# DIGITAL ORTHOPHOTO GENERATION 

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#### Abstract

A set of computer programs has been developed to perform the generation of orthophotos, using the terrain relief information (DTM) and the corresponding digital image. The procedure used takes the terrain position of each one of the orthophoto cells or pixels, next with the corresponding DTM, and the photograph orientation parameters, is determined the position of the orthophoto cell on the digital aerial photograph. In this way the corresponding tone or value for the cell is obtained. This paper explains in detail the procedure used, as well as criteria for the DTM requirements and possibilities for the cell tone interpolation. One of the benefits of the used procedure, is that minimise the image resampling, improving the visual quality of the resulting orthophoto.


## 1 INTRODUCTION

The photogrammetric central projection of a point of the terrain on the photographic negative plane is carried out by means of a projective line starting at terrain point, passing through de projection centre $O$ and intersecting the negative plane. To know the plane position of the point P on the terrain surface having only one photograph, it is necessary to reconstruct the corresponding projective line from the coordinates of the image point p and the projection centre. The length of the line L is determined from planialtimetric information of the point on the ground. The projective line is represented by the colinearity equations.

The planialtimetric information of the terrain necessary to found the intersection point P , is represented by a Digital Elevation Model (DEM). A DEM is determined from known characteristic points and interpolation algorithms to obtain the elevations of the each unknown points.

There are to different approaches to found the correspondence between image point and terrain point:

1. Transforming the position of each image point to its terrain position (DEM).
2. Transforming the position of each terrain (DEM) point to its image point.

There are important differences in the manner in which the process is carried out. In the first case, the georeferenciation of each image point on the DEM implies to determine iteratively its coordinates on the DEM to identify the cell that contains the point (Doysther and Hall,1995). In the second case, the DEM cell is already referenced, remaining only to find its position on the image and determine its grey value, which is simpler than to determine the Z value. In this paper is presented a procedure for the digital orthorectification based on the projection of the DEM on the image.

## 2 ORTHOPHOTO GENERATION PROCEDURE

The procedure is based on the determination of the six parameters of the colinearity equations, i.e., the spatial position of the projection centre $\left(\mathrm{X}_{\mathrm{O}}, \mathrm{Y}_{\mathrm{O}}, \mathrm{Z}_{\mathrm{O}}\right)$ and the camera orientation angles $(\omega, \varphi, \kappa)$ simultaneously, which permits to project the DEM points on the image. The process is developed in six stages as is showed in the Figure $1: 1$ )determination of the correspondence relation between the scanner system (reference system of the device used to digitise the photograph image) and the fiducial system, 2)determination of the correspondence relation between the terrain system and the fiducial system, 3)transformation of the DEM points to fiducial system, 4)transformation of the DEM from fiducial system to scanner system, 5)projective transformation of the densified DEM to scanner system and, 6)determination of the grey value for each rectified point.


Figure 1. Scheme of the procedure

### 2.1 Determination of the correspondence relation between the scanner system and the fiducial system

The Figure 2 shows the location the photograph respect to the reference system x , y of a digitizer device (scanner system). The relation between the scanner system and the fiducial system is established through an Affine transformation. The fiducial marks of the image are used to obtain the parameters of the Affine transformation because of its coordinates are known both the scanner system and the fiducial system (in the event that they are unknown, they can be determined). Then, the control points are transformed to fiducial system using the following expression:

$$
\begin{align*}
& x=a_{1} x+a_{2} y+a_{3}  \tag{1}\\
& y=a_{4} x+a_{5} y+a_{6}
\end{align*}
$$

where,
$\mathrm{x}, \mathrm{y}$ : are the coordinates in the fiducial system.
$\mathrm{x}, \mathrm{y}$ : are the coordinates in the scanner system.
$a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}$ : are the transformation parameters from the scanner system to the fiducial system.

Analogously, the parameters to transform from fiducial system and scanner system are computed using the Equations 1, to convert later the corners of the DEM cells to the scanner system.

### 2.2 Determination of the correspondence between terrain system and the fiducial system

In the figure 3 can be observed that the terrain system $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ has been moved parallel from G to O a distance $\mathrm{X}_{\mathrm{O}}, \mathrm{Y}_{\mathrm{O}}, \mathrm{Z}_{\mathrm{O}}$, defining the system $\mathrm{X}^{\prime}, \mathrm{Y}^{\prime}, \mathrm{Z}^{\prime}$, which is rotated the angles $\omega, \varphi, \kappa$ respect to $\mathrm{x}, \mathrm{y}, \mathrm{z}$. The determination of the parameters between terrain system and the fiducial system requires the control points and colinearity equations. In general, the colinearity equations are expressed as:

$$
\left[\begin{array}{l}
x  \tag{2}\\
y \\
-c
\end{array}\right]=k * A^{T} *\left[\begin{array}{l}
X-X_{o} \\
Y-Y_{o} \\
Z-Z_{o}
\end{array}\right]
$$

where:
$\mathrm{x}, \mathrm{y},-\mathrm{C}$ : are the image point coordinates referenced to the fiducial system.
C : is the principal distance of the camera.
$\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ : are the control point coordinates in the terrain system.
$\mathrm{X}_{\mathrm{O}}, \mathrm{Y}_{\mathrm{O}}, \mathrm{Z}_{\mathrm{O}}$ : are the projection centre coordinates O in the terrain system. k : the scale module between the vector $l$ and the vector $L$ for each point.
A: orthogonal rotation matrix defined by the rotation of the photograph


Figure 2. Relation between the projection of the fiducial system of the photograph and the scanner system.


Figure 3. Relation between the fiducial system and the terrain system.

The Equations 2 can be written as follows:

$$
\left[\begin{array}{l}
x  \tag{3}\\
y \\
-c
\end{array}\right]=k^{*}\left[\begin{array}{l}
a_{11}\left(X-X_{0}\right)+a_{21}\left(Y-Y_{0}\right)+a_{31}\left(\mathrm{Z}-\mathrm{Z}_{0}\right) \\
a_{12}\left(X-X_{0}\right)+a_{22}\left(Y-Y_{0}\right)+a_{32}\left(Z-Z_{0}\right) \\
a_{13}\left(X-X_{0}\right)+a_{23}\left(Y-Y_{0}\right)+a_{33}\left(Z-Z_{0}\right)
\end{array}\right]
$$

Such as is showed in the Figure 3, the vector I, defined from projection centre $O$ to the point p on the photograph, and the vector $L$, defined from the projection centre $O$ to the point $P$ on the terrain, are collinear (i.e. $I=k^{*} L$ ). Usually, the value k of each point is not know and can be eliminated dividing the first equations in the third of the equations 3 , obtaining:

$$
\begin{align*}
& \frac{\mathrm{x}}{-\mathrm{c}}=\frac{\mathrm{a}_{11}\left(\mathrm{X}-\mathrm{X}_{0}\right)+\mathrm{a}_{21}\left(\mathrm{Y}-\mathrm{Y}_{0}\right)+\mathrm{a}_{31}\left(\mathrm{Z}-\mathrm{Z}_{0}\right)}{\mathrm{a}_{13}\left(\mathrm{X}-\mathrm{X}_{0}\right)+\mathrm{a}_{23}\left(\mathrm{Y}-\mathrm{Y}_{0}\right)+\mathrm{a}_{33}\left(\mathrm{Z}-\mathrm{Z}_{0}\right)}  \tag{4}\\
& \frac{\mathrm{y}}{-\mathrm{c}}=\frac{\mathrm{a}_{12}\left(\mathrm{X}-\mathrm{X}_{0}\right)+\mathrm{a}_{22}\left(\mathrm{Y}-\mathrm{Y}_{0}\right)+\mathrm{a}_{32}\left(\mathrm{Z}-\mathrm{Z}_{0}\right)}{\mathrm{a}_{13}\left(\mathrm{X}-\mathrm{X}_{0}\right)+\mathrm{a}_{23}\left(\mathrm{Y}-\mathrm{Y}_{0}\right)+\mathrm{a}_{33}\left(\mathrm{Z}-\mathrm{Z}_{0}\right)} \tag{5}
\end{align*}
$$

The Equations 4 and 5 are rigorous and not lineal in terms of the six parameter unknowns $X_{0}, Y_{O}, Z_{0}, \omega, \varphi, \kappa$.

## 2. 3 Transformation of the DEM to scanner system

The transformation DEM to scanner system requires previously projecting the DEM, referenced in the terrain system, to fiducial system by using the Equations 4 and 5:

$$
\begin{align*}
& x=-c \frac{a_{11}\left(X-X_{0}\right)+a_{21}\left(Y-Y_{0}\right)+a_{31}\left(Z-Z_{0}\right)}{a_{13}\left(X-X_{0}\right)+a_{23}\left(Y-Y_{0}\right)+a_{33}\left(Z-Z_{0}\right)}  \tag{6}\\
& y=-c \frac{a_{12}\left(X-X_{0}\right)+a_{22}\left(Y-Y_{0}\right)+a_{32}\left(Z-Z_{0}\right)}{a_{13}\left(X-X_{0}\right)+a_{23}\left(Y-Y_{0}\right)+a_{33}\left(Z-Z_{0}\right)} \tag{7}
\end{align*}
$$

where,
$\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ : are the coordinates of the DEM nodes in the terrain system.
$\mathrm{X}_{0}, \mathrm{Y}_{0}, \mathrm{Z}_{0}$ : are the coordinates of the projection centre in the terrain system.
$x, y$ : are the image coordinates of DEM nodes in the fiducial system.
c : is the principal distance of the camera.
Then, the DEM nodes are transformed from fiducial system to scanner system, using the parameters of the Affine transformation, computed in the step 2.1 (Figure 4).

### 2.4 Projective transformation of the densified DEM from the terrain system to scanner system

Until now, it has been only projected in the scanner system the cell corners of the DEM. Since the DEM has a resolution lower than the digitised image, each DEM cell must be divided in sub-cells whose size is the same that the image cells (or image pixels). If the sub-cell size is greater than the cell size of the digitised image, it is lost information; if the sub-cell size is lower than the cell size of the digitised image, it is not obtained new details.

The geometric relation between each DEM sub-cell and the photograph can be determined in two ways: 1) using colinearity equations or 2)using projective transformation. In the first way, it is necessary to know the coordinate $Z$ of each sub-cell, and it is only available in the cell corners of the DEM (i.e., the DEM might be generated to the same resolution that the photograph image). This involves a lot of the computations and storing information. In the second way, it is not required the coordinate Z because of the sub-cell is projected directly on the digitised image, considering each DEM cell a plane (Figure 5). This reduces the computing process.


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Each sub-cell might project on the digitised image through a projective transformation whose equations are as follows:

$$
\begin{align*}
& x=a_{0}+a_{1} X+a_{2} Y+a_{3} X Y \\
& y=b_{0}+b_{1} X+b_{2} Y+b_{3} X Y \tag{8}
\end{align*}
$$

where,
$\mathrm{x}, \mathrm{y}$ : are the coordinates of the sub-cells in the scanner system.
$\mathrm{X}, \mathrm{Y}$ : are the coordinates of the sub-cells in the terrain system.
$a_{0}, a_{1}, a_{2}, a_{3}, b_{0}, b_{1}, b_{2}, b_{3}$ : are the parameters of the projective transformation
To solve the Equations 8 it is required at least four points in both systems, this is, the four cell corners of the DEM . Knowing the parameters of projective transformation, the coordinates X, Y of each sub-cell corner are transformed to scanner system.

### 2.5 Determination of the grey tone.

In this stage a grey tone is assigned to the cells of the rectified photograph image, i.e., each DEM sub-cell projected on the digitised photograph image. Commonly, there are a partial overlapping of a sub-cell on the several pixels of digitised photograph image (Figure 6). Therefore, it is necessary to interpolate the grey tone of each DEM sub-cell from the grey tone of the pixels of the photograph image overlapping. Nearest neighbour, proportional areas, significant areas are the methods usually used to interpolate.


Figure 6. Projection of a DEM sub-cell on the digitised photograph.

## 3 APPLICATION

To show the application of this procedure, an area in the Mérida State was selected. The photography of the zone was to a scale of 1: 10000 , taken with a normal angle camera of principal distance 150.48 mm and format 23 cm x 23 cm . The area relief is mountainous, whose heights oscillate between 1490 m and 1930 m above sea level, generating large relief displacements.

The DEM was generated from a topographical map of the zone at scale $1: 10000$ with contour interval of 10 m (Figure 7). The grid size was equal to 15 m , which guaranteed a height RMS (Root mean square) equal 1 m , with a maximal residual
value of 8.9 m , and obtained $1.3 \%$ of residual values greater than $1 / 3$ of contour intervals. The DEM of the area was generated using the Kriging interpolation procedure with the software Surfer for Windows ${ }^{\mathrm{TM}}$.


Figure 7. Perspective DTM view of application area.

The aerial photograph was scanned with an optical resolution of 300 dpi and were vectorized the control points and fiducial marks on the monitor using the software Map Maker ${ }^{\mathrm{TM}}$. The Affine parameters were then obtained using the coordinates of three fiducial marks in both monitor system and fiducial system and Equations 4, which allowed to transform the seven control points (Table 1) and features to the fiducial system.

Using the coordinates of the seven control points in both fiducial system and terrain system (Table 2), and the colinearity
$a_{1}=-126.727, \quad a_{2}=0.084, \quad a_{3}=0.004$
$a_{4}=-114.257, a_{5}=-0.004, a_{6}=0.085$

| Fiducial <br> marks | M onitor system |  | Fiducial system |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $x$ | $y$ | $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ |
| 1 | 103.53 | 1407.00 | -112.642 | 4.321 |
| 2 | 1433.00 | 2761.00 | 4.408 | 113.496 |
| 4 | 1450.00 | 76.59 | -4.412 | -113.599 |
| Control <br> points |  |  |  |  |
| 1 | 963.00 | 1146.00 | -41.311 | -21.202 |
| 2 | 384.00 | 801.00 | -91.354 | -48.056 |
| 3 | 377.00 | 420.00 | -93.397 | -80.249 |
| 4 | 585.00 | 109.00 | -77.081 | -107.386 |
| 5 | 1132.00 | 259.00 | -30.476 | -96.896 |
| 6 | 1378.00 | 368.00 | -9.358 | -88.665 |
| 7 | 1059.00 | 675.00 | -35.031 | -61.421 |


| Control <br> points | Fiducial system |  | Terrain system |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | $\mathrm{X}(\mathrm{m})$ | $\mathrm{Y}(\mathrm{m})$ | $\mathrm{Z}(\mathrm{m})$ |
| 1 | -41.311 | -21.202 | 40730.00 | 18347.00 | 1692.65 |  |
| 2 | -91.354 | -48.056 | 40211.00 | 17908.00 | 1699.89 |  |
| 3 | -93.397 | -80.249 | 40277.00 | 17548.00 | 1731.53 |  |
| 4 | -77.081 | -107.386 | 40541.00 | 17285.00 | 1760.32 |  |
| 5 | -30.476 | -96.896 | 41040.00 | 17382.00 | 1566.36 |  |
| 6 | -9.358 | -88.665 | 41267.00 | 17640.00 | 1718.25 |  |
| 7 | -35.031 | -61.421 | 40887.00 | 17857.00 | 1620.88 |  |

Table 2. Coordinates of the control points for colinearity transformation

Coordinates of the projection centre:
$\mathrm{X}_{\mathrm{O}}=41130.087 \mathrm{Y}_{\mathrm{O}}=18752.329 \mathrm{Z}_{\mathrm{O}}=3481.881$
Rotation matrix A:
Table 1. Affine transformation from monitor System to fiducial system
equations (Equation 2), the orientation parameters were determined. From these parameters, the DEM nodes were projected on the photo using Equations 4 and 5. The image resulting of the rectification performed is shown in the Figure 8, coinciding with photogrammetric restitution, which evidences the correctness of this procedure.


Figure 8. Comparison between the photogrammetric restitution and the rectified image

## 4 CONCLUSIONS

The proposed method allows the digital rigorous rectification of photograph from a known DEM. It is based on the projection of a DEM onto a photograph image using the colinearity equations and the projective transformation. The determination of image tones is only carried out in the latest step of the process. The procedure was tested in areas of mountain relief, producing satisfactory results, showing its potential use.

## REFERENCES

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