ASSESSMENT OF A DEVELOPED COMBINED POINT/LINE–BASED PROJECTIVE EQUATIONS

H. F. Barakat [a], H. A. Emam [b], M. S. Abdel-Wahab [b]

[a] Public Works Department, Faculty of Engineering, Cairo University, Giza, Egypt - hhbarakat@hotmail.com
[b] Survey Research Institute, National Water Research Center, Talbia, El Ahram Street, Giza, Egypt – hany_a_emam@hotmail.com, engmohamed25@hotmail.com

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ABSTRACT:

This paper presents the results of the assessment of the developed techniques for line-based and combined point/line-based eight parameters projective equations. The developed equations were implemented in a general least squares adjustment technique where both lines and/or points can be used for the rectification of both aerial and satellite images.
Several experiments were performed using the developed line-based and combined point/line-based techniques to rectify the various types of satellite images and also aerial photographs. The experiments always started first with rectification of aerial photographs before applying to satellite imagery, since the developed projective equations were derived based on frame geometry assumptions. Then, the experiments were performed using several types of satellite imagery, such as Landsat7, SPOT4, IRS-1D, and IKONOS. In all experiments, comparisons were performed between line-based and the commonly used point-based projective equations. It was found that mostly similar results were obtained from both techniques. However, with points, usually a large number of control points are needed to achieve accurate results (around 1-2 pixel resolution). Approximately, around 30 control points are needed for most cases of satellite images. On the contrary, with lines, only 5 control lines are needed to achieve the same accuracy figures of 1-2 pixels. This can be justified since control lines can be considered as sequence of control points, and thus a line will provide much more control information than a point.
The above conclusion presented a great reduction in the number of control features required for satellite image rectification. It provides a great save in the field collection process and, thus, a more economic technique for mapping from satellite imagery. In addition, the combined point/line-based developed technique will provide the flexibility, and thus speed, needed for the satellite images rectification process through the use of any available control features. Finally, conclusions and recommendations for future research were summarized.

1. INTRODUCTION

The use of satellite imagery for mapping applications has seen great interest in recent years, especially with the availability of high-resolution satellite images of IKONOS and QuickBird. Therefore, the need for using fast, economic, and, obviously, accurate mathematical models for satellite images rectification has also increased.

Generally, photogrammetic methodology, and in particular image rectification, has been based primarily on point features. Recently, line-based techniques have seen increased interest. This is because straight lines are usually easier than points to extract from digital imagery using automatic algorithms. Also, the use of straight lines makes it possible to use different line segments on the various overlapping images and consequently, eliminates the need for exact correspondence as in the case of points. In addition, straight lines are generally more available than points in imagery of human infrastructure. The inclusion of linear features, alone or in a combination with point features, into photogrammetic reduction algorithms requires careful development and analysis.

In a previous work by one of the authors (Barakat 1997; Barakat et al 1995), new development of line-based projective equations and combined point/line-based projective equations were introduced. Mathematical formulations of the developed equations are presented in section 2 in this paper, along with the classical point-based projective equations, for the sake of completeness of the paper. In addition, least square adjustment programs were developed using Mathlab version 6 for the point-based, line-based, and combined point/line-based projective equations.

The well-established point-based eight parameters projective equations have been commonly used for image rectification process. In this paper, the newly developed techniques of line-based and combined point/line-based projective equations are used for the image rectification as well. Comparative analysis of the results of the three techniques was also discussed.

In section3, results from several experiment using aerial photograph and satellite images are provided for all three techniques of point-based, line-based, and combined point/line-based projective equations. Various sources of low, medium, and high resolution satellite images (Landsat7, SPOT4, IRS-ID, and IKONOS) were used to test the performance of the developed techniques, especially concerning the effect of using straight lines for the rectification of those time-dependant sensor imagery. Ground control features (points and/or lines) were obtained using either Global Positioning System (GPS) measurements or Egyptian Survey Authority maps of scale 1:50,000 and 1:25,000.

Finally, conclusion is drawn regarding the performance of the developed techniques, and recommendations made for future research.

2. MATHEMATICAL MODELS

2.1 Point-Based Projective Equations

In the case of plane-to-plane central projection, the 8-parameter projective equations are used to relate the coordinates of a point in one plane to the coordinates of its projection in the second plane. Using points, only two independent projective equations can be constructed as follows,

$$x' = \frac{e_1 x + f_1 y + g_1}{e_o x + f_o y + 1} \qquad (1)$$
$$y' = \frac{e_2 + f_2 y + g_2}{e_o x + f_o y + 1}$$

Where x', y' = point coordinates in the first plane,
x, y = point coordinates in the second plane,
$(e_o, f_o ......, g_2)$ = eight projective parameters.

The eight parameters are estimated using a minimum of four points with known coordinates in both systems, provided that no three of which are collinear. Then, for any additional point, its unknown coordinates on the first plane can be computed using its known coordinates on the second plane, the two projective equations and the estimated eight parameters.

The equations demonstrate the non-linear nature of the projective transformation. Therefore, a least squares adjustment program was developed in order to uniquely estimate the eight parameters $(e_o, f_o ......, g_2)$ and to calculate the coordinates of any additional point.

2.2 Line-Based Projective Equations

A very important concept in projective geometry is the principle of duality. In the projective plane, points and lines are said to be dual. Any theorem applying to points also applies to lines and vice versa. By this principle, centrally projected straight lines may be used to establish the projective equations. The line equation using Euclidean coordinates is given in the following form,

$$a_i x + b_i y + 1 = 0 \qquad (2)$$

Where a_i, b_i = line parameters.

Equations (1) are the classical projective equations based on point correspondence. A comparable pair of projective equations for two protectively related straight lines is given by,

$$a' = \frac{r_1 a + s_1 b + t_1}{r_o a + s_o b + 1},$$
$$b' = \frac{r_2 a + s_2 b + t_2}{r_o a + s_o b + 1} \qquad (3)$$

Where a', b', a, b are the line parameters in the first and second plane, respectively,
$(r_o, s_o, ...t_1, t_2)$ are the eight projective parameters.

This equation is derived using equations (1) and (2). Solving the 3 equations (two of (1) and one of (2)) simultaneously, equation (3) is obtained using a simple algebraic manipulation.

The relationships between the line projective parameters $(r_o, s_o, ...t_1, t_2)$ and the point projective parameters $(e_o, f_o ......, g_2)$ are presented in subsection 2.3.Since the equations are nonlinear, a least squares adjustment program was developed to solve for the eight parameters using a minimum of 4 straight lines with known line parameters in both planes, then the parameters of any additional line can be computed, similar to the point case.

2.3 Combined Point/Line-Based Projective Equations

Equations (1) and (3) can be used together in the same least squares adjustment to effect projectivity between two planes based on the combination of points (1) and lines (3). However, in this case, two sets of 8-parameters, $(e_o, f_o ......, g_2)$ and $(r_o, s_o, ...t_1, t_2)$ need to be carried in the adjustment. Since there are only 8 independent projective equations in a two-plane projectivity, the following 8 constraint equations must be carried in the adjustment,

$$r_1 = (f_2 - f_o g_2)/(e_1 f_2 - e_2 f_1)$$
$$s_1 = (e_o g_2 - e_2)/(e_1 f_2 - e_2 f_1)$$
$$t_1 = (e_2 f_0 - e_0 f_2)/(e_1 f_2 - e_2 f_1)$$
$$r_2 = (f_o g_1 - f_1)/(e_1 f_2 - e_2 f_1) \qquad (4)$$
$$s_2 = (e_1 - e_o g_1)/(e_1 f_2 - e_2 f_1)$$
$$t_2 = (e_o f_1 - e_1 f_o)/(e_1 f_2 - e_2 f_1)$$
$$r_o = (e_2 g_1 - e_1 g_2)/(e_1 f_2 - e_2 f_1)$$
$$s_o = (e_2 g_1 - e_1 g_2)/(e_1 f_2 - e_2 f_1)$$

These constraint equations are derived from the solution of the equations (1) and (3) simultaneously as mentioned earlier. A least squares adjustment solution with constraints was developed to implement the above equations.

3. EXPERIMENTS AND ANALYSIS

Several experiments were carried out to test and study the performance of the above derived equations regarding the image rectification process that is the process of transforming the image data from one grid system into another grid system using mathematical models such as (affine, polynomial, projective…).

The point-, line-, and combined point/line-based projective equations are used as the mathematical models of the image rectification in order to perform the conversion/transformation from the image coordinate system into the ground reference coordinate system. Rectifying image data involves the identification and locating of well distributed ground control points/lines, computation and estimation of projective parameters, creation of an output rectified image, and, then, examination using some check points (points with known ground coordinates which are not used in the estimation of the projective parameters).

In the following experiments, several image data sources were used. Aerial photographs were used first to check the performance of the developed programs, especially for the newly derived projective equations for lines and combined points/lines solutions, and to compare their results to the well-established point-based projective equations. Then, various satellite image data of LANDSAT7, SPOT4, IRS-ID, and IKONOS were used to investigate the applicability of the developed programs using points, lines, and combined points/lines and to test the accuracy of the developed program using point-based projective equations by comparing its results to two well-known commercial software programs; Erdas Imagine version. 8.5 and Intergraph (Z/I Imaging) I/RASC version 8.4 as shown in the following subsections.

3.1 Aerial Photography

It was essential to use aerial photographs first, before applying to satellite imagery, since the projective equations are derived based on frame geometry. An aerial photograph of a 1:4500 scale for an urbanized city center was used in this experiment (Figure 1). A number of 5 control points, and 5 check points were selected to be used with the point-based projective equations program. Then, 5 control lines, and the same 5 check points were used with the line-based projective equations program. Finally, 3 control lines, 2 control points and the same 5 check points were used with the combined point\line program. Table 1 shows the different Root Mean Square (RMS) results of the 5 check points when using the projective equations of point, line, and combined point/line. The results show a superior performance of the line and combined point/line solutions over the point-based solution for this case of only 5 control points.

Figure 1. Aerial Photograph of an Urban City Center with Control (blue) and Check (red) Features Distribution

RMS	Point-based Program	Line-based Program	Combined Point/Line
X (cm)	4.28	2.38	2.44
Y (cm)	4.17	2.27	2.18

Table 1. RMS Results of 5 Check Points for Aerial Photography

3.2 LANDSAT7

A LANDSAT7 satellite image was used in this experiment. The panchromatic band, with its 15-m resolution, was selected (Figure 2). The image covers an area around Lake Nasr in the southern part of Egypt.

In the first experiment, the developed program for point-based projective equations was used with various number of control points in order to achieve the best possible accuracy limits. RMS values of 25 check points are listed in Table 2 where it is clear that 25 control points are needed to achieve the acceptable accuracy of around 1.1 of a pixel.

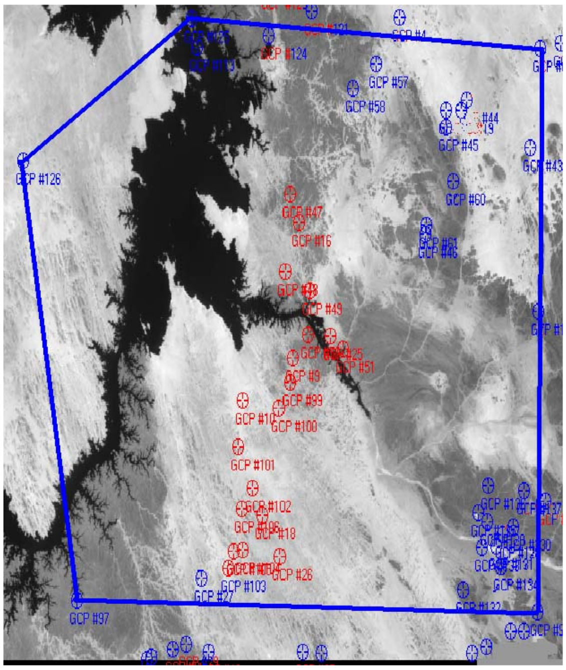

Figure 2. LANDSAT7 Image of Lake Nasr with its Control (blue) and Check (blue) Features Distribution

RMS	5 control points	10 control points	15 control points	20 control points	25 control points
X(m)	120.22	99.65	65.33	26.15	16.68
Y(m)	121.05	99.80	66.03	26.14	16.72

Table 2. RMS Values of 25 Check Points for LANDSAT7 Panchromatic Image Using Point-based Technique with Various Number of Control Points

Next, the point-based projective equations program was compared to that of Erdas Imagine and Z/I Imaging I/RASC. Then, the line-based projective equations program was used with 5 control lines. Finally, the combined point/line-based projective equations program was used with 3 control lines and 2 control points. The RMS results of the same 25 check points are listed in Table 3. It is important to note the equivalence of the results of the 25 control points with only 5 control lines and/or 3 control lines and 2 control points. This clearly shows the importance and potential of using straight lines as control features in the rectification process.

RMS	Erdas Imagine	I/RASC	Point-based	Line-based	Combined Point/Line
X (m)	16.68	16.16	16.68	16.07	16.36
Y (m)	16.69	16.14	16.72	16.53	16.62

Table 3. RMS Values of 25 Check Points for LANDSAT7 Panchromatic Image Using Point, Line, and Combined Point/Line Techniques

3.3 SPOT4

Two SPOT4 images were used in this case. The first is a panchromatic image with 10-m resolution (Figure 3) and the second is multispectral with 20-m resolution (Figure 4).

The panchromatic image covers the East Cairo region. The developed program for point-based projective equations was used first to achieve the best accuracy limits. A number of 30 control points was needed to reach the 1.3 pixel accuracy. RMS values of 25 checkpoints are presented in Table 4.

RMS	5 control points	10 control points	15 control points	20 control points	30 control points
X (m)	110.22	94.65	77.45	38.54	13.36
Y (m)	112.05	96.80	76.99	37.99	13.13

Table 4. RMS Values of 25 Check Points for SPOT4 Panchromatic Image Using Point-based Technique with Various Number of Control Points

The comparison between the developed program of point-based projective equations with that of Erdas imagine and I/RASC is performed next and results are shown in Table 5. Then, the line-based and the combined point/line-based techniques were applied with various number of control features to reach the same accuracy as with 30 control points. Table 5 demonstrate the RMS values of the same 25 check points with only 5 control lines for the line-based technique and 3 control lines & 2-control points for the combined point/line technique.

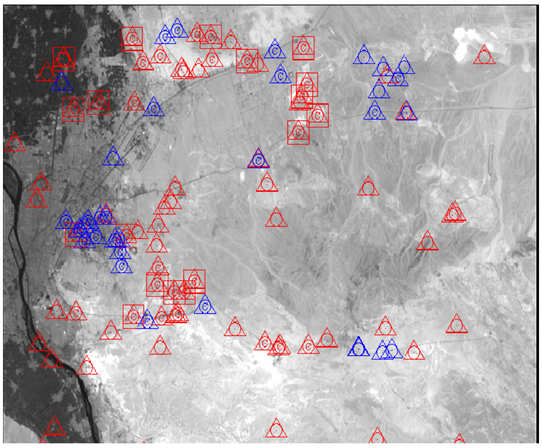

Figure 3. SPOT4 Panchromatic Image over East Cairo with its Control (red) and Check (blue) Points Distribution

RMS	Erdas Imagine	I/RAS C	Point-based	Line-based	Combined Point/Line
X (m)	13.36	13.32	13.36	13.35	13.35
Y (m)	13.14	13.17	13.13	13.17	13.14

Table 5. RMS Values of 25 Check Points for SPOT4 Panchromatic Image Using Point, Line, and Combined Point/Line Techniques

The second set of experiments was carried out using a multispectral image of SPOT4 covering the Rashid region in the northern part of Egypt. As shown in Table 6, which lists the RMS values of 25 check points, a number of 30 control points was needed to reach the best possible accuracy figure of 1 pixel.

The comparison between the developed program for point-based projective equations and those of Erdas Imaging and I/RASC was performed and results confirmed their equivalence as shown in table 7. Then, line-based and combined point/line-based projective equations developed programs were used with only 5 control lines and 3 control lines & 2 control points, respectively. RMS results of the same 25 check points are listed in Table 7. It is clear the overall equality of the results of the lines and combined point/line techniques with those of the point technique; however, using much fewer control features than the needed 30 control points.

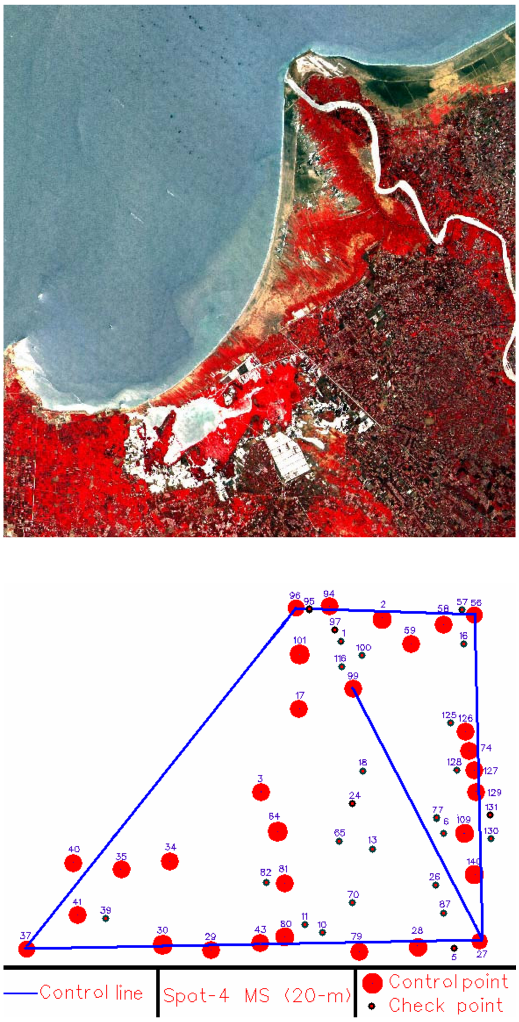

Figure 4. SPOT4 Multispectral Image over Rashid with its Control (red) Features and Check (black) Distribution

RMS	5 control points	10 control points	15 control points	20 control points	30 control points
X(m)	130.22	90.65	64.52	40.54	20.06
Y(m)	129.95	90.80	65.35	40.23	20.71

Table 6. RMS Values of 25 Check Points for SPOT4 Multispectral Image Using Point-based Technique with Various Number of Control Points

RMS	Erdas Imagine	I/RASC	Point-based	Line-based	Combined Point/Line
X m)	20.06	19.87	20.06	19.81	19.96
Y(m)	20.73	20.29	20.71	21.04	20.80

Table 7. RMS Values of 25 Check Points for SPOT4 Multispectral Image Using Point, Line, and Combined Point/Line Techniques

3.4 IRS-1D

An IRS-1D (Indian Remote Sensing satellite) panchromatic image with 5.8 m resolution was used in this experiment. The image covered the Toshka area in the southern west part of Egypt (Figure 5).

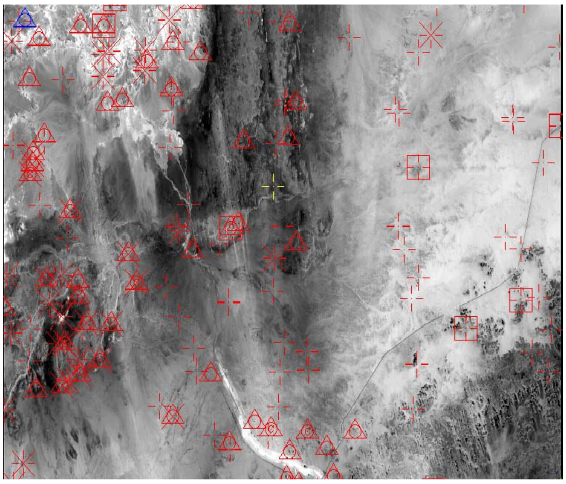

Figure 5. IRS-1D Panchromatic Image over Toshka with its Control and Check Points Distribution

The developed program for the point-based projective equations was applied first with different number of control points. The best possible accuracy of about 1.4 pixels was achieved using 27 control points. RMS values of 25 checkpoints are presented in Table 8.

RMS	5 control points	10 control points	15 control points	20 control points	27 control points
X (m)	94.65	64.87	45.95	28.64	8.43
Y (m)	95.78	64.08	46.10	27.90	7.50

Table 8. RMS Values of 25 Check Points for IRS-1D Panchromatic Image Using Point-based Technique with Various Number of Control Points

Next, the results of the developed program for point-based projective equations were compared with those obtained from I/RASC using 27 control points, as shown in Table 9, to confirm the accuracy of the developed program. Then, line-based and combined point/line-based developed programs were applied to check the required minimum number of control features to obtain the best possible accuracy figures. As shown in Table 9, only 5 control lines in the line-based projective equations and 3 control lines & 2 control points in the combined point/line projective equations, were needed to achieve the equivalent RMS values of the same 25 check points.

RMS	I/RASC	Point-based	Line-based	Combined Point/Line
X (m)	8.36	8.43	8.30	8.35
Y (m)	7.55	7.50	7.87	7.66

Table 9. RMS Values of 25 Check Points for IRS-1D Panchromatic Image Using Point, Line, and Combined Point/Line Techniques

3.5 IKONOS

The final satellite imagery experiment was performed on the high-resolution satellite; IKONOS. A panchromatic IKONOS image, of 1-m resolution, over the East Cairo region, was used in this experiment (Figure 6).

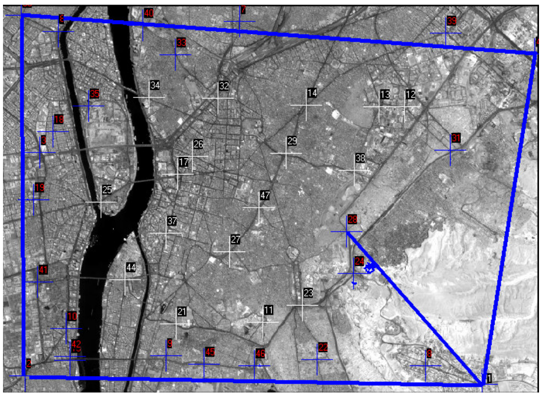

Figure 6. IKONOS Panchromatic Image over East Cairo with its Control (blue) and Check (white) Features Distribution

The developed program for the point-based projective equations was used to rectify the image using different number of control points. A number of 27 control points were the minimum required to achieve the best possible accuracy of about 1.7 pixels for this type of high-resolution image. RMS values of 25 checkpoints are presented in Table 10.

RMS	5 control points	10 control points	15 control points	20 control points	27 control points
X (m)	32.22	20.65	15.65	8.62	1.79
Y (m)	32.58	21.00	15.00	8.45	1.65

Table 10. RMS Values of 25 Check Points for IKONOS Panchromatic Image Using Point-based Technique with Various Number of Control Points

Next, comparison between the developed program for point-based projective equations and that of I/RASC was performed. Results of the same 25 check points are summarized in Table 11. It confirms the accuracy of the developed program.

Then, the developed programs for line-based and combined point/line-based projective equations were used with only 5 control lines and 3 control lines & 2 control points, respectively. RMS values of the same 25 check points are also presented in Table 11. It is clear that results of the line-based and combined point/line-based techniques are equivalent to those of point-based; however, with much fewer control features. This proves the potential of using such line techniques for the rectification of high-resolution imagery, and, thus, reducing tremendously the amount and expenses of fieldwork needed.

RMS	I/RASC	Point-based	Line-based	Combined Point/Line
X (m)	1.86	1.79	1.68	1.76
Y (m)	1.76	1.65	1.56	1.66

Table 11. RMS Values of 25 Check Points for IKONOS Panchromatic Image Using Point, Line, and Combined Point/Line Techniques

4. CONCLUSIONS AND RECOMENDATIONS

1- Linear image features are significant source of information to facilitate image rectification process since they are abundant in human-made infrastructure, and are amenable to automatic feature extraction.

2- Projective equations can be based on corresponding point features, corresponding straight lines features or combination of point/line features.

3- Least squares adjustment techniques were developed for point, line, and combined point/ line–based projective equations.

4- The developed techniques were tested and analyzed by performing several experiments using aerial photograph and satellite imagery with various resolutions (LANDSAT7 panchromatic (15m), SPOT4 panchromatic (10m) and mutlispectral (20m), IRS-ID panchromatic (5.8m), and IKONOS panchromatic (1m)).

5- Results of the point-based projective equations technique were compared to those obtained from two well-known commercial software packages; Erdas Imagine version 8.5 and Intergraph (Z/I Imaging) I/RASC version 8.4, and they proved equivalent. This confirmed the accuracy of the developed program.

6- Experimental results of the line-based and combined point/line-based projective equations techniques were equivalent to those of the point-based projective equations. However, only 5 control lines (for the line-based technique) and 3 control line and 2 control points (for the combined point/line-based technique) were used to achieve those results, while an average of 25-30 control points were used to achieve the same results (for the point-based technique). This is very important to realize since it proves that, with line-based or combined point/line-based techniques, the number of control features can be reduced tremendously and still yield the same accuracy figures. This will lead to the reduction in the amount of field survey work and, thus, its cost and duration.

7- Experimental results show the potential of using linear features for the rectification of the time-dependent satellite imagery. Study of specific geometry configuration of the line features is important.

Research is continuing on the following:

1- Use of geometric constraints between various linear features. This will provide substantial information in support of photogrammetic restitution and image rectification process.

2- Develop affine and polynomial equations (with different orders) using straight lines features.

3- Develop mathematical models (affine, polynomial, and projective) using linear features in general and not only straight lines, such as conic sections (e.g., a circle on the ground and its projection as an ellipse on the image).

4- Development of line-based and combined point/line-based for other photogrammetic equations/conditions and mathematical models for image rectification.

5- Experimentation to study the effects of various configurations of control lines (vertical, horizontal, parallel, etc), and combination of control points and lines on the performance of the developed technique.

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