GROSS ERROR DETECTION OF CONTROL POINTS WITH DIRECT ANALYTICAL METHOD

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ABSTRACT:

The paper demonstrates a new adjustment method together with a very effective gross-error detection in the space resection and in the on-line aerial triangulation.
The basic concept is well known for more than one hundred years, the only problem was that the algorithm is very complex and the implementation was very time consuming without modern computers. Now this barrier doesn't exist any more.
The adjusted value of the unknowns can be derived from the weighted mean value of solutions gained from minimally necessary number of control points and it is done in every combination. For example if we have five control points to solve the space resection, we can solve this task grouping three control points in every combination, and by this way we can calculate the orientation elements in ten different combinations. In this case the adjusted value of the unknowns will be the weighted mean value following the Jacobian Mean Theorem.
Why is it necessary to follow this way? Because we can pick up very effectively the points with a value of gross-error and it can be done before getting the adjusted value, which is a very remarkable issue comparing with the least square method or the robust estimators.
In practice, the first task is to determine the outer orientation elements, for this we usually use the collinear equations, which needs initial values of unknowns and an iteration process. In this paper I present a direct solution to solve the space resection in photogrammetry using geometrical considerations on the basis of three control points. The method doesn't need initial values and iterations, however, it is proved, that using only three control points more than one solutions are probable. To get a unique solution we need no less, than four control points. In this case we should do the resection in all possible combination and the differences gained from every solution can be adjusted using the Jacobian Mean Theorem and in parallel the procedure of the gross error detection can be done as well. I will illustrate by an example of calculation to check the validity of the presented method.

1. INTRODUCTION

1.1 Aims

To solve the space resection for one stereopair means a very important topic in photogrammetry since after this we can start to determine new ground points or together with a reliable stereo-correlation method even we can build DTM models.
The usual start point to solve the space resection is the use of collinear equations:

$$x = -c_k \frac{r_{11}(X - X_O) + r_{21}(Y - Y_O) + r_{31}(Z - Z_O)}{r_{13}(X - X_O) + r_{23}(X - X_O) + r_{33}(X - X_O)} \qquad (1)$$
$$y = -c_k \frac{r_{12}(X - X_O) + r_{22}(Y - Y_O) + r_{32}(Z - Z_O)}{r_{13}(X - X_O) + r_{23}(X - X_O) + r_{33}(X - X_O)}$$

where
x, y : image coordinates reduced to the principal point
X, Y, Z : ground coordinates
X_O, Y_O, Z_O : coordinates of projection center
r_{ij} : elements of rotation matrix,
where every $r_{ij} = f(\varphi, \omega, \kappa)$
c_k : focal length

After the Taylor linearization with the iterative solution we can face the following problems:

- It's no possible in every case to give approximate values of unknowns with such an accuracy which is enough to have a convergent iterative process (especially at terrestrial photogrammetry)
- To detect the points with gross errors is not so easy if there are several points with gross errors. The least square method distributes the gross errors to other points (even to good ones); and the robust estimators can become uncertain if the number of points with gross error is more than one.

This paper gives an alternative solution to avoid the above mentioned problems.

1.2 The procedure

First I give a direct solution for the space resection based on three control points (chapter 2.1). Than I explain how to get the adjusted values of unknowns using the Jacobian Mean Theorem (Gleinsvik, 1967), if the number of control points is more than three (chapter 2.2). And finally I present the basic formulas necessary to detect the points with gross error (chapter 2.3).

2. DIRECT ANALYTICAL METHODS

2.1 Space resection without adjustment

As it is seen on Figure 1. we have a tetrahedron with a,b,c sides. The ABC triangle is known and formed by the control points, The $A'B'C'P$ tetrahedron including the α,β,γ is also known after the measurement of image points. The goal is to determine the outer orientation elements (φ,ω,κ and X_O,Y_O,Z_O). It is wise to first calculate only the projection center coordinates and after this the rotation angles can be calculated with well known direct equations.

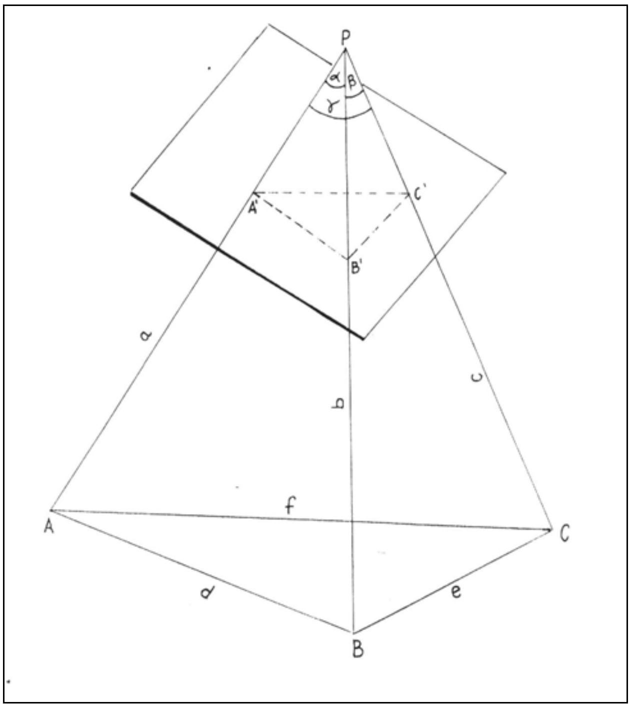

Figure 1. Space resection based on three control points

Let's derive equations for the a,b,c sides, since using these values we can calculate the P projection center coordinates using the well known distance equations from the coordinate geometry.
If we take the a side as a basic distance the sides of b and c will differ only with an n scale factor, so in this case we have only two unknowns (a and n).
We can setup three independent equations for the triangles ΔABC, ΔBCP and ΔACP using the cosine-theorem:

$$\begin{aligned} d^2 &= a^2 + (a\cdot n)^2 - 2a^2 n\cos\alpha \\ e^2 &= (a\cdot n)^2 + (a\cdot m)^2 - 2a^2 nm\cos\beta \\ f^2 &= (a\cdot m)^2 + a^2 - 2a^2 m\cos\gamma \end{aligned} \tag{2}$$

We can eliminate the side a from the equations reducing the three equations into one forth-degree equation (Jancso, 1994):

$$W_1 n^4 + W_2 n^3 + W_3 n^2 + W_4 n + W_5 = 0 \tag{3}$$

where $W_i = f(d,e,f,\alpha,\beta,\gamma)$

After solving the equation (3) we can calculate the unknown sides:

$$\begin{aligned} a &= +\sqrt{\frac{d^2}{1+n^2-2n\cos\alpha}} \\ b &= a\cdot n \\ c &= a\cdot m = \frac{e^2 - f^2 + a^2 - b^2}{2(a\cos\gamma - b\cos\beta)} \end{aligned} \tag{4}$$

Now we can calculate the projection center coordinates using the distance equations:

$$\begin{aligned} a^2 &= (X_P - X_A)^2 + (Y_P - Y_A)^2 + (Z_P - Z_A)^2 \\ b^2 &= (X_P - X_B)^2 + (Y_P - Y_B)^2 + (Z_P - Z_B)^2 \\ c^2 &= (X_P - X_C)^2 + (Y_P - Y_C)^2 + (Z_P - Z_C)^2 \end{aligned} \tag{5}$$

The solution of (5) is :

$$\begin{aligned} Z_P &= \frac{-v_a \pm\sqrt{v_a^2 - 4u_a w_a}}{2u_a} = \frac{-v_b \pm\sqrt{v_b^2 - 4u_b w_b}}{2u_b} = \frac{-v_c \pm\sqrt{v_c^2 - 4u_c w_c}}{2u_c};\ Z_P > 0 \\ X_P &= k_1 - k_2 Z_P \\ Y_P &= k_3 - k_4 Z_P \end{aligned} \tag{6}$$

where $u_{a,b,c} v_{a,b,c}, w_{a,b,c}, k_{1-4}$ parameters are functions of coordinates of the A,B,C control points and the sides of a,b,c (Jancso, 1994).

Finally we can calculate the rotation angles of φ,ω,κ from the rotation matrix with the well-known direct equations (Hirvonen, 1964):

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{7}$$

and

$$\begin{aligned} \varphi &= -\mathrm{arctg}(r_{31}/r_{33}) \\ \omega &= \arcsin(r_{32}) \\ \kappa &= -\mathrm{arctg}(r_{12}/r_{22}) \end{aligned} \tag{8}$$

2.2 Space resection with adjustment

If we have more than three control points the space resection should be solved with an adjustment. We adjust only the projection center coordinates. The rotation angles can be calculated separately in one step at the end. Let's list the steps of the adjustment procedure.

STEP 1. Let the number of control points be n . We will group them by three in every possible combination and for each group we solve the space resection directly (formulas 2-6). In general case, at each group we will get more than one solutions for the projection center.

STEP 2. From each group we choose common solutions, it means we choose those solutions where the sum of square differences is minimal.

STEP 3. By the error propagation law we calculate the M_{y_i} covariance matrices for each solution by the following formula:

$$M_{y_i} = F^T{}_i M_x F_i \tag{9}$$

where

M_x : covariance matrix of control points considered at the geodetic measurements.

The F^T dispersion matrix can not be derived directly with partial derivation from the equations 3-6, so we construct this matrix from the differences gained from the original solution and from solutions where each image coordinate is incremented with a small value one by one. Finally we got the F^T matrix with the dimensions of 3x6, this matrix well approximates the matrix containing the partial derivatives.

STEP 4. We determine the weight matrices for each solution by the following equation:

$$P_i = Q^{-1}{}_i = c^2 \left(M_{y_i}\right)^{-1} \tag{10}$$

where c is a scalar factor, at the space resection we take its value as 1/1000.

STEP 5. We calculate the adjusted values of unknowns by the Jacobian Mean Theorem as follows:

$$X = \left(\sum P_i\right)^{-1} \times \sum \left(P_i L_i\right) \tag{11}$$
$$X = Q_{xx} \times \sum \left(P_i L_i\right)$$

where

$$L_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} \tag{12}$$

and it contains the solutions from each group.

2.3 Gross error detection

During the adjustment we can detect the control points with gross errors. A gross error can exist in the ground coordinates or in the image coordinates. By the following procedure we can detect them affectively and no matter where the gross error is.
Let's group four control points in every combination and solve the space resection with adjustment (11). Since we made the space resection in every possible combination, our duty now to determine which group has a gross error and finally we can conclude exactly which point or points caused the gross error. For example if we have 5 control points, we can group them by four as follows:

$$\{1,2,3,4\} \quad \{1,2,3,5\} \quad \{1,2,4,5\} \quad \{1,3,4,5\} \quad \{2,3,4,5\}$$

Let's suppose that the control point No. 1 has a gross error, it means that the first four solutions will be wrong and only the $\{2,3,4,5\}$ group gives a good solution. So, by this logic we conclude that only the point No. 1 can be the cause for a gross error. A similar logic can be proved when the number of points are more than 5 or the number of points with gross-error is more than one. The only limitation for the detection is that finally at least four error-free control points should remain (otherwise no reason to do the space resection with adjustment).
Here is the procedure which helps to decide whether a space resection made by four control points has a gross error or not:

STEP 1. After getting the adjusted projection center we can calculate the residuals by the following:

$$V_i = X - L_i \text{, where } V_i = \begin{bmatrix} v_{x_i} \\ v_{y_i} \\ v_{z_i} \end{bmatrix} \tag{13}$$

STEP 2. After this we can calculate the m_0 weight unit error (14) and the RMS for each unknown (15) with the help of the Q_{xx} covariance matrix:

$$m_0 = \sqrt{\frac{\sum \left(V_i^T P_i V_i\right)}{3n-3}} \tag{14}$$

where n means the number of control points.

$$\left.\begin{aligned} m_X &= m_0 \cdot \sqrt{q_{xx}^{1,1}} \\ m_Y &= m_0 \cdot \sqrt{q_{xx}^{2,2}} \\ m_Z &= m_0 \cdot \sqrt{q_{xx}^{3,3}} \end{aligned}\right\} \tag{15}$$

STEP 3. The errors of (15) can be estimated before the adjustment by the following formulas:

$$\tilde{m}_X = \sqrt{\frac{\sum\left(M_{y_i}^{1,1} \times P_i^{1,1}\right)}{\sum P_i^{1,1}}}$$
$$\tilde{m}_Y = \sqrt{\frac{\sum\left(M_{y_i}^{2,2} \times P_i^{2,2}\right)}{\sum P_i^{2,2}}} \quad (16)$$
$$\tilde{m}_Z = \sqrt{\frac{\sum\left(M_{y_i}^{3,3} \times P_i^{3,3}\right)}{\sum P_i^{3,3}}}$$

STEP 4. The space resection is free from gross errors if

$$m_X \le \tilde{m}_X$$
$$m_Y \le \tilde{m}_Y \quad (17)$$
$$m_Z \le \tilde{m}_Z$$

Otherwise we should setup a null- hypothesis to compare two RMS values (Detrekoi, 1991):

$$H_0 : \sigma_1 = 3.3\sigma_2 \quad (18)$$

At $p = 0.95\%$ probality level with rank of freedom equal to 3, we get the statistical value as $F_{0.95(3,3)} = 9.28$. Let's consider it as a theoretical value and we symbolise it with F_t. On the other hand the value F can be calculated by the following equations for each coordinate:

$$F_{sz_X} = \frac{m_X^2}{\tilde{m}_X^2} \cdot \frac{1}{3.3^2}$$
$$F_{sz_Y} = \frac{m_Y^2}{\tilde{m}_Y^2} \cdot \frac{1}{3.3^2} \quad (19)$$
$$F_{sz_Z} = \frac{m_Z^2}{\tilde{m}_Z^2} \cdot \frac{1}{3.3^2}$$

It means the space resection has no gross-error if the following equations will be fulfilled together:

$$F_{sz_X} \le F_t$$
$$F_{sz_Y} \le F_t \quad (20)$$
$$F_{sz_Z} \le F_t$$

Otherwise, we can deny the null-hypothesis and we should consider a gross-error in the space resection.

3. CONCLUSIONS

3.1 Space resection

Regarding the procedure of (1)-(6) we can notice that more than one solutions are probable for the projection center. If we have only three control points the maximally possible number of solutions is 8. Hence we get the tetrahedron sides from a forth-degree equation (3) and the equations of (6) will double it. Of course we will eliminate the complex and negative solutions, but still in this case we can get more than one solutions. So, to have a unique solution we need at least four control points, but in this case we should do the resection with an adjustment.

3.2 Gross error detection

The gross-error can be detected by formulas of (15)-(20) and even we can tell exactly which coordinate has a gross error. See the example in Appendix I.

It still needs more investigation to determine the exact F^T matrix from a real partial derivation (9), which probably results more accurate and better based gross-error detection from theoretical point of view.

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APPENDIX A. EXAMPLE CALCULATION

Let's go through an example where four control points are and the point No. 11 has a real gross-error in Y:

```
Ck=75.00
Number of points: 4

No     x           y              X          Y          Z
-----------------------------------------------------------
11  -14.99085   71.32913      0.200  1401.000      0.200
12   40.44218   71.30058    550.000  1400.000      3.000
27  -14.34352  -68.94081      0.200     0.200      0.200
28   40.35546  -68.87416    550.000     0.200      6.000
```

Wrong point: 11 $\varepsilon_Y = +1.0\ \mathrm{m}$ (a real gross-error)

Table 1 Dataset of control points

From the equations of (3)-(6) we get:

	11-12-27	11-12-28	11-27-28	12-27-28
x1	44.513645613	1.063867943	0.998597778	0.9386478218
x2	-19.64260979	0.682547252	-0.869298619	0.3995205349
x3	1.2413725904	0.020988643	0.013778544	-0.3670483527
x4	1.0637556915	0.049113251	0.008824024	-0.9758124510
Xp	-10.713	141.295	140.207	140.000
Xp	-10.649	148.910	156.005	155.818
Xp	141.509	560.135	-10.547	560.226
Xp	149.141	560.191	-10.417	560.343
Yp	1401.968	700.581	699.525	700.000
Yp	1401.968	697.374	699.525	696.787
Yp	700.036	1402.183	-0.752	-1.986
Yp	700.036	1402.159	-0.752	-2.010
Zp	6.416	750.711	750.515	750.000
Zp	-6.127	-745.827	-746.994	749.476
Zp	750.271	8.537	6.249	11.693
Zp	-748.393	-2.443	-6.075	0.533
n	44.513646	1.063868	0.998598	0.938648
n	44.513646	1.063868	0.998598	0.938648
n	1.063756	0.020989	0.008824	0.399521
n	1.063756	0.020989	0.008824	0.399521
m	111.284608	1.062497	1.061676	0.998044
m	111.284608	1.062497	1.061676	0.998044
m	0.999264	2.503617	0.399876	0.008492
m	0.999264	2.503617	0.399876	0.008492

Table 2. Solutions from every combination

where

x1,x2,x3,x4 : roots of the equation, gained by the equation (3)
Xp,Yp,Zp : projection center
n,m : scalar factors

After this we chose the common solutions:

```
      1 xp=   141.509 yp=   700.036 zp=   750.271
      2 xp=   141.295 yp=   700.581 zp=   750.711
      3 xp=   140.207 yp=   699.525 zp=   750.515
      4 xp=   140.000 yp=   700.000 zp=   750.000
```

Table 3. Common solutions

Now, let's determine the F dispersion matrix for each solution incrementing each x,y image coordinates with a small value (see Table 4.).

```
Incremental value: 0.005 mm

1
  0.0130   0.0742  -0.0136  -0.0640   0.0130   0.0742
  0.1460   0.0026  -0.1237   0.0710   0.1460   0.0026
  0.0024   0.0133   0.0025   0.0121   0.0024   0.0133

 2
 -0.0370   0.0635   0.0367  -0.0926  -0.0370   0.0635
  0.1396   0.0293  -0.1372   0.0505   0.1396   0.0293
 -0.0203   0.0347   0.0067  -0.0166  -0.0203   0.0347

 3
  0.0501   0.0104   0.0133  -0.0765   0.0501   0.0104
  0.0226  -0.0232  -0.1478   0.0018   0.0226  -0.0232
 -0.0046   0.0253   0.0023  -0.0138  -0.0046   0.0253

 4
  0.0502  -0.0288  -0.0372  -0.0659   0.0502  -0.0288
  0.0029  -0.0297  -0.1413   0.0294   0.0029  -0.0297
  0.0141   0.0183  -0.0205  -0.0362   0.0141   0.0183
```

Table 4. The F matrices in each group

To each solution we can calculate now the weight matrices by the equation (10):

```
1
 0.0020    0.0001   -0.0048
 0.0001    0.0004   -0.0012
-0.0048   -0.0012    0.0629
2
 0.0068    0.0006   -0.0138
 0.0006    0.0005   -0.0005
-0.0138   -0.0005    0.0362
3
 0.0024   -0.0001   -0.0020
-0.0001    0.0011    0.0014
-0.0020    0.0014    0.0196
4
 0.0035   -0.0007   -0.0042
-0.0007    0.0012    0.0005
-0.0042    0.0005    0.0140
```

Table 5. Weight matrices for each group

By the equation (11) the adjusted coordinates of the projection center are:
xp= 140.549 m
yp= 700.226 m
zp= 750.301 m

By the equation (14) we got mo = 0.026. Applying the equations (15) the RMS values for each coordinate are:
mx= 0.262 m
my= 0.465 m
mz= 0.087 m

We gain the estimated RMS values before the adjustment by the formulas (16) as follows:
mx~= 0.025 m
my~= 0.037 m
mz~= 0.008 m

Setting up the null-hypothesis by the formula (18) we got the following statistical value: Ft=9.28.

Also we calculate the F values by (19) for each coordinate and compare them with Ft:
Fx= 9.52 > Ft
Fy= 13.79 > Ft
Fz= 10.08 > Ft

The null-hypothesis is not fulfilled, so we can declare that the space resection has a gross-error.