NEW APPROACH TO SOLVING MATCHING PROBLEMS IN PHOTOGRAMMETRY

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ABSTRACT

Typically in photogrammetric applications, an adjustment is used to estimate the parameters of a mathematical model relating corresponding entities of two data sets. In the proposed technique, a statistical approach is used to robustly estimate the parameters of the mathematical model when the correspondence is unknown. This method uses only the conjugate entities between the data sets, filtering out "outliers", for a robust solution. As a result, the correspondence between entities is implicitly determined. This approach is general enough to be applied to several photogrammetric applications including single photo resection, automatic relative orientation and surface matching. The use of this technique offers new capabilities in these applications. In surface matching, change detection is facilitated. In single photo resection, extracted linear features can be used to estimate the exterior orientation parameters. In automatic relative orientation, the conjugate points are determined using the true mathematical relationship between conjugate points, yielding a more robust solution than traditional image matching techniques.

1. INTRODUCTION

Many automated photogrammetric applications rely on matching techniques to establish the correspondence of entities in two data sets. Once the correspondence is determined, the conjugate entities are usually used as observations in a mathematical adjustment. The robustness of the matching technique greatly influences the accuracy of the parameter estimation.

Well-established matching techniques such as area based, relational, and feature based matching use a similarity or cost measure to determine correspondence. In area-based matching, the similarity measure may be correlation; in least squares matching, the radiometric and geometric differences between two templates are minimized. In relational matching, features are decomposed into primitives, and cost functions are tailored to these primitives (Schenk, 2000). In feature-based matching, the generalized Hough transform (for example) can be used to measure the similarity of extracted features. In all of these matching methods, the true mathematical model of the photogrammetric application is not considered; rather, matches are based on an assumed similarity measure.

In many applications, we must estimate the parameters of a mathematical model relating entities in two data sets. To do this without manually identifying corresponding entities is crucial in the automation of photogrammetric processes. Specifically, we will look at three applications in which the parameters of a mathematical model relating two data sets are estimated, while simultaneously determining the corresponding entities of two data sets. This is accomplished by applying the modified Hough transform for robust parameter estimation.

2. BACKGROUND

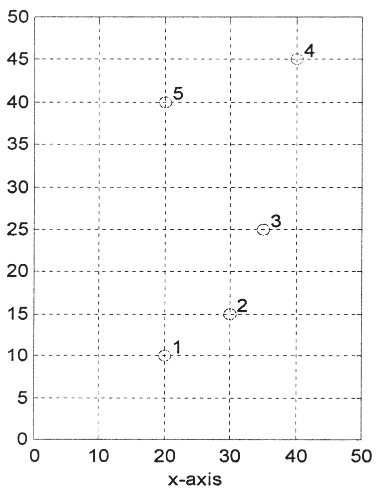
2.1. Hough Transform Techniques

Hough(1962) introduced a method of determining parameters by way of a voting scheme. The basic principal of his approach was to switch the roles of parameters and spatial variables. To illustrate this approach, consider the following example. Suppose we want to detect points that lie on a circle of known radius, r. A circle can be defined by:

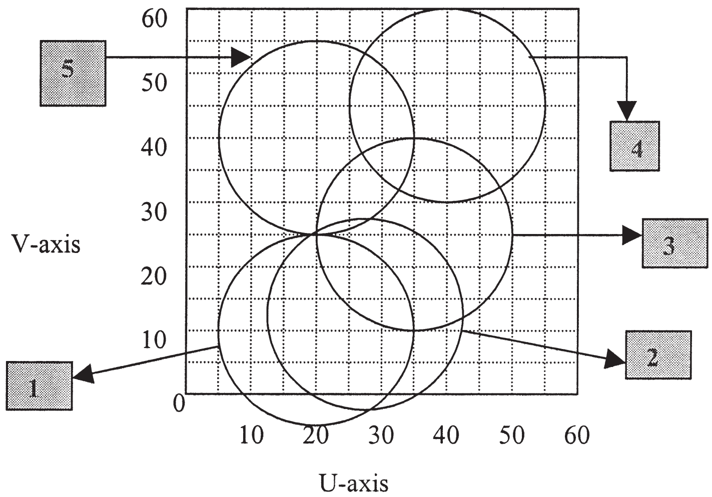
$$(x-u)^2 + (y-v)^2 - r^2 = 0 \tag{1}$$

With x,y the spatial variables and u,v the parameters (center) of the circle in the spatial domain. Now, let us introduce the parameter space, represented by the coordinate system u,v. A point x_i,y_i in the spatial domain corresponds to a circle in the parameter space centered at x_i,y_i. For every point in the spatial domain, there exists a circle in the parameter space, and vise versa. The intersection of circles in the parameter space identifies centers of circles in the spatial domain. The number of intersecting circles in the parameter space is directly related to the number of points that lie on this circle (see Figure 1).

The Hough method is usually implemented by an accumulator array, which is an n-dimensional, discrete space, where n is equal to the number of parameters. In our example with circles of known radii, the parameter space is two-dimensional. Each circle is discretely represented in the parameter space. To keep track of all the circles, we simply increment all of the cells that are turned on by every circle. After having processed all points in this fashion, we analyze the accumulator array and determine the number of hits per cell. Every hit casts one vote for a point lying on that particular circle. The cell with the maximum number of hits, m, yields the center of the circle in the spatial domain that passes through m points. Similarly, other peaks in the accumulator array identify additional circle centers. Tracking the points contributing to the peak in the accumulator array identifies the points lying on the circle of known radius.



(a)



(b)

Figure 1: Illustration of finding circles through data points. A point in the spatial domain (a) corresponds with a circle in the parameter space (b) and vice versa. The intersection of circles in the parameter space determines the center of the sought circles in the spatial domain. The intersection of four circles at u = 20, v = 25 identifies points 1,2,3 and 5 as belonging to a circle whose center c in the spatial domain is c = (20,25).

The notion of a parameter space and voting scheme have led to variations of the Hough transform, which are suited to particular applications.

2.2. The Modified Hough Transform for Robust Parameter Estimation

The modified Hough transform is used to estimate the parameters of a mathematical model relating entities of two data sets. In this approach, we assume no knowledge of correspondence and do not require complete conjugacy of entities. As a result of the parameter estimation, the correspondence is implicitly determined. The method is outlined as follows.

First, a hypothesis is generated that an entity in the first data set corresponds to an entity in the second data set. The relation between entities of the data sets is expressed by a mathematical function, and by using the hypothesized match, this function yields an observation equation. The parameters of the mathematical relation can be estimated simultaneously or sequentially, depending on the number of hypothesized matches simultaneously considered. All possible entity matches are evaluated, and the results (parameter estimations) are represented in an accumulator array. The accumulator array will exhibit a peak at the location of the correct parameter solution. By tracking the matched entities that contributed to the peak, the correspondence is determined.

The number of parameters being simultaneously solved determines the dimension of the accumulator array. In order to solve n parameters simultaneously, one must utilize the number of hypothesized entity matches needed to generate the required n observations. However, this approach is not practical. To evaluate all permutations of entities quickly leads to combinatorial explosion. In addition, the memory requirements of an n dimensional accumulator array create another problem.

An alternative is to solve for each parameter sequentially in an iterative manner, updating the approximations at each step. Consequentially, the accumulator array becomes one-dimensional and the memory problem disappears. Also, if there are i elements in data set 1 and j elements in data set 2, the total number of evaluated entity matches becomes $i{\cdot}j$, reducing the computational complexity of the problem. After each iteration, the approximations are updated and the cell size of the accumulator arrays can be reduced. In this manner, the parameters can be estimated with high accuracy. This approach is dependent on the seperability of the parameters. Highly non-linear transformations have a slower convergence rate and would require more iterations.

The basic steps to implementing the modified Hough transform for robust parameter estimation are as follows:

1. A mathematical model is established that relates the entities of two data sets (see Figure 2). The relation between the data sets can be described as a function of its parameters: $f(x_1, x_2, \ldots x_n)$.
2. An accumulator array is formed for the parameters. The accumulator array is a discrete tessellation of the range of expected parameter solutions. The number of parameters to be simultaneously solved will designate the dimension of the accumulator array.
3. Approximations are made for parameters which are not yet to be determined. The cell size of the accumulator array depends on the quality of the initial approximations; poor approximations will require larger cell sizes.
4. Every possible match between entities of the two data sets is evaluated, incrementing the accumulator array at the location of each solution.
5. After all possible matches have been evaluated, the maximum peak in the accumulator array will indicate the correct solution of the parameter(s).
6. After each parameter is determined, the approximations are updated.
7. For the next iteration, decrease the cell size of the accumulator array, and repeat steps 2-6.

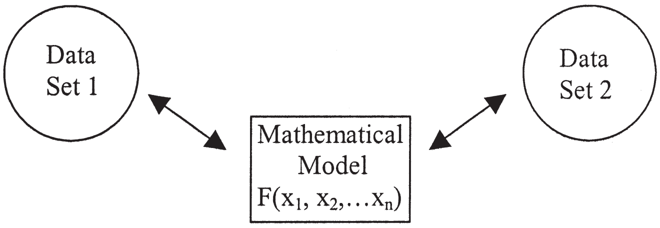


Figure 2: Mathematical model relating two data sets.

3. APPLICATIONS

3.1. Single photo resection with extracted linear features

The objective of single photo resection is to determine the six exterior orientation parameters associated with an image. The relationship between conjugate image and object points is given through the collinearity equations (Eq. 2).

$$\begin{bmatrix} x_i - x_p \\ y_i - y_p \\ -c \end{bmatrix} = \lambda \; R^T(\omega, \phi, \kappa) \begin{bmatrix} X_I - X_O \\ Y_I - Y_O \\ Z_I - Z_O \end{bmatrix} ; \qquad \lambda \ldots \text{ scale} \tag{2}$$

where $(x_i, \; y_i)$ are image coordinates, and $X_I, \; Y_I, \; Z_I$ are the corresponding object coordinates of point I.

In digital imagery extracted edges contain a large number of image points, and are often associated with characteristic features in object space. In this application, we estimate the EOPs and establish the correspondence between extracted image points and 3-D points along linear features in object space. The 3-D points may be acquired from a GIS database, mobile mapping system, or as a result of digitization of existing maps or manuscripts. This technique does not require full conjugacy of matching entities.

The mathematical model relating the two data sets is the collinearity model. For parameter estimation using the modified Hough transform, we use an iterative, sequential approach to avoid computational complexity and memory problems.

An attempted match between one image point with one object point is evaluated. One match forms two collinearity equations, allowing for the solution of two EOPs at a time. Initial approximations of the remaining four parameters are required. A two-dimensional accumulator array is created for each pair of parameters. The cell size is chosen based on the quality of the initial approximations. All possible pairings of image and object points are evaluated. For each attempted match, the parameters are computed and the accumulator array is updated. A peak in the accumulator array indicates the parameter values to be used as approximations for the next iteration. To refine the solution, the cell size of the accumulator array is decreased after each iteration.

A possible sequence could be, for example, to solve for (X_0, Y_0), (Z_0), (ω, ϕ) and (κ) sequentially. It should be apparent that the iterative technique is a more realistic approach. If we had attempted to solve all six parameters simultaneously, we would have to evaluate every permutation of three object points matched with three image points, leading to combinatorial explosion.

By tracking the indices of the matched entities that contributed to the peak in the accumulator array, the correspondence between image points and object points is determined.

3.2. Automatic Relative Orientation

Relative orientation establishes the relative relationship between the images of a stereopair in such a way that it is similar to the relative relationship of the two images at the moment of exposure. The perspective centers associated with the images of a stereopair and a single point on the ground define the epipolar plane (Figure 3). Epipolar lines are defined by the intersection of the epipolar plane with the focal planes of the images. The coplanarity constraint confines conjugate points in a stereopair to lie on the epipolar plane. The relative orientation parameters can be determined using this constraint. The coplanarity condition is utilized by constraining the normal to the epipolar plane to be perpendicular to the image base vector. This condition is defined as follows:

$$(\vec{p}_r \times \vec{p}_l) \bullet \vec{b} = 0 \qquad (3)$$

where p_l is the vector from the perspective center of the left image to image point a_l, p_r is the vector from the perspective center of the right image to the conjugate image point a_r, and b is the baseline vector between the two perspective centers of the stereopair.

Each pair of conjugate points contributes one coplanarity constraint equation according to Eq. (3). At least five conjugate light rays must intersect in object space to define a stereo model. These points are selected at the Von Gruber locations. During relative orientation, we solve only five out of the twelve EOPs. The remaining seven EOPs are later determined through absolute orientation which removes the datum deficiency for that particular stereo model.

The proposed technique is well suited for use with extracted image points associated with linear features. In this case, the two data sets are the image points from the left and right images, respectively. The mathematical model is the coplanarity condition.

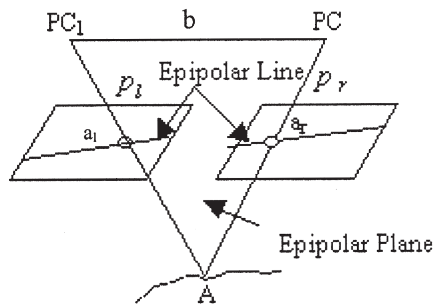


Figure 3: Epipolar geometry of a stereopair.

In empirical relative orientation, the parameters are determined sequentially by eliminating the y-parallax at the Von Gruber locations in a particular sequence. This is due to the fact that some parameters are more sensitive to measurements at certain locations on the image. In the iterative approach, we solve the parameters in a similar way- at one Von Gruber location at a time, according to this established sequence.

In the iterative parameter estimation approach, each match of one point in the left image with one point in the right image is evaluated. Such a pairing forms one coplanarity equation, allowing for the solution of one ROP at a time. Initial approximations of the remaining five ROPs are required. A one-dimensional accumulator array is utilized for each parameter. Every possible match of conjugate points is evaluated in the determination of each parameter. The accumulator array is updated for each solution. The parameters are calculated in the same sequence as in empirical relative orientation. Each parameter solution is used to update the parameter approximations for the next iteration. To refine the solution, the cell size of the accumulator array is decreased after each iteration. By tracking the indices of the points that contribute to the peak in the accumulator array, the matching of image points is implicitly solved. For more details, the reader can refer to Habib (1999).

3.3. Surface Matching

Given are two sets of points that describe the same surface. Let $S_1 = \{p_1, p_2, \ldots p_n\}$ be the first set, and let $S_2 = \{q_1, q_2, \ldots q_n\}$ the second set, $n \neq m$. Suppose the points are randomly distributed (no point to point correspondence). The problem is to describe how well the two data sets agree describing the same surface.

The approach (Habib and Schenk 1999) is to compute the difference between the two sets along surface normals and at the original point location, to avoid interpolation. Suppose now that local surface patches for S_1 are generated. The simplest approach would be to create a TIN model. Let surface patch SP in S_1 be defined by the three points p_a, p_b, p_c and let q_i be a point in the second set. Then, Eq. (4) is the shortest distance between a point in the second data set and the corresponding surface patch, as illustrated in Figure 4. If we want to impose the condition that q_i lies on the surface patch (coplanarity condition), then we have D=0 in Eq. (5).

$$d_i = \frac{D}{\sqrt{D_1^2 + D_2^2 + D_3^2}} \tag{4}$$

$$D = \begin{vmatrix} xq_i & yq_i & zq_i & 1 \\ xp_a & yp_a & zp_a & 1 \\ xp_b & yp_b & zp_b & 1 \\ xp_c & yp_c & zp_c & 1 \end{vmatrix} \tag{5}$$

We generalize the surface comparison problem by allowing that the two data sets S_1 and S_2 are in different reference systems. We assume that there is a known functional relationship between the two sets but with unknown parameters. An example would be the knowledge that the two sets are related by a 3-D similarity transformation (Eq. 6); and the seven parameters should be determined without identical points. This situation exists when merging two data sets that may be affected by uncompensated systematic errors. Calibrating laser systems is a classical case; here, the surface defined by laser points from an uncalibrated system is compared with a known surface (control surface). One should note that any functional relationship between the two data sets can be used in our proposed matching scheme. We have:

$$q_i' = s \cdot R \cdot q_i + t \tag{6}$$

It would be possible to solve the parameters in an adjustment procedure, using Eq. (5) as the target function. Such a procedure would determine the transformation parameters that minimize the distance d_i according to the least squares principal. However, to compute the distance d_i, a correspondence between the points q_i and the surface patches must be established. This matching problem is no longer trivial because the two sets are in different reference systems. The proposed technique will solve the problem in parameter space.

To determine the seven parameters of the similarity transformation, seven equations of the type of Eq. (5) are required. Since there is no redundancy, we introduce the condition d_i=0. That is, Eq. (5) becomes the coplanarity condition. Theoretically, we can select seven points q in set S_2 and match them with all possible surface patches of S_1. For every such combination, a set of seven equations is found and solved. The discretized solution yields those cell addresses of the 7-D accumulator array that need to be incremented. Once all possible combinations are explored, we select again seven points q and repeat the procedure. The correct solution will emerge as a peak in the accumulator array.

The iterative approach will avoid the combinatorial explosion and memory problems associated with solving the parameters simultaneously. By tracking the matches that have contributed

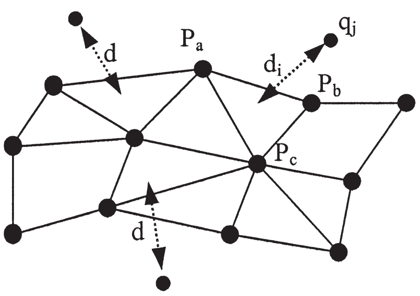


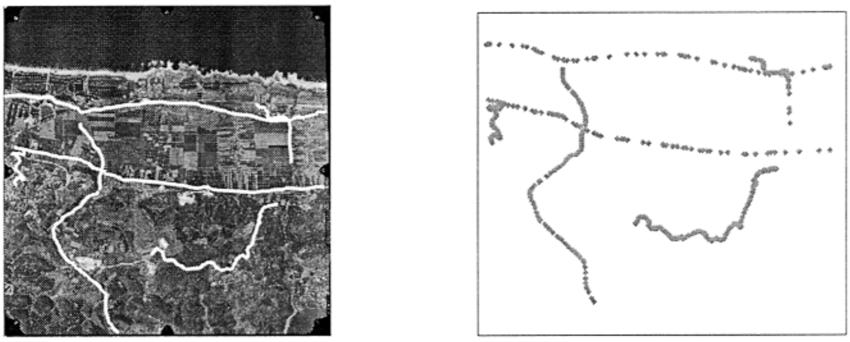
Figure 4: Comparison of two data sets that describe the same surface. The points of one set are shown in relation to surface patches of another set.

to the peak in the accumulator array, the correspondence of points to surface patches is determined.

4. EXPERIMENTS/RESULTS

4.1. Single Photo Resection

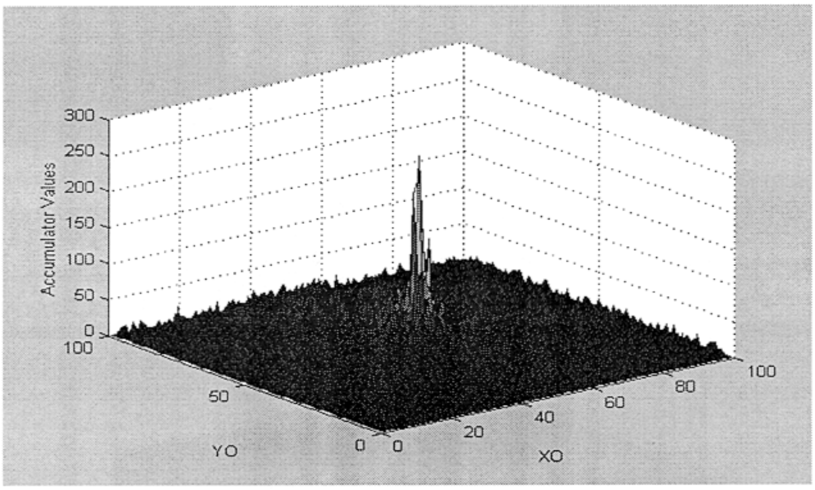
To test the single photo resection using the modified, iterated Hough transform, 3-D points were measured along linear features in a stereopair with known exterior orientation (Figure 5a). Next, an edge operator was used to detect edge pixels in one of the images. The proposed technique was applied to estimate the exterior orientation parameters and determine the matches between image and object points. Figure 5b shows the resulting image points that matched with 3-D object points.



(a) (b)

Figure 5: Aerial image with digitized linear features superimposed (a) and the matches resulting from the algorithm (b).

In implementation, two parameters are solved at a time. Figure 6 shows the accumulator array used to solve for X_0 and Y_0 for the final iteration. The distinct peak indicates a robust solution. Figure 6b shows the quality of the estimated parameters and the statistics (Figure 6c) of the experiment are also included.



(a)

Estimated Variance Component:0.236032

Xo	3102.811398m	±4.143860m
Yo	9876.683891m	±3.169007m
Zo	6520.202995m	±0.943299m
Omega	-0.019162deg	±84.490722sec
Phi	1.740318deg	±121.592419sec
Kappa	86.314970deg	±29.663966sec

(b)

Total number of image points	839
Total number of object points	579
Total number of matched points	471
Percentage of matched points	81.347

(c)

Figure 6: The accumulator array for Xo and Yo (a), parameter estimation for single photo resection (b) and statistics of experiment (c).

4.2. Automatic relative orientation

The modified Hough transform technique for automatic relative orientation was applied to a stereopair that was acquired digitally. An edge detection operator was applied to the images, and the edge pixels were used as input to the relative orientation algorithm. Figure 7 shows a pair of overlapping Von Gruber subimages with the resulting matched points superimposed. Table 1 compares the results of the proposed parameter estimation technique to the traditional manual method.

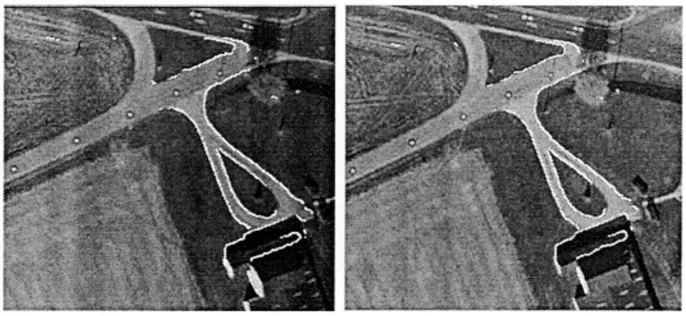


Figure 7: Von Gruber sub-images with matched points superimposed.

Estimated Variance Component: 0.50118			
			St. Dev.
phi1 =	-2.640998	deg	0.000021
kapa1 =	12.807528	deg	0.000078
omega2 =	-1.104487	deg	0.000018
phi2 =	1.89773	deg	0.000062
kapa2 =	12.236085	deg	0.000076

(a)

Estimated Variance Component = 0.710853			
			St. Dev.
phi1=	-2.501762	deg	0.004837
kapa1 =	13.138726	deg	0.023809
omega2 =	-0.99605	deg	0.005525
phi2 =	1.552138	deg	0.00521
kapa2 =	12.630122	deg	0.023612

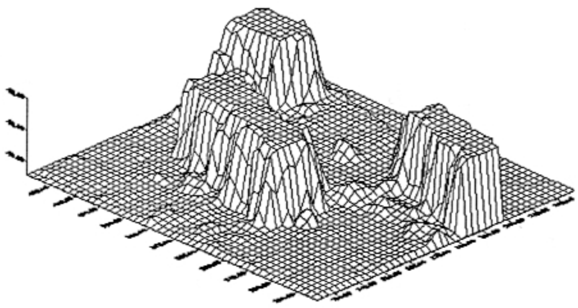
(b)

Table 1: The estimated relative orientation parameters from the proposed method (a) and from manual relative orientation (b).

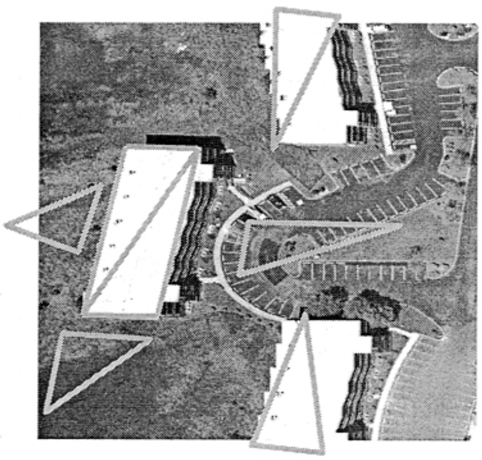
4.3. Surface matching

The surface matching technique was tested with laser data sets provided by NASA Wallops, and aerial imagery of Ocean City, MD flown by NGS (National Geodetic Survey). The data provide an excellent opportunity to test the proposed procedure on a real world problem: determining how well a laser surface agrees with a photogrammetrically derived surface.

A TIN was created using photogrammetrically derived points (Figure 8b). The laser data points (Figure 8a) were compared to the surface of this TIN. The parameters found by our approach indicate very good agreement between the data sets. A more meaningful check is to perform the transformation with the parameters found, followed by computing the distance of the transformed points to the surface patches S_1. The average distance of 0.03m between the laser and stereo surface confirms the accuracy potential of both methods. Table 2 shows the results of the parameter estimation.



(a)



(b)

Figure 8: Mesh plot of laser data points (a) and the TIN points superimposed on the image of the test site (b).

XT =	0.3996	±3.439537	(m)
YT =	-0.8882	± 1.176025	(m)
ZT =	0.028	± 0.097754	(m)
S =	1.0016	± 0.001382	
Omega =	0.0135	± 0.000181	(deg)
Phi =	-0.0363	± 0.000332	(deg)
Kappa =	0.0977	± 0.008508	(deg)

Table 2: Parameter estimation from surface matching algorithm.

In the area examined, a group of laser points did not correspond to any surface patch. Such points are labeled as blunders. A closer examination reveals that the laser points are on top of a tree. The planar surface patch, determined by photogrammetry, is on the ground. Hence, the distance from the laser points to the surface patch exceeded the tolerance.

5. CONCLUSIONS/RECOMMENDATIONS

A new approach for matching entities in two data sets has been developed. The matching is recovered as a by-product of solving the parameters involved in the mathematical model relating conjugate entities in the two data sets. This approach can filter out blunders (non-corresponding entities) and prevent them from contributing towards parameter estimation- making this a robust technique. The suggested algorithm has been used in three different applications. First, object and image linear features have been used to estimate the exterior orientation parameters of the involved imagery. Second, using extracted edges in a stereopair, we estimated the relative orientation parameters together with solving the correspondence problem. Finally, this approach was used to estimate the transformation parameters between two surfaces (for change detection and calibration purposes). The results from real data proved the feasibility and robustness of the proposed algorithm.

Future work will concentrate on the following:

- Efficient handling of the accumulator array,
- Applying a coarse-to-fine strategy in a way that is similar to scale space processing of digital imagery,
- Finding more useful applications for this algorithm.

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