-5	0	65	35	12	36	52	10	41	49
P-6	55	10	35	33	8	59	31	9	60
P-7	20	40	40	19	16	65	16	25	59
P-8	5	30	65	5	32	63	5	29	66
P-9	10	15	75	18 1	2	70	18	9	73
P-10	10	15	75	11	26	63	11	21	68
P-11	30	10	70	30	7	63	30	9	61
P-12	20	15	65	18	18	64	18	21	61
P-13	15	60	25	35	7	58	34	5	61
P-14	45	25	30	43	0	57	38	13	49
P-15	10	60	30	18	36	46	11	33	56
P-16	20	40	40	19	28	53	13	29	58

Table 4: Comparison of the mixed model results with the ground data for the 39 sites used to derive the pure classes.

Name	Ground			LSE			Hough		
	S	AP	V	S	AP	V	S	AP	V
BAR-T1	5	0	95	0	57	43	1	24	75
BAR-T2	0	0	100	0	56	44	1	0	99
LAV-T1	15	65	20	52	19	29	50	18	32
LAV-T2	10	50	40	41	28	31	37	36	27
PEN-T1	35	35	30	24	47	29	33	45	22
PEN-T2	30	50	20	10	44	46	5	48	47
PEN-T3	60	10	25	50	13	37	39	39	22
PEN-T4	5	55	40	30	48	52	1	45	54
PAT-T1	25	15	60	14	19	67	14	21	65
PAT-T2	15	10	75	17	16	67	17	15	68
PAT-T3	30	0	70	10	26	64	10	25	65
PAT-T4	10	75	15	15	24	61	21	13	66
PAT-T5	70	20	10	34	10	56	32	17	51
PAT-T6	70	0	30	43	0	57	41	5	54

Table 5: Comparison of the mixed model results with the ground data for the 14 testing sites.

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