

# CREATION OF DISTORTION MODEL FOR DIGITAL CAMERA (DMDC) BASED ON 2D DLT

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**Abstract:**

It is once again presented and discussed in this paper that the method of distortion calibration based on 2D Direct Linear Transformation (DLT) is easy and feasible. It is emphasized that it is fit for the creation of distortion model for solid-state cameras (CCD, CID, PSD) to compensate for the system errors of various image points. The application of the distortion model presented in this paper is convenient and reliable.

**KEYWORDS:** digital, distortion, model, CCD, calibration, error

**Introduction:**

It is important for professionals in photogrammetry to do the work of calibration for system errors of various image points including diversified distortion errors. In the solutions of single-image spatial resection, direct linear transformation and self-calibration correction for bundle adjustment, it is able to compute the parameters of the system error model, such as the factors of various distortions  $k_1, k_2, \dots, p_1, p_2$  or additional parameters in self-calibration correction, while computing the interior and exterior orientation elements. The Analytical plumb-line calibration method by Brown D.C.[S] can be used as well only to compute the above mentioned distortion coefficients.

The author proposed a simple method to detect the optical distortion in 1988, of which the object is the 120 camera using film. The digital camera is widely used in the present time. The flatness of CCD chips can reach  $\pm 1\mu m$  without the problem of “negative film distortion”. It is of great importance and possibility to measure the optical distortion of CCD camera in advance. This paper proposes again the method of distortion error calibration correction based on 2D DLT. It does not compute the parameter values. Instead, it directly computes the system error values caused by various factors (mainly optical distortion) on each pixel on the CCD chips. A series of digital distortion models can be measured with different focuses in an easy and

convenient way. Thus, the method is quite fit for the zooming digital cameras with severe distortion in the condition of close-range photographing.

**1. Optical Distortion Calibration based on 2D DLT**

**1.1 2D DLT and its characteristics**

The expression for 3D spatial DLT is as follows:

There are eleven factors in the expression, which respectively has a strict mathematic relational expression with the interior orientation factor, exterior orientation factor, scale inconsistent factor  $ds$  and non-orthogonal factor  $d\beta$  (11 factors altogether).

When the object to be measured is 2D and Z is a constant, the expression (1) can be simplified as the 2D DLT relational expression (2) after inferring, deduction and simplifying symbols.

$$\left. \begin{aligned} x + \frac{l_1 X + l_2 Y + l_3 Z + l_4}{l_9 X + l_{10} Y + l_{11} Z + 1} &= 0 \\ y + \frac{l_5 X + l_6 Y + l_7 Z + l_8}{l_9 X + l_{10} Y + l_{11} Z + 1} &= 0 \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} x + \frac{l_1^* X + l_2^* Y + l_3^*}{l_7^* X + l_8^* Y + 1} &= 0 \\ y + \frac{l_4^* X + l_5^* Y + l_6^*}{l_7^* X + l_8^* Y + 1} &= 0 \end{aligned} \right\} \quad (2)$$

With the deduction of the above expression, the relation between coefficients of 2D  $l^*$  and of 3D  $l$  can be got . A

matrix of  $L^*$  can be defined as follows.

$$L^* = \begin{bmatrix} l_1^* & l_2^* & l_3^* \\ l_4^* & l_5^* & l_6^* \\ l_7^* & l_8^* & 1 \end{bmatrix} = \frac{1}{l_{11}Z+1} \begin{bmatrix} l_1 & l_2 & l_3Z+l_4 \\ l_5 & l_6 & l_7Z+l_8 \\ l_9 & l_{10} & l_{11}Z+1 \end{bmatrix} \quad (3)$$

When  $Z=0$ , the expression would be:

$$L^* = \begin{bmatrix} l_1^* & l_2^* & l_3^* \\ l_4^* & l_5^* & l_6^* \\ l_7^* & l_8^* & 1 \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_4 \\ l_5 & l_6 & l_8 \\ l_9 & l_{10} & 1 \end{bmatrix}$$

$$= \frac{1}{\gamma_3} \begin{bmatrix} f_x & -f_x \tan \beta & -x_0 \\ 0 & \frac{f_x}{(1+d)\cos \beta} & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & b_1 & \gamma_1 \\ a_2 & b_2 & \gamma_2 \\ a_3 & b_3 & \gamma_3 \end{bmatrix} \quad (4)$$

where  $\gamma_3 = -(a_3 X_s + b_3 Y_s + c_3 Z_s)$ .

As a result, the meaning of coefficients  $l^*$  in 2D transformation and its relationship with coefficients  $l$  in 3D transformation is as follows.

$$\left. \begin{aligned} l_1^* &= \frac{1}{\gamma_3} (a_1 f_x - a_2 f_x \tan \beta - a_3 x_0) = l_1 \\ l_2^* &= \frac{1}{\gamma_3} (b_1 f_x - b_2 f_x \tan \beta - b_3 x_0) = l_2 \\ l_3^* &= \frac{1}{\gamma_3} (\gamma_1 f_x - \gamma_2 f_x \tan \beta - \gamma_3 x_0) = l_4 \\ l_4^* &= \frac{1}{\gamma_3} \left( \frac{a_2 f_x}{(1+d)\cos \beta} - a_3 y_0 \right) = l_5 \\ l_5^* &= \frac{1}{\gamma_3} \left( \frac{b_2 f_x}{(1+d)\cos \beta} - b_3 y_0 \right) = l_6 \\ l_6^* &= \frac{1}{\gamma_3} \left( \frac{\gamma_2 f_x}{(1+d)\cos \beta} - \gamma_3 y_0 \right) = l_8 \\ l_7^* &= \frac{a_3}{\gamma_3} = l_9 \\ l_8^* &= \frac{b_3}{\gamma_3} = l_{10} \end{aligned} \right\} \quad (5)$$

The function of the above deduction lies in two ways:

- The 2D DLT establishes a strict projection relational expression between the object space 2D surface and film surface (digital camera chip surface). It

establishes a relational expression for two surfaces that need no image interior orientation elements and fiducial marks. When using this expression, the image space coordinate can use any point as the origin and any direction as the axe. Theoretically speaking, it is not required that the image surface be paralleled with the object space 2D surface.

- There will be no effect on the distortion error calibration whether x axe and y axe is perpendicular or not ( $d\beta$ ), or whether the chip pixel is square or not ( $ds$ ).

Suppose  $ds=0$ ,  $d\beta=0$ ,  $\varphi=\omega=k=0$ ,  $f_x=f$ , the geometric meaning of each  $l^*$  factor would be:

$$L^* = \begin{bmatrix} l_1^* & l_2^* & l_3^* \\ l_4^* & l_5^* & l_6^* \\ l_7^* & l_8^* & 1 \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_4 \\ l_5 & l_6 & l_8 \\ l_9 & l_{10} & 1 \end{bmatrix}$$

$$= \frac{1}{Z_s} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & X_s \\ 0 & 1 & Y_s \\ 0 & 0 & Z_s \end{bmatrix}$$

$$= \frac{1}{Z_s} \begin{bmatrix} f & 0 & fX_s \\ 0 & f & fY_s \\ 0 & 0 & Z_s \end{bmatrix} = \begin{bmatrix} \frac{1}{m} & 0 & \frac{X_s}{m} \\ 0 & \frac{1}{m} & \frac{Y_s}{m} \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Thus, in the above mentioned conditions,  $l_1^*$  and  $l_5^*$  are the image scales;  $l_2^*$ ,  $l_4^*$ ,  $l_7^*$  and  $l_8^*$  are the decimal values;  $l_3^*$  is the expression value for  $X_s$  according to the scale, while  $l_6^*$  is the expression value for  $Y_s$  according to the scale. Select a suitable object spatial coordinate system D-XYZ, when  $(X_s, Y_s)$  is decimal,  $l_3^*$  and  $l_6^*$  would be decimal too.

## 1.2 Methodology to establish digital camera distortion models

By using control points (A, B, C, D), the self-created plane control network E and digital camera chip surface P may realize the 2D DLT. After getting factor 1, the ideal coordinates  $(x_i, y_i)$  of other control points on the chip

can be obtained. The difference between image point measured coordinate  $(x_i, y_i)$  and their ideal coordinates  $(\bar{x}_i, \bar{y}_i)$  is the total of all system errors caused by imaging. Proved by several experiments, the distribution regularity of the systematic error including its value and direction, is in accordance with the distribution regularity of optical radial distortion.

The surface degree of CCD chips may maintain at the level of  $\pm 1 \mu m$ . The object space plane control network is produced on the AV10 TAB plotting table by Leica Company, Swiss. Aided by the computer plotting functions, a 1.1m×0.9m control grid is created on the 0.1mm thick polystyrene film. The interval of the grid is 5cm. There are 399 black solid round symbols with a diameter of 15mm on the intersection of grids. Thus, the network might have enough intensity, as Figure 1 shows. With regards to experiences, the central error of each symbol is no more than  $\pm 0.05mm$ . With the similar method, similar plane control networks can be created on the 8mm thick adhesive backing glass.

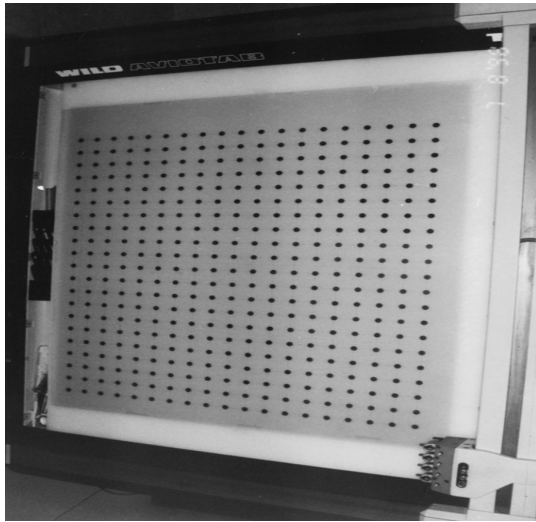


Fig.1 2D control field

The Nikon E2 (1280×1000 pixels, size  $10 \mu m$ ) and Kodak DCS 4800 (2160×1440 pixels) digital cameras are used as the test camera to implement “vertical photography” for the control network. Select four points that are close to the image edge as the control points. Establish the projection

relationship between the oblique image and plane control network and work out the coefficients l.

Set the measured coordinates of symbols as  $(x_i, y_i)$ . The ideal coordinates would be  $(\bar{x}_i, \bar{y}_i)$  aided by factor l. Their difference  $(\Delta x_i, \Delta y_i)$  is caused by various optical distortion errors  $(\Delta x_i, \Delta y_i)$ .

$$\left. \begin{aligned} \Delta x_i &= x_i - \bar{x}_i \\ \Delta y_i &= y_i - \bar{y}_i \end{aligned} \right\} \quad (6)$$

Where  $(\bar{x}_i, \bar{y}_i)$  is the theoretical value, while  $(\Delta x_i, \Delta y_i)$  is the distortion error.

Presented method does not touch any relationship between the physical factors of the error. The physical factors include radial distortion and decentring distortion in objective-lens optics, radial distortion coefficients changing with the focuses, radial distortion change of the object point that is not on the focus, and other geometric imaging errors caused by electronic and optical factors, and etc. Take the test done with Kodak DCS 4800 as an example. Rectified according to four corner angle points (001,015,136,150), the image point distortion error  $\Delta r$  of each control point is shown as Figure 3. Its size and direction is in accordance with the general deduction of distortion error of radial distortion. The error vector directs to the center of the image frame (it should directs to PPA: Principal point of autocollimation). Moreover, it is easy to analyze that the distortion of the whole image presents as from the image edge to the center, when the distortion of the four corner points is not zero.

A primary model of distortion error  $\Delta r$  can be created with these data, as Figure 4 shows. The model is created by the distortion error  $\Delta r$  of the 150 points on the digital camera chip. The unit of each direction (x, y,  $\Delta r$ ) is pixel. The primary model on direction x is shown as Figure 5, while the model on direction y is shown as Figure 6.

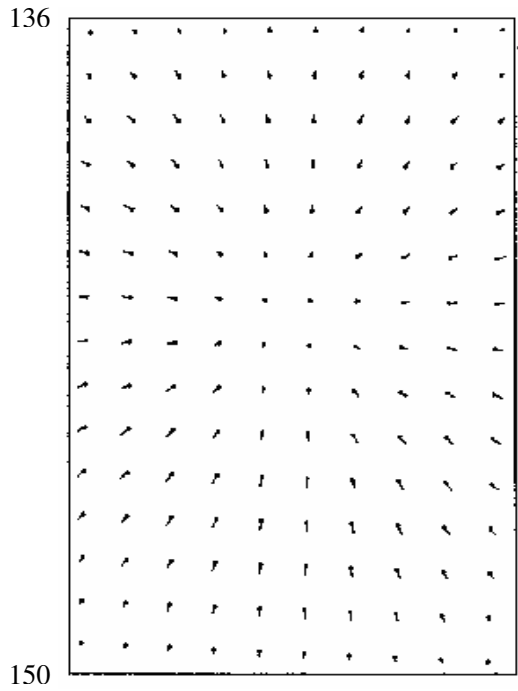


Fig.3 A distortions distributed chart on the CCD



Fig.4 Distortion primary model

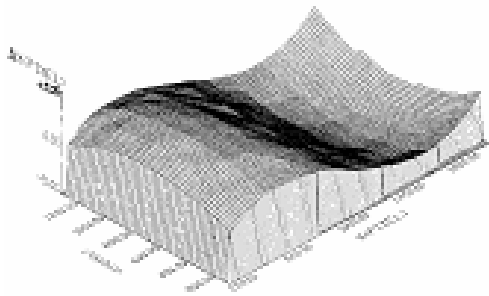


Fig.5. Distortion primary model for X



Fig.6. Distortion primary model for Y

By using ordinary camera calibration methods, the

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observation value is limited for the distortion factors (such as  $k_1, k_2, p_1, p_2$ ). The relationship between the unknown factors will have an effect on the measure quality of the distortion coefficients. However, with the method discussed in this paper, the measure result directly reflects the real status of distortion, because only imaging system error values are to be measured with several hundreds of observation values. For real applications, according to the known distortion error values on the “dispersed points”, establish distortion superior model by interpolating and computing the distortion of all pixel positions; or by temporarily interpolating the distortion of a certain image point. The interpolation principle of the two methods is consistent. The required storage capacity should at least be equal to the resolution of the chip, i.e., several G or tens of G, in correspondence with the distortion superior models on each focus plane. The method of temporary interpolation is of more benefit.

With regards to the interpolation of the distortion error of a certain focus plane, theoretically speaking, the computation should be executed by the following expression:

$$\left. \begin{aligned} \Delta x &= k_0 x + k_1 x r^2 + k_2 x r^4 + \dots \\ \Delta y &= k_0 y + k_1 y r^2 + k_2 y r^4 + \dots \end{aligned} \right\} \quad (8)$$

In addition, referred to the creation process of DEM, the interpolation method can be selected.

## 2. Conclusion

- 1) The distortion error calibration method based on 2D DLT proposed in this paper, is a method to calibrate distortion errors, like the analytical plumb-line calibration method by D.C.Brown. This method has no relationship with any physical meanings. There are no measured distortion error coefficients. Instead, the errors on each pixel are measured directly. Those errors including various optical distortion errors and other kinds. Compared with measured optical distortion factors (such as  $k_1, k_2, \dots, p_1, p_2, \dots$ ) method, this one seems to be more practical. The analytical plumb-line calibration method is more suitable for cameras with films as the carrier. This method is more suitable for digital camera.
- 2) This method is fit for CCD cameras with no square pixels.
- 3) The technical key is to establish accurate plane control network and accurately and automatically extract the

round symbol center.

- 4) This method is easy and rapid. It can measure the errors for different groups with different focuses. Considering the effect of the changing focuses on distortion, it is specially fit for the close-range camera calibration. It takes about five minutes to establish a group of models.
- 5) The investment and space occupation to establish a plane control network is smaller than to establish a plumb-line control network. And it is easier for maintenance.

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