THE ACCURACY OF THE OBJECTS POSITION MEASURING IN AN IMAGE

V.I.Lebedev

Etalon Plant, 2A Novatorskaya Str., 220053 Minsk, Belarus – vil_lebedev@yahoo.com

Commission 1, WG 1/2

ABSTRACT:

A discrete location of photo-sensitive elements in CCD and in other matrixes leads to the distortions of shape, reduction of sizes, shift, etc. of the objects in an image. As a result, the distortions influence on the accuracy of the objects coordinates measuring. In this paper, we estimate the corner points coordinates measurement errors for arbitrary oriented rectangle and the other objects with mutually perpendicular boundary segments. We derive the formulas for calculation of the errors depending on the angle orientation of the figures. Estimations are based on the properties of digital straight segments and geometrical constructions. Some applications of the results are discussed.

1. INTRODUCTION

In image reading, using the scanners on the basis of CCD and the other matrixes of photo-sensitive elements, on their object coordinates measuring accuracy influences to a considerable extent the discrete location of the elements in a matrix. The influence of geometrical distortions are less significant even in low-cost scanners (Cramer, Bill, and Glemser, 1994). On coordinates measuring accuracy also influences the point spread function of a digitizer. For example, if binary image is digitized, the result is a gray scale image, that is its boundary is fuzzy. But the methods for recovering the binary images solve this problem (see, for example, Pavlidis and Wolberg, 1986; Li et al. 1986; Medioni and Yasumoto, 1986). Then the errors remain because of the finite resolution or super-resolution of the scanner (see Pratt, 1978; Cohen and Dinstein, 2000). In this paper, we estimate the corner points coordinates measuring errors of the rectangular-shaped objects that are arbitrary oriented relative to the axes OX, OY of the Cartesian coordinate system XOY superposed with the digitizer coordinate system. Corresponding estimations we derived (see Lebedev and Zolotoy, 2001; Lebedev, 2003), but for special angles of orientation and side lengths of the figures.

2. DIGITIZATION

We assume that an image is read by the receptors field being the integer valued grid with the constant step equal 1 and the pointed photo-sensitive elements are located in its nodes (see fig.1). Let the image is quantized in these points on two levels (0 is a background, 1 is an object), and contour element of the object is an unit element with four neighbouring elements in vertical and horizontal directions having at least one zero element. The shortest closed broken line connecting the contour elements (assuming that they are eight-connected) constitutes the object contour. Let name this contour as the black contour (according Freeman, 1970). Similarly, introduce the white contour notation, i.e. the shortest closed line connecting the eight-connected zero elements bordering in the sense of the four-connection with the black contour elements. The domain lying between the black and white contours determines the so-called zone of non-sensitivity (Pisarevsky et al., 1988). The expressions for description of such zone has been given (Dorst and Smeulders, 1984). The width β of this zone influences on the measuring errors $\Delta = (\Delta x, \Delta y)$ of the rectangle corner points coordinates. In one's turn, the width β depends on the angle of the rectangle orientation φ. In next section we will be derive the equations for this dependence.
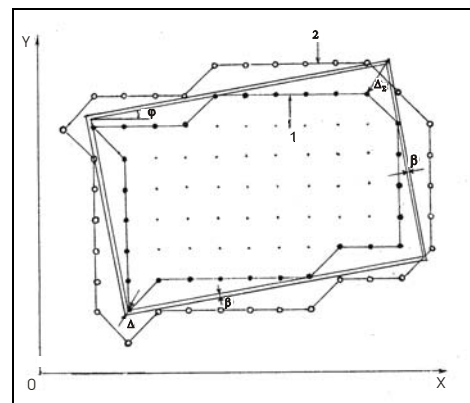

Figure 1. A digitized rectangle (1 is black contour; 2 is white contour)

3. THE WIDTH OF NON-SENSITIVITY ZONE

Let a side of a rectangle lies in the first octant of the Cartesian coordinate system and its slope is $\mathrm{tg}\varphi = p/q$, where p and q are integers and p/q is irreducible fraction ($p \le q$). Then the chain code of the directions of digital straight segment belonging to black or white contour and corresponding the side of a rectangle is the periodic Freeman's sequence $L = l_1,...,l_q$ (or its reverse sequence $L_r = l_q,...,l_1$), where l_j is 0 or 1, j = 1,2,…,q (Pham, 1986). Using the proof of the proposition 3 from the Pham's work (see Pham, 1986), we can show that

$$L = 0,...,0, l_{i+1},…, l_q \ (\text{or } L_r = 0,…,0, l_{q-i},…, l_1) \qquad (1)$$

if the end points of a rectangle side lie in the nodes of a grid, where maximum value i equal

$$i_{max} = \ ]\frac{q-p}{p}[\ , \qquad (2)$$

]A[is the smallest integer not smaller than A (ceiling-function).

Take advantage the formula (2) and geometrical constructions it is easy to show that

$$\beta = \frac{q -]\frac{q-p}{p}[\ p}{\sqrt{p^2 + q^2}} \qquad (3)$$

if $\varphi \in \left[0, \text{arctg}\frac{1}{2} \ \right]$, and

$$\beta' = \frac{q -]\frac{p}{q+p}[(q-p)}{\sqrt{p^2 + q^2}} \qquad (4)$$

if $\text{arctg} \ \frac{1}{2} < \varphi \leq \frac{\pi}{4}$.

When horizontal projection a_x of the rectangle side a is less than corresponding projection L_x of the sequence (1), the width of non-sensitivity zone is

$$\beta* = \ \cos \varphi - j_0 \sin \varphi \ \ (j_0 = 0, 1,...,q-2) \qquad (5)$$

for $0 \leq \varphi \leq \frac{\pi}{4}$.

4. ESTIMATIONS OF THE LOCATION ERRORS

Let all corner points of a rectangle lie in the nodes of a grid. Then we have (see Lebedev and Zolotoy, 2001)

$$\Delta = \sqrt{2} \ \beta \qquad (6)$$

where β is calculated by (3) or (4).

In the other cases we have (for $a_x \geq L_x$)

$$\Delta_\Sigma = \Delta + \Delta_0 \qquad (7)$$

where Δ_0 is minimal distance between the corner point of a rectangle and its black contour points.

Using geometrical constructions we can show that is correct the next estimations for Δ_0:

$$\overline{\sqrt{2}} \ (\cos \varphi - \beta) \leq \Delta_0 \leq 2(\cos \varphi - \beta). \qquad (8)$$

Then according to (6) and (7) we have the next estimation for Δ_Σ from (8):

$$\sqrt{2} \ \cos \varphi \ \leq \Delta_\Sigma \leq 2\cos \varphi - \ (2-\sqrt{2} \ ) \beta. \qquad (9)$$

Note that the function Δ defined by the expression (6) and the functions $\Delta'_\Sigma = 2\cos\varphi - \left(2 - \sqrt{2}\right)\beta$, $\Delta''_\Sigma = \sqrt{2} \ \cos\varphi$ defined by the inequalities (9) are symmetric functions relative to $\varphi_n = (\pi/4)n$ in intervals $[(\pi/2)(n-1), (\pi/2)n]$ $(n = 1,2,...)$. Besides, these functions are periodic with the minimal period in $\pi/2$.

If the width of non-sensitivity zone is determined by the equality (5), the estimation of the corresponding error is

$$\Delta * = \sqrt{2\cos^2 \varphi - 2j_0 \sin \varphi \cos \varphi + \ j_0^2 \sin^2 \varphi} \qquad (10)$$

where index j_0 corresponds the side of a rectangle that oriented under the angle φ for its lower right and upper left angles, and oriented under the angle $\varphi + \pi/2$ for its lower left and upper right angles.

Estimations (6), (9), and (10) are the same for the other figures with mutually perpendicular boundary segments. For acute-angled objects these estimations are lower estimations, and for the obtuse-angled objects they are upper estimations. Proper formulas can be derived in analogous manner.

5. PLOTS OF THE ERRORS

The functions $\Delta'_\Sigma = 2\cos\varphi - \left(2 - \sqrt{2}\right)\beta$, $\Delta''_\Sigma = \sqrt{2} \ \cos \ \varphi$, and $\Delta = \sqrt{2} \ \beta$ are plotted in figure 2 (the curves 1,2 and 3 correspondingly). We can see that maximal error (curve1) exceeds minimal error (curve3) approximately in 32 times. But the probability that all corner points of a rectangle lie in the nodes of a grid is hardly possible. Thus, the worst case (curve1) or the middle case (curve2) are more likely. We can also see that the expressions for the errors estimation can be used for the optimization of the images reading (aerospace pictures, drawings, schemes, etc.). In particular, the positional accuracy of the rectangles (and other figures with mutually perpendicular boundary segments) reading can be increased in the manner of certain orientation of the figures on a grid. For example, if the object was oriented along the axes OX, OY of coordinate system XOY combined with the scanner coordinate system, the error is maximum and equal $\Delta = \sqrt{2}$. It can be decreased in 23 times if the figure angle orientation will be 35° (see curve 3 in fig. 2). For the other cases (see curves 1 and 2 in fig. 2) optimal angle orientation of the objects is 45°. It permits us to reduce the errors to step of a grid. Correspondingly, maximum error for a centroid of a rectangle is in this case $\Delta = \sqrt{2}$ /2. It is in $\sqrt{2}$ times less than for the angle orientation $\varphi = 0°$. The scattering of the coordinates x_c, y_c of a centroid will be also less for $\varphi = 45°$. For example, if these coordinates satisfy a uniform distribution, the variance D = 1/24 for $\varphi = 45°$, and D = 1/12 for $\varphi = 0°$. In all cases the positional accuracy is increased if we estimate the corner points position by both black and white contour points coordinates and put them as an average.
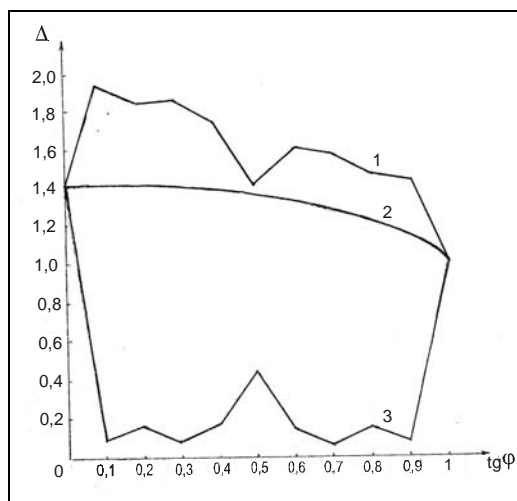
Figure 2. The errors in the corner points of a rectangle
as a function of the slope

6. INFLUENCE OF A NOISE

On the scattering of the coordinates of the objects also influences a noise. If an image is represented as the poisson random point field, the variance in the coordinates x_c, y_c of the centroid and in the side a of a rectangle is equal (Bakut, Troitsky, and Ustinov, 1976)

$$D^* = \frac{2}{n_s^2 (r-1)^2} \, a^2$$

for the angle orientation $\varphi = 0^o$, where r is the ratio signal / background ($r \gg 1$), n_s is the average value of signal points located within a rectangle.

Depending on the angle of orientation and the ratio of the sides of a rectangle, the sides length, and the ratio signal/background, the variance D^* can be either less or more then corresponding variance D that conditioned by the effect of digitization.

7. SOME APPLICATIONS OF THE RESULTS

The derived results let us to minimize geometrical errors of aerial pictures scanning for photogrammetry and the other applications by means of optimum orientation of the lines of scanning or the objects. Such approach can be used to select the templates with optimal angle of the orientation in stereo matching and other registration problems for mapping (see, for example, Erosh and Zolotar, 1999). The estimations of the errors can be also used to define more exactly a distance between the objects, their dimensions, shape and geometrical features such as area, perimeter (Pisarevsky et al., 1988), shape-factors (Warl, 1983). The other fields of theirs application are the objects reconstruction, for example, the straight lines and buildings reconstruction in 3-D (Suveg and Vosselman, 2002), the man-made objects detection (the roads and their intersections (Lu et al, 2002), the buildings, means of transport, etc.), synthesis of the images, computer graphics, and so on.

8. CONCLUSION

The results of this paper enable to minimize geometrical errors of aerospace pictures scanning for photogrammetry and the other applications. The derived results can be spread to the objects with non-mutually perpendicular boundary segments and the other grids (rectangular, hexagonal, etc.). It allows us to extend the sphere of applications of the results on arbitrary objects in aerial and other images.

REFERENCES

Bakut, P.A., Troitsky, I.N., Ustinov, N.D., 1976. Potential accuracy of estimating the parameters of optical images. *Trans. Acad. Sci. USSR, Eng. Cybern.,* No.4, pp. 148-150.

Cohen, B, Dinstein, I., 2000. Polyphase back-projection filtering for image resolution enhancement. *IEE Proc. Vision, Image and Sygnal Processing*, No.4, pp. 318-322.

Crammer, M., Bill.,R., Glemser, M., 1994. Investigations of low-cost devices for digital photogrammetric systems.*GIS,* 7(1), pp.2-8.

Freeman, H., 1970. Boundary encoding and processing. In: Picture Processing and Psychopictorics, B.S.Lipkin and A.Rosenfeld, eds. Academic Press, New York, pp. 241-266.

Dorst, L.,Smeulders, A.W.M., 1984. Discrete representation of straight lines. *IEEE Trans. Pattern Anal. Machine Intell,* PAMI-6(4), pp. 450-463.

Erosh, I. L, Zolotar, A.V., 1999. Construction of 3D-model from two pictures. In: *Proc.10th Conf. Extremal Robot Technique* , St. Petersburg, Russia, pp. 329-335 (in Russian).

Lebedev, V.I., 2003. The accuracy of the objects position measuring in discrete raster. In: Digital Image Analysis, S.V.Ablameyko, ed. United Inst. of Information Sciences of NAS, Minsk, Vol. 2, pp. 194-197 (in Russian)

Lebedev, V. I., Zolotoy, S. A., 2001. Estimation of the scanner resolution influence on the accuracy of the object coordinates definition in aerospace pictures decoding. In: Digital Image Processing, S.V.Ablameyko, ed. Inst. of Engineering Cybernetics of NAS, Minsk, Vol.5, pp. 7-10 (in Russian).

Li, X., Shanmugamani, C., Wu, T.,Madhavan, R.,1986. Correlation measures for corner detection. In: *Proc. CVPR'86, IEEE Comput. Soc. Conf. Comput. Vision and Pattern. Recogn.,* Washington, USA, pp. 643-646.

Lu, J., Yang, M., Wang, H., Zhang, B., 2002. Vision-based real-time road detection in urban traffic. *Proc. SPIE,* 4666, pp. 75-82.

Medioni, G., Yasumoto, Y., 1986. Corner detection and curve representation using cubic B-splines. In: *Proc. IEEE Int.Conf. Rob. and Autom,* Washington, USA, vol.2, pp. 764-769.

Pavlidis, T., Wolberg, G., 1986. An algorithm for the segmentation of bilevel images. In: *Proc. CVPR'86, IEEE Comput. Soc. Conf. Comput. Vision and Pattern. Recogn.,* Washington, USA, pp. 570-575.

Pham, S., 1986. Digital straight segments. *Computer Vision, Graphics, and Image Processing,* 36(1), pp. 10-30.

Pisarevsky, A. N., et al., 1988. *Machine Vision Systems,* A.N. Pisarevsky and A.F. Chernyavsky, eds. Mashinostroenie, Leningrad (in Russian).

Pratt, W. K., 1978. *Digital Image Processing,* John Willey and Sons, New York.

Suveg, I., Vosselman, G., 2002. Automatic 3D building reconstruction. *Proc. SPIE,* 4661, pp. 59-69.

Warl, F. M., 1983. A new distance mapping and its use for shape measurement on binary patterns. *Computer Vision, Graphics and Image Processing,* 23(2), pp. 218-226.