# SPATIAL RELATION OPERATIONS BASED ON V9I MODEL 

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#### Abstract

Operations and description for spatial relations are important for GIS. At the present, investigations about them are mainly based on 9 -intersection (9I) model for spatial relations. However, due to the infinity of the complements of objects in the model, it is difficult to directly capture the complements of objects. Therefore, even though most of spatial relations could be defined in the SDTS with aggregates of two or more 9 -intersection primitives, it is difficult to calculate these primitives from spatial data, and further operations on spatial relations are also difficult. A new Voronoibased 9-intersection (V9I) model has been developed by some researchers, which redefines the exterior of spatial object with its Voronoi region instead of its complement. Since the exterior of the model is not infinite any more, it is possible to directly calculate and operate them, thereafter, it also becomes rather easy and realizable to manipulate spatial relations in a GIS system with this model. Based on the above idea, with the help of this model, this paper sets up a mechanism for manipulating and acquiring spatial relations from spatial data stored in a GIS database, and then defines and describes corresponding operations. Finally, this paper gives an experimental tool with Visual C++ on Windows platform.


## 1 INTROUDCTION

Spatial relations generally refer to relations with spatial characters between spatial objects such as distance relations, direction and topological relations[Egenhofer, 1991]. Operations and description for spatial relations are crucial to spatial query and spatial analysis in a GIS system[Chen et.al, 1999]. The 9-intersection model is such one most popular model for describing spatial relations[Egenhofer,1991]. In the 9 -intersecion model, the spatial relations between two simple spatial objects $A$ and $B$ are defined in terms of the intersections of A's boundary $(\partial A)$, interior $\left(A^{0}\right)$ and exterior $\left(A^{-}\right)$with A's boundary $(\partial B)$, interior $\left(B^{0}\right)$ and exterior $\left(B^{-}\right)$. However, the exterior of a object is defined as its complement in this model, so it is quite difficult to directly capture the exterior and compute the intersections with the exterior since the complement are infinite. Therefore, even though most of spatial relations could be defined in the SDTS (Spatial Data Transform Standard) with aggregates of two or more 9-intersection primitives, it is difficult to calculate these primitives from spatial data, and further operations on spatial relations are also difficult [Mark, 1995, Chen et.al.,1997].

A new Voronoi-based 9-intersection (briefly V9I) model has been developed by Chen et.al (1997), which redefines the exterior of spatial object with its Voronoi region instead of its complement. It can be represented by the following $3 \times 3$ matrix,

$$
V 9 I(\mathrm{~A}, \mathrm{~B})=\left[\begin{array}{lll}
\partial A \cap \partial B & \partial A \cap B^{o} & \partial A \cap B^{V} \\
A^{O} \cap \partial B & A^{o} \cap B^{o} & A^{o} \cap B^{V} \\
A^{V} \cap \partial B & A^{V} \cap B^{o} & A^{V} \cap B^{V}
\end{array}\right]
$$

Where, $\partial \mathrm{A}, \partial \mathrm{B}$ are respectively object $\mathrm{A}, \mathrm{B} \mathrm{s}$ boundary $\mathrm{A}^{\circ}, \mathrm{B}^{\circ}$ are respectively $\mathrm{A}, \mathrm{B} \mathrm{s}$ interior $\mathrm{A}^{\mathrm{V}}, \mathrm{B}^{\mathrm{V}}$ are respectively A, B s Voronoi region. Voronoi regions can be got from the Voronoi diagram of spatial objects. Voronoi diagram is a subdivision of geographical space according to the nearest-neighbor rule: each object is associated with the region of the geographical space closest to it, while each region associated with a object is called its Voronoi region. Through Voronoi regions of spatial objects, we can easily and quickly know which objects are adjacent to a given object.

In this new model V9I, since the exterior of the model is not infinite any more, it is possible to directly calculate and operate them, hence, it also becomes rather easy and realizable to manipulate spatial relations in a GIS system with this model. In addition to the investigation of the model itself, we have also done a series of investigation about Voronoi diagram, for instance, the method of generating Voronoi diagram and Voronoi solution to adjacent problems in GIS[Li et.al, 1998, 1999, Zhao et.al.,1999].

Based on these investigations, this paper mainly discusses how to calculate and determine spatial relations with the new model. Section 2 sets up a mechanism for manipulating and acquiring spatial relations from spatial data stored in a GIS database, and then, Section 3 defines and describes corresponding operations. In order to examine the above basic principle, Section 4 gives an experimental tool with Visual C++ on Windows platform and the further investigations.

## 2 SPATIAL RELATIONS MAPPING BASED ON V9I MODEL

Spatial relations can be regarded as a kind of high level semantic information which can be understood by endusers, while corresponding spatial data stored in databases can be regarded as a kind of low level information because the information can not often be easy to be understand by end-users. Usually, all of spatial relations among spatial objects can not explicitly be stored in database since large space are required even we store spatial relations among a small of sets of spatial objects [Egenhofer, 1990]. It is necessary to set up a mechanism for acquiring spatial relations from spatial databases. In this paper, we call this kind of mechanism spatial relation mapping. As introduced in Section 1, we will use V9I model and Voronoi diagram instead of 9I model to set up the mechanism. There are three levels in this mechanism: shown in figure 1, $\square$ the spatial objects level, to extract spatial objects from databases and generate Voronoi Diagram on them; $\square$ the middle level, to compute V9I model values $\square$ the semantic information level, to acquire


Fig.1: mechanism for mapping spatial relations based on V9I spatial relations among spatial objects.

### 2.1 Generating Voronoi Diagram with Dynamical Distance Transform

Generating Voronoi diagram is one of most important preconditions of setting up the spatial relation mapping based on V9I model. There are many methods for generating Voronoi diagram. In the work about this paper, we mainly adopt a raster method to generate Voronoi Diagrams based on dynamic distance transform. This method is developed from the basis of traditional raster methods by Li et.al (1999) for improving the precision of raster Voronoi Diagram. The key benefits is : the distance of each pixel can be high-precision computed by so called dynamic distance transform, that is to say, we can dynamically choose the structure element with the lest error to compute the distance from several different structure elements. The maximum distortion resulting from this method is about 1 pixel. Within a given spatial region and precision, the time cost of this method will only depend on the number of raster cells. Figure 2 shows us that the process of the method.


Fig.2: Voronoi diagram by raster method

### 2.2 Computing V9I Model Values

The main task of the second level is to compute V9I values with the result of the first level, the last result will be directly used for the third level. The efficiency of computing V9I values is a key factor of speeding up the extraction of spatial relations. In fact, it is found that the nine elements of V9I imply some links by further investigation, this will be helpful to improve the speed of computing V9I values. Just based on this point, we establish computational rules and logical control flow to compute V9I values, seen in the following figure 3. The flow chart can be used to improve the efficiency of computing V9I model values since it can avoid much direct complex intersection computation.


Fig3: rules and logical follow for computing V9I values

### 2.3 Acquiring Spatial Relations

This level is used to acquire spatial relations fromV9I model values computed in the second level by computation and
Table1: Rules for determining spatial relations based on V9I

| No. | V91 values | Spatial relations | Graphic representation | No. | V9I values | Spatial relations | Graphic representation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R1 | $\left[\begin{array}{lll}0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1\end{array}\right]$ | $B$ inside the hole of A | A $\bigcirc_{\square}^{\square}$ | R6 | $\left[\begin{array}{lll}1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$ | A equals B |  |
| R2 | $\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1\end{array}\right]$ | B inside the hole of A and touch | A | R7 | $\left[\begin{array}{lll}1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1\end{array}\right]$ | A touch B | ${ }^{\text {A }} \vdots \vdots \bigcirc \bigcirc^{\text {B }}$ |
| R3 | $\left[\begin{array}{lll}0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$ | A contains B | A $\because \bigcirc \bigcirc$ | R8 | $\left[\begin{array}{lll}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1\end{array}\right]$ | A immediately adjacent to B | ${ }^{\text {A }}$ |
| R4 | $\left[\begin{array}{lll}1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$ | B inside A and touch | A B | R9 | $\left[\begin{array}{lll}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{array}\right]$ | A Voronoi separate B |  |
| R5 | $\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$ | A intersect B | $\mathrm{A} \square^{\mathrm{B}}$ | Rn |  |  |  |

inference. Therefore, it is necessary to set up spatial relation rules based on V9I model which make the digital V9I model values map into the semantic spatial relations information understood by end-user. Table 1 gives a part of rules between two area objects and corresponding graphic representation. According to these rules, it is easy to determine spatial relations from V9I values got from the second level and give corresponding graphic representation to help user understand spatial concepts more intuitively. As a result, the basic spatial relations can be acquired with the mapping method based on V9I model, furthermore, more complex spatial queries and analysis can be extended on the basis.

## 3 DEFINING SPATIAL RELATION OPERATIONS

The above mechanism of spatial relation mapping can be realized by corresponding operations, while these operations are indispensable for spatial databases at the same time and form the base of spatial query and analysis. These operations are based on V9I model, consist of three main types: basic operations, basic relation operations and compound operations. For the sake of the standardization and facility of development of systems, it is necessary to formalize and define these operations. In this paper, Universal Algebra is adopted to formalize spatial relation operations.

### 3.1 Spatial Data Basic Operations

This kind of operations is mainly used to extract spatial data for the computation of spatial relations. Spatial data handled here can be abstracted into four basic types: point data, line data, region data, compound data which are composed of point, line and region data. These operations can accomplish such functions as extracting positions of objects ( boundary and interior ), type, dimension, Voronoi regions and Voronoi Diagram on sets of objects and so on, formally represented by the following with Universal Algebra :

```
Construct Operations:
Type data Obeject=New({Point|Line|Region|Mixed})
Observers Operations:
GetObjectType:->Point|Line|Region|Mixed // Get object s type
B(Object):-> Point|Line|Region|Mixed // Get Object's Boundary
I(Object):-> Point|Line|Region|Mixed // Get Object's interior
V(Object):-> Region|Mixed // Get Object's Voronoi region
Dim(Object)->Integer // Get Object's Dimension
```


### 3.2 Operations of Acquiring V9I values

Computing V9I values can be implemented by these operations together with the spatial relation rules described in Section 2, which primarily include the following operators:

```
Intersect(ObjectA,ObjectB):-> NULL|Point|Line|Region|Mixed //Get the result of intersection of object A and B
Same(ObjectA,ObjectB):->BOOL //If Object A and Object B is same.
GetV11:=Intersect(B(ObjectA),B(ObjectB)):-> NULL|Point|Line|Region|Mixed
    //Get the result of intersection of the boundary of object A and B
GetV12:=Intersect(B(ObjectA),I(ObjectB)):-> NULL|Point|Line|Region|Mixed
    // Get the result of intersection of the boundary of A and the interior of B
GetV33:=Intersect(V(ObjectA),V(ObjectB)):-> NULL|Point|Line|Region|Mixed
                            // Get the result of intersection of the Voronoi region of A and the Voronoi region of B
```


### 3.3 Basic Relation Operations

In this kind of operations, the operated objects involve more than two spatial objects. Most of basic spatial relations can be got through this kind of operations which is realized using spatial relation rules in section 2 . For example, the operations on spatial relations between two objects described in the Table 1 can be described as the following:

```
Distance(ObjectA,ObjectB):->Real // Distance
```

SDisjoint(ObjectA,ObjectB):= Intersect(V $\square \mathrm{A} \square, \mathrm{V} \square \mathrm{B} \square)$ :->BOOL // disjoint (using rule R9)
VAdjacency(ObjectA,ObjectB):=Sdisjoint(ObjectA,ObjectB)=False
$\square$ Intersect(ObjectA,ObjectB)=False // Immediate adjacency (using rule R8)
In(ObjectA,ObjectB):= Sdisjoint(ObjectA,ObjectB)=False
$\square$ Same $($ Intersect $(\mathrm{V}($ ObjectA $), O b j e c t B)), \mathrm{V}($ ObjectA $))=$ True
$\square$ Same(Intersect(V(ObjectA),ObjectB)),V(ObjectB))=False //A include B (using rule R3)

### 3.4 Compound Operations

Compound operations involve several kinds of spatial relations and three or more than three objects. For instance, the combination of spatial data operations, distance operations and topological operations can realize many complex query and analysis operations such as the Nearest, the Lateral Adjacency, and the set of Voronoi Adjacent Objects and so on.

```
GetVAdjacency (ObjectA):-> {Object|Vadjacency(Object,ObjectA)} // Get a set of adjacent objects
    MostAdjacency(ObejctA,ObjectB):=VAdjacency(ObejctA,ObjectB)=TRUE
                            \square M i n ( D i s t a n c e ( O b j e c t A , ~ G e t V A d j a c e n c y ~ ( O b j e c t A ) ) ~ / / ~ G e t ~ t h e ~ m o s t ~ a d j a c e n t ~ o b j e c t
LateralAdjacency(ObjectA,ObjectB):= VAdjacency(ObejctA,ObjectB)=TRUE
                                    \square ( G e t O b j e c t T y p e ( O b j e c t A ) = L i n e ~
                            | GetObjectType(ObjectB)=Line) // Get the lateral adjacent objects
```


## 4. AN EXPERIMENTAL TOOL

An experimental tool - VTKit is designed and realized with Visual $\mathrm{C}++$ on Windows operation on the basis of the above basic mapping principle and operations. In order to examine this mapping principle easily, the architecture of the tool is designed to be composed of three levels corresponding to the three levels of the mechanism of spatial relation mapping, as shown in figure 4.

The basic function of VTKit is to process query among spatial relations, spatial objects and V9I model: two objects are given, their V9I value and their spatial relation can be computed; or a relation is given, all spatial objects satisfying the relation can be found according to the corresponding V9I value. In figure 5, we can see that the spatial relation and V9I value are immediately acquired while object A and B are selected in a map or a attribute table; or , a object and V9I values ( or spatial relations) are given, the objects satisfying the relations can be obtained with the tool.


Fig.4: the Architecture of VTKit


Fig.5: the result of a part of experiments
(a) the V9I value and spatial relation between object A and object B ;
(b) the set of immediately adjacent objects of object L;
(c) the most adjacent object of object L;
(d) the lateral adjacent objects of object L ;
(e) the second adjacent object of object $L$;

The realization of the tool and experiments indicate that the mechanism of spatial relation mapping based on V9I model can bridge low level spatial data stored in databases and high level semantic spatial relation information. From the viewpoint of operation, Voronoi region of a object can be captured more easily than its complement, therefore, spatial relation operations occupy characters of locality and operable. In addition, the formalization of operations facilitates the development and standardization. On the other hand, from the viewpoint of efficiency, due to the locality of operations and the adoption of computational rules for V9I values and spatial relation rules, the spatial range of operations and the computational works reduced much, therefore, spatial relations can be acquired more efficiently.

Further work about this paper will mainly be to develop a better practical tool and extend the functions based on V9I model, for example, extend V9I model to determine complex spatial relations and apply V9I to recognize and extract terrain features.

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