# 3D MODELING AND REPRESENTATION OF "IDEAL CITY" PAINTED BY PIERO DELLA FRANCESCA 

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#### Abstract

: Recently, digital archives for cultural heritage or arts and crafts has received attention from possibility that people can appreciate or experience the archaeological objects, the past space or arts and crafts through the computer at any time and without going to the museum or the spot. In particular, 3 D reconstruction and visualization of the lost objects or past word using antique maps or pictures have been receiving more attention since reconstructed 3D world gives important information for understanding those days. In these circumstances, 3D reconstruction for picture was investigated using "Ideal City" painted by Piero della Francesca at 1475 in this paper.


## 1. INTRODUCTION

3D reconstruction and visualization of the lost objects or past word using antique maps or pictures have been receiving more attention since reconstructed 3D world gives important information for understanding those days.
From these viewpoint, 3D reconstruction of historical city Kawagoe where locate in Saitama Japan, and Palatitsa palace where was the residence of great Alexander's father were investigated using antique maps and surveying maps by the authors (Sakamoto \& Chikatsu, 2002, Suzuki \& Chikatsu, 2003).

Similarly, 3D reconstruction of painted world as picture is interesting issue. With this motive, 3D reconstruction "Ideal City" painted by Piero della Francesca at 1475 was investigated. Ideal City was painted as ideal urban in those days using perspective geometry. 3D reconstruction using photogrammetric geometry, and visualization and landscape animation for the "Ideal City" are demonstrated in this paper.

## 2. PERSPECTIVE PROJECTION METHOD

Piero della Francesca was famous Italian painter during Renaissance period but also he was a famous painter based on perspective geometry. Figure 1 shows "Ideal City" which was painted based on the perspective projection.
The perspective projection is basic technique for expressing 3dimensional object in 2-dimensional plane, and the method which was established in Italy in the 15th century is used in CG or architectural Perth. Figure 2 shows the concept of the perspective projection, and the basic points for the perspective projection are as follows. The scale and the object position are changed by the distance and the angle from the eye point, and the horizontal line is drawn as the straight line to show the height of the eye point. Moreover, even if the position of the eye point is changed, the horizontal line is drawn in the height of the eye point.
There are some perspective method depend on number of the eye points, but the "Ideal city" painted by the one point perspective projection, and Figure 2 shows also the concept of the one point perspective projection.


Figure 1. "Ideal City"


Figure 2. Schematic of perspective projection
Where $E$ shows eye point, $V C$ shows the view center, $L H$ shows the horizon line, $P P$ shows the painting plane, and $G P$ shows the ground plane.

The basic points for the one point perspective projection are as follows. The depth is expressed toward the center in the picture, and the straight line toward the center that extends from right to leave intersects at one point. This point becomes the vanishing point in the painting, and lines which join to the same height point make the straight line always that expresses the depth faces the vanishing point.
The origin point in ground coordinate system are defined at the eye point, similarly origin point in photo coordinate system are defined at the vanishing point as shown in Figure 2.

## 3. ACQUISITION OF 3D INFORMATION

As previously stated, the one point perspective projection expresses geometrical space in 2 -dimension plane. In order to acquire 3-dimentional information from 2-dimensional plane, perspective geometry should be considered, and detail procedures are as follows.

### 3.1 Estimation of vanishing point

In order to acquire depth information from the eye point to an object space, vanishing point which is able to detect the point where the extensions of the lines concentrate should be detect. The vanishing point of the "Ideal City" was detected on the front door of the shrine painted at the center as shown in Figure 3-1.
The vanishing point is defined as origin point in photoho coordinate system as mentioned above, and origin point means principal point in photogrammetry.


Figure 3-1. Estimation of vanishing point

### 3.2 Estimation of focal length

In order to compute the actual depth distance to the object is necessary to estimate distance ( $f$ ) from the eye point to the painting plane. Painting plane and the distance $f$ are correspond to focal plane and focal length in photogrammetry respectively. Figure 3-2 shows relationship between focal length $f$ and angle $\theta$, and these relationships is expressed as Eq. (1) using $w$ in figure 3-2.

$$
\begin{equation*}
f=\frac{w}{\tan \theta} \tag{1}
\end{equation*}
$$

The size of the "Ideal City" is 2000 mm in width and 600 mm in vertical, then $w$ becomes 1000 mm . Furthermore, let assume angular field of view (2 $\theta$ ) equal 60 under Cone's theory, $f$ is computed from Eq. (1) as 1733 mm .


Figure 3-2. Relationship between focal length and field angle

### 3.3 Estimation of Depth Distance

As the next step, it is necessary to estimate the depth distances from the eye point to objects. Then, let assume $L$ is ground distance in a real space; and $l$ is photo distance on the painting in figure 3-3, the depth distance $Z$ is computed by Eq.(2).
In order to compute the depth distance, the width of the door at the center of picture was assumed as 6 ft , and the photo distance for the width is measured as 38 mm . Therefore, the depth distance is computed as 83.4 m using Eq.(2), and the plane include the door was defined as datum plane in photogrammetry in this paper. Then, it can be said that the depth distance 83.4 m is flying height above datum plane in photogrammetry.

$$
\begin{equation*}
Z=\frac{f \cdot L}{l} \tag{2}
\end{equation*}
$$



Figure 3-3. Outline of depth distance

### 3.4 Estimation of Shrine Size

Utilizing photo and ground distance of the height and width for the front entrance gate, the depth distance to the center of both left and right side entrance gates are computed as figure 3-4, and the depth distance to the center of the shrine is estimated as the depth distance to the center of both left and right side entrance gates. As the results, diameter of the shrine is estimated as 35.4 m .
On the other hand, in order to compute the height of shrine, photo coordinate for the center of the shrine are requested. Therefore, oval general form are used to compute the photo coordinate for the center of the shrine since the circle on a real space is expressed as an oval on the picture, and a quadrangle diagonal intersection that encloses the oval become center of a circle.
However, an oval center is not projected to center of a circle in the perspective projection. Then, in order to compute a center of circle using oval general form, major axis (a), minor axis (b), and center coordinates $\left(o_{l}\right)$ are requested. The diagonal intersection means a point projected as center of circle $\left(o_{2}\right)$ in figure $3-5$, and 28.2 m is computed as the height of the shrine. Furthermore, height of the eye point is computed as 3.3 m from the difference of photo coordinates in y direction, and exposure position is also estimated as $(0,0,0)$.


Figure 3-4. Depth distance to the center of the shrine


Figure 3-5. Relationship between oval and circle center

### 3.5 3D Coordinate of the Objects

Let assume the photo and ground coordinates for two points which locate on the vanishing line, $a\left(x_{a}, y_{a}\right), b\left(x_{b}, y_{b}\right)$. And $A$ $\left(X_{A}, Y_{A}, Z_{A}\right)$ and $B\left(X_{B}, Y_{B}, Z_{B}\right)$ respectively in Figure 3-6. The depth distance to the plane including point $A$ is computed by Eq.(2) since the horizontal line have the same value with the height of eye point, therefore $X_{A}$ and $Y_{A}$ is computed by Eqs (3).


Figure 3-6. Photo coordinates and 3D coordinate

$$
\begin{align*}
& X_{A}=\frac{x_{a}}{f}\left(Z_{A}-Z_{0}\right)+X_{0}  \tag{3}\\
& Y_{A}=\frac{y_{a}}{f}\left(Z_{A}-Z_{0}\right)+Y_{0}
\end{align*}
$$

Where, ( $X_{0}, Y_{0}, Z_{0}$ ) is 3D coordinate of the eye point (exposure station), and $x_{a}, y_{a}$ is photo coordinate for the vanishing point.

$$
\begin{equation*}
Z_{B}=\frac{y_{a}}{y_{b}} Z_{A} \tag{4}
\end{equation*}
$$

On the other hand, take into account that the $Y$ coordinate on the same vanishing line have equal value, and $Z_{0}$ equal 0 in this paper, following equation is obtained from Eqs.(4), and $X, Y$ coordinate for the point $B$ are also computed by Eqs.(3). Similarly, 3D coordinate for the other objects are computed by the same procedures, and the height of right and left buildings are about 20 m . Figure 3-7 shows the plane figure for the " Ideal City".


Figure 3-7. Plane figure for the "Ideal City"


Figure 3-8. Reconstructed 3D model

### 3.6 3D Modelling

3D modelling for the "Ideal City" is performed by using 3D CG software (3D stadio max), Figure 3-8 is reconstructed 3D model from the eye point position, and it can be seen that the same picture is reconstructed compare with Figure 1.

Figure 3-9, 3-10 are reconstructed 3D model from right and left side view points with the height of a man's eye level.


Figure 3-9. 3D model from the left side


Figure 3-10. 3D model from the right side

## 4. CONCLUSION AND FURTHER WORK

3D modelling from picture was investigated using "Ideal City" which was painted by the one point perspective projection in this paper, and 3D modelling was performed using only basic geometry in photogrammetry. However, in order to reconstruct accurate 3D model, exact information such as some height or width for objects are necessary.
It should be noted that 3D model was reconstructed using only the width for the front door under the assumption that the width of the door is 6 ft , and the eye position ( 3.3 m height), size of the shrine ( 35.4 m diameter, 28.2 m height) and the size other objects were estimated.
Piero della Francesca was famous painter based on perspective geometry. In particular, it is supposed that his painting technique from the human's eye position developed perspective painting during Renaissance period. However, it is estimated that the height of eye position is 3.3 m . It may be concluded that more detail considerations are needed as a further works, but it is concluded that 3D modelling procedure investigated in this paper will promise development of 3-dimentional visualization from 2-dimentional pictures.

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