# THE NEW PHOTOGRAMMETRIC METHOD FOR CLOUD REMOTE STUDIES 

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#### Abstract

The main objective of the suggested work is to improve predictions of cloud top entrainment phenomena with photogrammetric methods based on satellite data analyses. The new analysis methodology for the cloud top 2D-flow reconstruction from remote sensing data is developed and suggestions for further satellite missions and data sets is discussed. Validation examples based on available data is provided. Method uses a set of images of the cloud layer with known temporal and spatial resolution in order to obtain flow pattern. We consider visual movement and deformation of the picture as an atmospheric flow perturbation and in the simplest situation imply it to be identical. The identity of two points is determined by their neighboring vicinity. Thus the method, analyzing some vicinity of point at the first image, is to find an identical vicinity at the second one. Algorithm analyses the couple of consecutive In that way we build an auxiliary space in which small displacement of original image point will be in correspondence with that of auxiliary space point. Also two points with equal in the above sense vicinities have the same image point in the space. The solution is developed by use of iterative method.


## 1 INTRODUCTION

The main objective of the suggested work is to improve predictions of cloud top entrinment phenomena with photogrammetric methods based on satellite data analyses. The new analysis methodology for the cloud top 2D-flow reconstruction from remote sensing data is developed and suggestions for further satellite missions and data sets is discussed. Method uses a set of images of the cloud layer with known temporal and spatial resolution in order to obtain flow pattern. We consider visual movement and deformation of the picture as an atmospheric flow perturbation and in the simplest situation in order to obtain flow pattern. We consider visual movement and deformation of the picture as an atmospheric flow perturbation and in the simplest situation imply it to be identical. The identity of two points is determined by their neighboring vicinity. Thus the method, analyzing some vicinity of point at the first image, is to find an identical vicinity at the second one. It is natural that we can not find the exact transformation because of liquid particle deformation. However, assuming that the above deformation is small enough we can disregard such a problem. Algorithm analyses the couple of consecutive monochromic images in some spectral range and releases the field of deformation which translate the first image into the second one. The field of deformation in hand is developed as a solution of a variation problem. In that way we build an auxiliary space in which small displacement of original image point will be in correspondence with that of auxiliary space point. Also two points with equal in the above sense vicinities have the same image point in the space. The solution is developed by use of iterative method. A tangent hyperplane is considered and found is the optimum directions of movement toward the solution. Further the iterative process is repeated, and as we can control both the movement of image point and displacement itself, the error of iteration is not accumulated. Therefore the solution can be obtained with adequate accuracy. Then it extracts regions free of clouds and calculates the velocity field. Numerical experiments show major parameters affecting the solution to be the size of point vicinity and how thoroughly it is described, that can be determined by number of reference points, and therefore by dimension of auxiliary space. It is interesting that both parameters have their optimum values depending on image character. This technique is subjected to time resolution of input data, so there are some restrictions for experiments methods. In analysis of cyclones in Earth atmosphere with the scale about thousand kilometers it requires temporal resolution about some minutes while at the present time such resolution is about hour depending on satellite orbit. The further possible development of the method may follow several directions, such as to take into consideration the vertical movement of the cloud layer using stereoimages, to improve trajectory search algorithm in case when exact coincidence is impossible, to automate evaluation of optimum algorithm parameters in accordance with statistical properties of images being aligned.

## 2 PRELIMINARY NOTES

The motion of air masses is identified with that of clouds, i.e. with modifications of a shadow pattern of cloudy layer observable from space. Generally speaking, this supposition requires the separate proof, however considering of this problem goes out for frameworks of a subject in hand. One remark motion of cloudy layer on peculiarity motion of a flow pattern, that is population of shadow marks or other peculiarities and heterogeneities. Images that are capable to reveal such heterogeneities on required scales are suitable for handling in any spectral band. It can be the shadow patterns of cloudy layer in a visible band, shooting in lines of water on large scales, overseeing by transition of impurities in conditional colours etc.

It is evident, that we could place correspondence between sequential images and the distinction between them should be reduced to a minimum. As in turbulent flow streamlines can diverge on arbitrary distance, thus strongly changing a flow pattern, then critical parameter from the point of view of reliability of operation is the interval between sequential images $\Delta \mathrm{T}$, which should be, whenever possible, diminished. As it is a priori mathematically impossible to determine a degree of "similarity" or "non-similarity" of a couple of the images, for in fact this problem is a subject of consideration, then it is possible to consider that parameter $\Delta \mathrm{T}$ should satisfy to a relation

$$
\begin{equation*}
\Delta T=k \text { Lchar } / \text { Vchar or } \Delta S=k \text { Lchar } . \tag{1}
\end{equation*}
$$

Here Lchar and Vchar define a characteristic linear scale of heterogeneities and characteristic velocity on scales of these heterogeneities, $\Delta \mathrm{S}$ sets expected shift, and value $k$ equals $0.1 . .0 .5$. Generally speaking, the basic complexity is represented by essential strains of the images, at which the topology of lines of identical luminosity varies. In ordinary transition of all pattern, the reconstruction is theoretically possible at any value of displacement. During testing algorithm on the synthetic images, in a series of cases the reliable outcome was achieved at values $\mathrm{k}=2 . .3$ and more.

In reconstructing of a global pattern of atmospheric flow, we shall neglect vertical motions of air masses. In spite of the fact that these transitions can lead to essential deformations of a shadow pattern observed from the satellite, the singularities with characteristic scales about several hundreds kilometres are maintained long enough. Thus, for reconstruction of a flow pattern on a series of the sequential images it is necessary to find in each point of the image a vector of an air flow velocity laying in a plane that is perpendicular to ray of sight. In case if for each point of the first image the similar point on second will be retrieved, we can construct a field of a deformation of the image and, if knew a temporal interval between two images, field of velocities.

## 3 STATING A PROBLEM

Further we shall speak only of the monochromic images implying that a colour pattern we can set with the help of three monochromic channels - red, green and dark blue, according to RGB colour model. As a matter of fact, such approach does not restrict us to the images, and permits to consider any functions assigning 2D-allocation of any magnitude, for example temperatures, luminosity, concentrations etc. For the sake of a determinancy all such function we shall define as "images".

It is supposed, that the two-dimensional image is determined by restricted continuous together with the first derivative function, given on a singly connected domain. The rather feeble requirements to a smoothness were defined proceeding from the minimum theoretical requirements. Basically, the requirements to a smoothness can be made more rigorous, and the quality of operation of algorithm as a whole seems to be better. As the images, which are subject to handling, are discrete representation of the analogue images, i.e. set netting function, the requirements to a smoothness render serious influence on used algorithms of approximating. These problems are not bound directly to a primal problem.

Thus, the starting problem of reconstruction of an instantaneous pattern of flow is reduced to the following: it is required to revive a deformation field of a couple of the images translating first image in second one. The deriving of a deformation field of is to determine a vector of transition of each point of the first image, i.e. determination of analogue of this point on the second image. Therefore for further operation it is required formally to spot concept of "resembling" of points of the image.

Intuitively trying to place correspondences between two images, we are guided by any interior, subjective estimation. Thus in different situations the essential role is played by the various factors - colour, topological singularities of figures, statistical properties, the texture etc. For various objects we select various variants of matching. The legible mathematical criterion permitting to compare a points of the image or images and to select the most approaching couples is necessary for the purposes of program handling.

## 4 MATHEMATICAL DESCRIPTION

"Similarity" of two of the points is defined by their neighbourhoods - i.e. any characteristic peculiarities located near to a considered point. Distracting from the analysis of topological singularities, mathematically this measure of "similarity" or "non- similarity " can be expressed as:

$$
\begin{equation*}
R_{S}=\int_{S}\|f(x, y)-g(x, y)\| d x d y \tag{2}
\end{equation*}
$$

Where the integration is carried out on a neighbourhood $S$ of a point ( $x 0, y 0$ ), and the norm is picked by the most convenient mode for example square-law. If $f$ is identically equal $g$, i.e. the images in a neighbourhood of a point ( $\mathrm{x} 0, \mathrm{y} 0$ ) are identical, RS reverts in 0 . As for the real images the module $f$ is bounded above, then at given area of an integration $S$ value RS is also restricted. In further we assume, that $f$ belongs to a segment [ 0 , fmax].

When we speak that the images will be similar, if one of them to turn (to bias, to add luminosities, to stretch etc.), it signifies, that before matching in sense (2) it is necessary one of the images to subject to some transformation. Then

$$
\begin{equation*}
R_{S}=\int\left\|f(x, y)-F_{A} g(x, y)\right\| d x d y \tag{3}
\end{equation*}
$$

Where FA - some operator circumscribing some transformation of the image, maintaining its property, i.e. translating image in the image. In such situation we shall speak, that the images are similar on transformation FA, where A is a set of parameters of transformation (on how much luminosity has increased, turn angle etc.). However, in spite of the fact that the formulas (2-3) allow quantitatively to estimate a measure "non-similarity" of points of the image, we can not conveniently define concept of distance, as image is a point infinite-dimensional of function space. For further operation it is necessary to narrow down this class of functions and to proceed to finite-dimensional space, which precisely enough would feature the starting image and thus would allow to spot as it is necessary to distort the images for a diminution of a non-similarity degree. Most simple but not a unique method of build-up of such space is considered below.

Let's set a $\delta$ - neighbourhood of a given point x of the image $\mathrm{f}(\mathrm{x})$ with the aid of netting function

$$
\begin{equation*}
\xi_{i}(x)=f\left(x+a_{i}\right), \quad\left|a_{i}\right|<\delta, \quad i=1 . . N, \quad a_{i} \text { and } \delta \in \mathrm{R}^{2} \tag{4}
\end{equation*}
$$

The allocation $\left\{a_{i}\right\}$ in neighbourhoods of a point x can be arbitrary, however for a correct image reconstruction it is desirable, that they would distribute in a neighbourhood uniformly enough. Problem of an optimal method of allocation of nodal points by itself represents separate interest and will be considered later. In such approximation the concept of "proximity" will be stated as

$$
\begin{equation*}
R_{s}(x)=\left(\sum\left(f\left(x+a_{i}\right)-g\left(x+a_{i}\right)\right)^{2}\right)^{1 / 2} \tag{5}
\end{equation*}
$$

As while increasing of an amount of nodal points we can more and more precisely approximate values of function (classical theorem of the Weierstrass), the formula (5) on its sense becomes to the equivalent integrated formula (2), though the numerical values Rs will differ. Thus, points with neighbourhoods similar in indicated sense we shall define as "close on a set $\{a\} N N^{\prime \prime}$. Generally speaking, the points, close on one set $\{a\} N$ not necessarily are close on other set $\{b\} N$.

Interpreting (4) as map $\mathrm{R}^{2} \rightarrow \mathrm{R}^{\mathrm{N}}$ and vector $\xi_{i}$ as a vector of N -dimensional Euclidean space accordingly, we gain that (5) sets distance between points in this space.

The representation of the integral images in new space will have the following properties:

1. as the image is function of two variables, a representation of the image will be a two-dimensional surface;
2. by virtue of boundedness of values of the image, the fashion bodily belongs to a N -cube $[0, \mathrm{fmax}]$;
3. the representation has the same tangential properties, as function circumscribing the image;
4. the representation of points of the images having identical neighbourhoods, coincide;

Thus, the sleek initial image corresponds to a sleek restricted 2D-surface in conjugate N -dimensional space possessing composite topology. Further it is considered, that the set $\left\{a_{i}\right\}$, defining build-up of conjugate space, is given, and the ranges of representations, i.e. surfaces in conjugate space, are designated as $\Sigma_{f}$ and $\Sigma_{g}$ accordingly. Apparently, that two identical images $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}(\mathrm{x})$ generate identical $\Sigma_{f}$ and $\Sigma_{g}$. Moreover, if $\mathrm{f}(\mathrm{x})=\mathrm{g}(\mathrm{x}+\mathrm{s})$, then $\Sigma_{f}$ and $\Sigma_{g}$ appear to be identical too, though the interior coordinates on these surfaces differ.

Let's assume, that $f(x)=g(x)+\varepsilon(x)$, where $|\varepsilon(x)|<\varepsilon_{\max }$. Then distance Rs, defining resembling of two points of the image has the following property:

$$
\begin{equation*}
\left|R_{s}(x)\right|<\mathrm{N} \varepsilon_{\max } \quad R s(x) \rightarrow 0 \text { at } \varepsilon_{\max } \rightarrow 0 \tag{6}
\end{equation*}
$$

I.e. at continuous transformation of the starting image $\mathrm{g}(\mathrm{x})$ its representation also continuously varies.

## 5 OVERLAPPING OF THE IMAGES

In a general sense overlapping of the images implies searching parameters of some transformation, which allows to minimise (in an ideal to reduce to 0 ) value of Rs. Usually class of transformations is determined by a concrete problem and is set a priori. Generally, these transformations can be divided into two groups: deformation (image shaping), operating on argument $\mathrm{f}(\mathrm{x})$, and modification of luminosity (radiometric shift), changing its value.

$$
\begin{equation*}
f(x)=F_{A} g(x)=F^{(R)}\left(g\left(F^{(S)}\left(x, A_{S}\right), A_{R}\right)\right. \tag{7}
\end{equation*}
$$

Where FA is the generalized transformation with vectorial parameter Am, m=1.. M, and $F^{(S)}$ and $F^{(R)}$ are deformation and brightness components accordingly. In further we shall assume, that the transformation is continuous and sleek. Besides we shall assume, that the transformation is identical at $\mathrm{Am}=0$.

$$
\begin{equation*}
F_{A=0} f(x)=f(x) \tag{8}
\end{equation*}
$$

It is necessary to pay attention, that it is possible to bleed two classes of problems on overlapping the images further called dot and regional problems of overlapping. As we spoke, set of points assigning a neighbourhood of a point, should admit interpolation reconstruction of function in all neighbourhood, the obtained outcomes of transformation can be referred both to the point, and to a neighbourhood as a whole. Thus the size of a neighbourhood can be arbitrary, in particular it can coat the image bodily.

In the former case the object of examination is the point of the first image and the problem of searching of parameters of transformation permitting to combine the same point of the second image with first one is considered. Thus the parameters of transformation can vary from point to point and the basic interest represents a field of vectors circumscribing a modifications of this parameter. In case of reconstruction of a flow pattern such parameter is the displacement for a particular time term, which is easily resulted in a field of velocities.

The second regional problem puts by the purpose searching of all-purpose parameters, which allow on a given class of transformations to combine area of the images or image bodily. Such problems arise, for example, when it is required to spot a position of the camera with respect to exterior subjects. In this case parameters of visible deformation of these objects are enumerated in coordinates of the camera and angles assigning its orientation. By default all further reasonings concern to a problem of dot overlapping.

It is clear, that the problem of overlapping is reduced to definition of a vector $A=\left\{A_{S}, A_{R}\right\}$, assigning transformation parameters in (7). It is a standard problem of minimisation, which is solved by a method of the gradient descent. Having assumed that $A \rightarrow 0$, let decompose a right-hand part of expression (7) in a series on vectorial parameter A. For the sake of simplicity we first derive the separate formulas for $F^{(R)}$ and $F^{(S)}$.

$$
\begin{align*}
g\left(F^{(S)}\left(x, A^{(S)}\right)\right. & =g\left(F^{(S)}(x, 0)\right)+\nabla g(x) \nabla F^{(S)}(x, 0) A^{(S)}+\ldots  \tag{9}\\
F^{(R)}\left(g(x), A^{(R)}\right) & =F^{(R)}(g(x), 0)+\nabla F^{(R)}(g(x), 0) A^{(R)}+\ldots \tag{10}
\end{align*}
$$

That in aggregate, taking into account parameters Am to be rather small, gives the general formula to within linear terms:

$$
\begin{equation*}
f(x)=g(x)+\nabla F^{(R)}(g(x), 0) A^{(R)}+\nabla F^{(R)}(g(x), 0) \nabla g(x) \nabla F^{(S)}(x, 0) A^{(S)} \tag{11}
\end{equation*}
$$

In the terms of conjugate space:

$$
\begin{equation*}
\xi_{i}(x)=\eta_{i}(x)+R_{i}(x) A^{(R)}+S_{i}(x) A^{(S)}+e_{i}(x) \tag{12}
\end{equation*}
$$

Where $\xi_{i}(x)=f\left(x+a_{i}\right), \eta_{i}(x)=g\left(x+a_{i}\right)$ - column vectors circumscribing a point of the image in conjugate space, $A_{j}^{(R)}, A_{j}^{(S)}-$ column vectors assigning parameters deformation and brightness components of transformation, matrixes:

$$
\begin{align*}
& R_{i j}(x)=\partial F^{(R)}\left(g\left(x+a_{i}\right), 0\right) / \partial A^{(R)}{ }_{j}  \tag{13}\\
& S_{i j}(x)=R_{i k}(x) \partial g_{k}\left(x+a_{i}\right) / \partial x_{l} \partial F^{(S)}\left(x+a_{i}, 0\right) / \partial A^{(S)}{ }_{j} \tag{14}
\end{align*}
$$

and $e_{i}(x)$ designates a vector of errors, which is subject to minimization.
Having designated for $B$ a block matrix $\{R, S\}$ and having aggregated $A^{(R)}$ and $A^{(S)}$ in a single vector of parameters, we shall receive

$$
\begin{equation*}
r_{i}(x)-B_{i j}(x) A_{j}=e_{i}(x), \quad \mathrm{i}=1 . . \mathrm{N}, \quad \mathrm{j}=1 . . \mathrm{M}, \quad \mathrm{~N}>\mathrm{M} \tag{15}
\end{equation*}
$$

According to the Gauss - Markov theorem, the solution of such system with a zero right-hand part gives a vector A, minimising $\mathrm{e}(\mathrm{x})$ in sense of square-law norm. The visual performance of the formula (15) consists that the matrix B sets base of transformation $F_{A}$ at $A \rightarrow 0$ in conjugate space, and the parameters A determine decomposition coefficients of a radius-vector pairing a points of two images on this basis. Apparently, that each M-dimensional parameter generates Mdimensional basis, defining M-dimensional subspace in conjugate space. As the radius-vector r has dimensionality N , the expansion represents a projection of this vector to a subspace. Generally, when real transformation is tried to approximate by transformation with a smaller amount of parameters, the vector of an error appears to be nonzero.

Considering obtained value A as the first approximation, we can construct a series of iterations permitting to improve a required parameter value up to anyone beforehand given magnitude. Thus at the k-th step:

$$
\begin{align*}
& g_{k}(x)=F\left(g(x), A_{k-1}\right) \\
& r(x)-B(x) d A_{k}=0 \\
& A_{k}=A_{k-1}+d A_{k} \tag{16}
\end{align*}
$$

All intermediate parameters are calculated on the basis of function $g_{k}(x)$, obtained as a result of application to an assumed function of transformation with usage of an parameter value gained on the previous step. In case if the increment of parameter A appears less than given $\varepsilon$, it is supposed, that the solution is retrieved.

## 6 CONCLUSIONS AND PERSPECTIVE

1. The new approach to a revertive problem of reconstruction of deformation parameters - examining of a problem from the point of view of approximating infinite-dimensional spaces of the images finite-dimensional and reduction to a standard problem of the gradient descent.
2. The practical application of a designed method to a new class of problems - examination of behaviour dynamics of cloudy layer on various scales.
3. Development of a method for solution of a revertive problem of transition of a surface of cloudy layer in a threedimensional case. Thus at each stage the stereoimages will be utilised which allow to revive a vertical profile of high layer of a cloudiness.
4. Development of alternate approximating techniques for infinite-dimensional spaces of the images by finitedimensional, for example with usage of expansion of 2D-function of the image in series.
5. Development of the plan of automatic image analysis on various scales. Thus the outcomes obtained on wide scale singularities will be utilised as the first approximation for the analysis of shallow details.
6. Development of a method of the context-sensitive analysis of outcomes on the basis of the supposition that the flow parameters (or, generally, any strain of the image) feebly vary from point to point.
7. Study of influence of the images pre-treatment parameters on algorithm operation quality.
