# STRAIGHT LINES IN LINEAR ARRAY SCANNER IMAGERY 

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#### Abstract

Increased use of digital imagery has facilitated the opportunity to use features, in addition to points, in photogrammetric applications. Straight lines are often present in object space, and prior research has focused on incorporating straightline constraints into the bundle adjustment for frame imagery. In this research, we introduce a straight-line constraint in the bundle adjustment for linear array scanners. A linear array scanner scene is acquired using different geometry than frame cameras. A scene is composed of a sequence of images, each of which may be slightly shifted against each other due to slight changes in the system's trajectory. As a result, straight lines in object space do not appear as straight lines in image space. The proposed bundle adjustment constraint accommodates this distortion. The underlying principal in this constraint is that the vector from the perspective center to a scene point on a straight-line feature lies on the plane defined by the perspective center and the two object points defining the straight line. This constraint utilizes the perspective transformation model for point features in linear array scanner imagery. The proposed technique makes use of straight-line features in object space, and aids in the recovery of the exterior orientation parameters as well as adding to the geometric strength of the bundle adjustment. This constraint has been embedded in a bundle adjustment software application, developed at the Ohio State University, that models frame and linear array scanner imagery


## 1. INTRODUCTION

Most photogrammetric applications are based on the use of distinct points. These points are often obtained through measurements in an analog or digital environment. Recently, more attention has been drawn to linear features. There are several motivations for the utilization of linear features in photogrammetry:

- Points are not as useful as linear features when it comes to higher level tasks such as object recognition.
- Automation of the map making process is one of the major tasks in digital photogrammetry and cartography. It is easier to automatically extract linear features from the imagery rather than distinct points (Kubik, 1988).
- Images of a man made environment are rich with linear features.

Habib (1999) discusses the various options for straight-line representation in photogrammetric applications. The primary representation considerations are uniqueness and singularities. In this research, straight lines in object space are represented by using two points along the line. In this way, the line segment is well localized in the object space. This representation is attractive because such coordinates can be easily introduced or obtained from a GIS database.

Previous research on linear features in photogrammetry has focused primarily on frame imagery. Straight-line constraints can be incorporated into the bundle adjustment by utilizing the fact that the perspective transformation of a straight line is also a straight line. Mikhail and Weerawong (1994) proposed a straight-line constraint that constrains a unit vector defining the object space line, the vector from the perspective center to a point on the object line, and the vector from the perspective center to the image point to be coplanar. In their approach, the object line is represented as an infinite line segment. Habib (1999) proposed a straight-line constraint, which forces the plane defined by the image line to be coplanar with the plane defined by the perspective center and two object points defining the line. In this approach the object line is defined by two points, localizing it in space. These techniques rely on the use of frame imagery.

## 2. BACKGROUND

### 2.1. Linear array scanner imagery

With the increased use of digital photogrammetry, there are motivations to use digital cameras to facilitate the automation of photogrammetric tasks. To attain the same resolution as aerial frame photography, $20 \mathrm{~K} \times 20 \mathrm{~K} 2$-D digital array sensors would be necessary. However, at this time, the highest resolution commercially available is 4 K X 4 K . Linear array scanners simulate 2-D images by using a 1-D array of sensors operating with an open shutter on a moving platform.

Linear array scanners have one or more 1-D arrays of CCD sensors in the image plane. The electromagnetic energy incident upon these sensors at a given time will constitute an image. Movement of the platform and/or rotation of the lens configuration will enable successive coverage of different areas on the ground. A scene is defined as a sequence of linear array scanner images. Depending on the number of 1D-arrays, the scanning direction and the relation of the sensor with the flight direction, one differentiates between three-line, pushbroom and panoramic linear array scanners (see figures 1 and 2).


Figure 1: Perspective geometry of frame cameras (a) and pushbroom scanners (b).


Figure 2: Perspective geometry of three-line scanners (a) and panoramic linear array scanners (b).
Previous research at The Ohio State University has focused on modeling the perspective geometry of linear array scanners (Habib and Beshah, 1998). The collinearity model used for frame imagery has been modified in such a way that it is also valid for pushbroom, three-line and panoramic linear array scanners. This model accommodates the most general scenario for linear array scanners- panoramic linear array scanners. Collinearity models for the other scanner
types are easily derived from this model, and implemented by fixing some of the parameters of the panoramic model. The perspective transformation model for point features in linear array scanners is as follows.
$x_{a}^{t}=x_{P}+\operatorname{imc}(t)-c \frac{N_{x}}{D}$
$y_{a}^{t}=y_{P}-c \frac{N_{y}}{D}$
where
$N_{x}=r_{11}^{t}\left(X_{A}-X_{0}^{t}\right)+r_{21}^{t}\left(Y_{A}-Y_{0}^{t}\right)+r_{31}^{t}\left(Z_{A}-Z_{0}^{t}\right)$
$N_{y}=r_{12}^{t}\left(X_{A}-X_{0}^{t}\right)+r_{22}^{t}\left(Y_{A}-Y_{0}^{t}\right)+r_{32}^{t}\left(Z_{A}-Z_{0}^{t}\right)$
$D=r_{13}^{t}\left(X_{A}-X_{0}^{t}\right)+r_{23}^{t}\left(Y_{A}-Y_{0}^{t}\right)+r_{33}^{t}\left(Z_{A}-Z_{0}^{t}\right)$
$x_{a}^{t}, y_{a}^{t}: \quad$ image coordinate measurement of point a at time t
$X_{A}, Y_{A}, Z_{A}$ : object coordinates of point A
$x_{P}, y_{P}, c:$ calibrated principal point position and principal distance of the camera
$r_{11}^{t}, r_{12}^{t} \ldots r_{33}^{t}: \quad$ time dependent elements of the combined rotation matrices $R^{T}\left(\alpha_{t}\right) R^{T}\left(\omega_{t}, \phi_{t}, \kappa_{t}\right)$
$\alpha_{t}$ : scan angle for panoramic sensor at time $t$
$\operatorname{imc}(t)$ : image motion compensation at time t
$X_{0}^{t}, Y_{0}^{t}, Z_{0}^{t}$ : time dependent object coordinates of the perspective center

### 2.2. Straight-line constraint for frame imagery

Habib (1999) introduced a straight-line constraint for the bundle adjustment with frame imagery. This function constrains the perspective center of any image, the image line and any object point that belongs to that line to be coplanar. The constraint is implemented in the following way.

In one image, two points are measured to define the object line (see Figure 3). The image and object points are related by the collinearity equations. These points do not need to be visible in subsequent images. In each subsequent image $i$, the image line is represented by polar coordinates (Eq. 2).

$$
\begin{equation*}
x \cdot \cos \theta_{i}+y \cdot \sin \theta_{i}=\rho_{i} \tag{2}
\end{equation*}
$$

Which can also be written as:

$$
\left[\begin{array}{c}
x  \tag{3}\\
y \\
-c
\end{array}\right] \cdot\left[\begin{array}{c}
c \cdot \cos \theta_{i} \\
c \cdot \sin \theta_{i} \\
\rho_{i}
\end{array}\right]=0
$$

The two points defining the object line can be projected into the image space of the $i^{\text {th }}$ image by the collinearity equations (Eq. 4) and must satisfy the equation of the line (Eq. 3).

$$
\left[\begin{array}{c}
x_{a}  \tag{4}\\
y_{a} \\
-c
\end{array}\right]=\lambda \cdot R_{i}^{T} \cdot\left[\begin{array}{c}
X_{A}-X_{0 i} \\
Y_{A}-Y_{0 i} \\
Z_{A}-Z_{0 i}
\end{array}\right]
$$

Finally, the constraint relating the $i^{\text {th }}$ image line with any of the points along the object line can be defined as follows.

$$
R_{i}^{T} \cdot\left[\begin{array}{c}
X_{A}-X_{0 i}  \tag{5}\\
Y_{A}-Y_{0 i} \\
Z_{A}-Z_{0 i}
\end{array}\right] \cdot\left[\begin{array}{c}
c \cdot \cos \theta \\
c \cdot \sin \theta \\
\rho
\end{array}\right]=0
$$

Here, $\lambda$ is ignored because $\lambda \neq 0$.
In summary, there are two steps to implementing this constraint. First, in one image, the straight line is defined by measuring two points (A\&B), generating four collinearity equations (Figure 3). Next, in each subsequent overlapping image, that line is defined in terms of polar coordinates- adding two independent constraint equations for A and B . It is important to note that in each overlapping image, no more than two independent constraint equations can be generated for a particular line.


Figure 3: Geometry of the straight-line constraint for frame imagery.

## 3. STRAIGHT LINES IN LINEAR ARRAY SCANNER IMAGERY

The underlying principal in the straight-line constraint for linear array scanner imagery is that the vector from the perspective center to a scene point on a straight-line feature lies on the plane defined by the perspective center and the two object points defining the straight line. In object space, straight lines are to be represented by two points along the line. The corresponding line in image space will be represented as a sequence of points that may not lie on a straight line.


Figure 4: Perspective geometry and straight lines in linear array scanner imagery.
As shown in Figure 4, two points in one scene are used to define a straight line in object space. The object point, the corresponding image point and the perspective center of the exposure station lie on a single light ray. Therefore, the generalized collinearity equations (Eq. 1) can be applied to each of the two points defining the line.

For each image, the vector from the perspective center to any image point along the line can be defined with respect to the ground coordinate system as:
$\vec{V}_{1}=R\left(\omega^{t}, \phi^{t}, \kappa^{t}\right)\left[\begin{array}{c}x-x_{p} \\ y-y_{p} \\ -c\end{array}\right]$

The multiplication with the rotation matrix R , transforms the vector into the ground coordinate system.
The vector from the perspective center to the first object point along the line is defined as:
$\vec{V}_{2}=\left[\begin{array}{c}X_{1}-X_{o}^{t} \\ Y_{1}-Y_{o}^{t} \\ Z_{1}-Z_{o}^{t}\end{array}\right]$
The vector from the perspective center to the second object point along the line is defined as:
$\vec{V}_{3}=\left[\begin{array}{c}X_{2}-X_{o}^{t} \\ Y_{2}-Y_{o}^{t} \\ Z_{2}-Z_{o}^{t}\end{array}\right]$
Both vectors (Eq. 7 and 8 ) are defined with respect to the ground coordinate system.
As illustrated in Figure 5, the vectors from the perspective center to each scene point along the line should lie on the plane that is defined by the perspective center and the two object points defining the straight line. This condition can be formulated as:

$$
\begin{equation*}
\left(\vec{V}_{2} \times \vec{V}_{3}\right) \bullet \vec{V}_{1}=0 \tag{9}
\end{equation*}
$$



Figure 5: Plane defined by two object points and the perspective center.
This constraint for straight lines in aerial triangulation is a function of the following parameters.

$$
\begin{equation*}
f\left(X_{1}, Y_{1}, Z_{1}, X_{2}, Y_{2}, Z_{2}, X_{O}^{t}, Y_{O}^{t}, Z_{O}^{t}, \omega^{t}, \phi^{t}, \kappa^{t}, x, y\right)=0 \tag{10}
\end{equation*}
$$

The unknown parameters are the EOPs of the images and the ground coordinates of the two points (A\&B) defining the line. These two points are measured in one scene. In each scene that the line is visible, the constraint (Eq. 9) can be applied to all points measured along the line, regardless of whether or not the defining points are visible. Because the EOPs change from one point to the next, the number of independent constraints will equal the number of measured points along the line. The ground coordinates of the supplemental points along the straight line are not determined during the bundle adjustment. These measured points only contribute to increase the geometric strength of the adjustment.

The constraint can also be applied to frame imagery. In this case, only two independent constraints can be generated per image, due to the fact that there is only one set of EOPs associated with a single image.

## 4. EXPERIMENTS/RESULTS

The proposed straight-line constraint in aerial triangulation has been tested for frame, three-line and panoramic linear array scanner imagery, using the MSAT (Multi Sensor Aerial Triangulation) software, developed at the Ohio State University. The adjustments were performed with and without the straight-line constraint, using tie points in both cases. The RMS values between the adjusted and geodetic ground coordinates of the tie points are shown in Tables 24. An overview of the test data configurations is shown in Table 1.

|  | Number of images | Number of tie lines |
| :--- | :--- | :--- |
| Frame imagery | 12 | 8 |
| Three-line scanners | 6 | 2 |
| Panoramic linear array scanners | 4 | 7 |

Table 1: Configuration for bundle adjustment with points and straight lines
For frame imagery, the test field in Figure 6 was used to form a block of twelve images. Eight lines were used in the overlapping area. Aerial triangulation was performed with and without the linear feature constraint. The results are shown in Table 2.


Figure 6: Test field with equally spaced signalized targets
Six scenes with two tie lines were used to test the algorithm for three-line scanners. The six scenes were captured in two flight lines ( 3 per flight line) with almost $100 \%$ overlap, and $60 \%$ sidelap. GPS observations at the exposure stations were utilized in the adjustment. The results are shown in Table 3.

The configuration for panoramic linear array scanner imagery is shown in Figure 7. Again, GPS observations at the exposure stations were utilized in the adjustment. The results are shown in Table 4.


Image 2


Image 1


Image 3


Image 4

Figure 7: Layout of four panoramic linear array scanner scenes with seven tie lines.

The following processing settings were applied for all of the simulated data sets:

- The threshold $\sigma$ for terminating the iteration process was set to $1.0 \mathrm{E}-7$.
- The approximations of the tie points were displaced from their actual positions by approximate values of 100 m (horizontally) and 10 m (vertically) to verify their adjustment.

| FRAME IMAGERY | Without linear features | With linear features |
| :--- | :--- | :--- |
| Rms x [m] | 0.024 | 0.040 |
| Rms y [m] | 0.031 | 0.034 |
| Rms z [m] | 0.106 | 0.071 |

Table 2: Rms-values of the bundle adjustment of frame imagery

| THREE-LINE SCANNERS | Without linear features | With linear features |
| :--- | :--- | :--- |
| Rms x $[\mathrm{m}]$ | 2.639 | 1.688 |
| Rms y $[\mathrm{m}]$ | 1.292 | 0.825 |
| Rms z $[\mathrm{m}]$ | 0.550 | 0.559 |

Table 3: Rms-values of the bundle adjustment of three-line scanners

| PANORAMIC LINEAR ARRAY | Without linear features | With linear features |
| :--- | :--- | :--- |
| SCANNERS |  |  |
| Rms x [m] | 0.522 | 0.376 |
| Rms y [m] | 0.797 | 0.300 |
| Rms z [m] | 0.935 | 0.553 |

Table 4: Rms-values of bundle adjustment of panoramic linear array scanners
As illustrated in Table 1, the straight-line constraint did not improve the RMS values in the case of frame imagery. However, in the case of linear array scanner imagery, there was a noticeable improvement. With linear array scanner imagery, many EOPs are involved in the adjustment, and the added equations constrain the solution, aiding in the determination of the EOPs. Therefore, it is advantageous to evaluate as many image points along the straight line as possible.

## 5. CONCLUSIONS/RECOMMENDATIONS

A new approach was developed to handle object space straight lines in linear array scanner imagery. Because of the nature of line cameras, straight lines in object space may not appear as straight lines in the captured scene. In the proposed constraint, object space lines are defined by two points, which must be identified in at least one scene.

Using this technique with linear array scanner imagery, one independent constraint equation is added to the adjustment for each image point evaluated. The added constraint equations aid in the recovery of the many exterior orientation parameters associated with linear array scanner imagery. It is therefore advantageous to evaluate as many image points along the straight line as possible. This constraint is also valid in the case of frame imagery. Also, the incorporation of this constraint into existing bundle adjustment software is straightforward.

Testing with simulated data proved the superiority of this technique over aerial triangulation with disconnected points.

## Recommendations for future work:

- More testing with real data. We would like to use an available GIS database and data collected by terrestrial mobile mapping systems, e.g. road networks, to provide control for aerial triangulation.
- Automatic extraction and matching of linear features from imagery.


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