A THREE-DIMENSIONAL ERROR MODEL OF GRID DEM
BASED ON THE RECONSTRUCTED CONTOURS

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ABSTRACT:

In this paper, a three-dimensional error model of grid digital elevation model (DEM) is proposed based on the reconstructed contours. Though analyzing the construction of three-dimensional error model, the paper disassembles the three-dimensional error to a series of triangular pyramids and quadrangular pyramids, and it can make the calculation simpler. The three-dimensional error is continuous and unique, and the proposed model can evaluate the accuracy of raster DEM totally. The model can avoid the nonuniqueness in the root mean square of error (RMSE) where different checking groups have different results. The proposed approach to evaluate raster DEM is more reasonable and more reliability than to evaluate by RMSE. At last, the proposed model is validated and analyzed by the some experiments.

1. INTRODUCTION

The digital elevation model (DEM) is widely used in remote sensing, geographical information system (GIS), and virtual reality and so on. Evaluating the accuracy of DEM is an essential issue for spatial data, particularly for the applications that require high accuracy of DEM. Therefore, it is very important and necessary to develop the effective tools to evaluate the accuracy of DEM.

The accuracy of DEM is subject to many factors, such as the accuracy of source data, human errors due to sampling and interpolating, and errors in the mathematical model used for interpolating the DEM surface. For the accuracy of DEM, there were a few studies. Kumler (1994) developed a method to study the cause of errors in DEM compiled by different approaches. Gao (1997) studied the resolution and accuracy of terrain representation by regular grid DEM at a micro-scale. Rees (2000) studied the accuracy of DEM interpolated to higher resolutions and showed that simple bilinear or cubic convection was an appropriate approach for DEM applications. Li et al. (2000) systematically discussed the theory of DEM, especially the models of accuracy analysis, and deduced the accuracy model of the linear models based on raster data. Zhu et al. (2002) simplified the DEM data by multi-band wavelet and studied the accuracy. Hu et al. (2002) studied the accuracy of DEM from the aspect of function approximation. Kidner (2003) argued that higher-order techniques of interpolation were always more accurate than the general bilinear interpolation. Shi et al. (2005) obtained the formulae of error propagation in terms of the mean elevation error of DEM constructed by both biquadrate and cubic interpolations. It is concluded that the biquadrate and cubic interpolation algorithms are more accurate than the bilinear interpolation in terms of their total error including both the propagation error and the model error of the

generated DEM surface. Zhu et al. (2005) estimated the mean elevation error in the DEM surface that is linearly constructed from a TIN, and a mathematical formula for the error estimation was derived theoretically. Zhu et al. (2006，2008) proposed an error model of DEM based on the reconstructed contours, and evaluated the accuracy of DEM by the ratio of the area error between the original contours and the reconstructed contours to the length of the original contours.

At present, the main accuracy indicator is the root mean square error (RMSE). But the RMSE depends on the position and the number of checking points. There are different RMSE for different checking data. Hence, it is not credible to evaluate the accuracy of DEM by RMSE. The error model of DEM based on the reconstructed contours is a new accuracy method, which is proposed by Zhu et al. However, some experiments indicates that the DEM error based on the reconstructed contours is decreasing with the increasing of terrain slope, which is inconsistent with the results of RMSE and other experiments. So it is necessary to develop a new effectual and reliable accuracy method.

In this paper, a new accuracy model of DEM based on the reconstructed contour lines is developed by studying the error of grid DEM thoroughly.

2. THE THREE-DIMENSIONAL ERROR MODEL OF GRID DEM BASED ON THE RECONSTRUCTED CONTOURS

In this section, the three-dimensional error model of grid DEM based on the reconstruct contours is defined. Its main idea is that: Firstly, the Grid DEM is built based on the triangulated irregular network (TIN), which corrects the planar triangles

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through adding the terrain feature points. Secondly, the contour lines are reconstructed on the surface of Grid DEM, and are projected on the surface of TIN vertically to generate a series of spatial lines (projected contours). Lastly, we analyze the projected contours, the reconstructed contours and the original contours, and propose a new indicator to evaluate the DEM accuracy. Now, some concepts are defined as follows:

1. Original contour
They are the basic data to build DEM. Supposed they have no error.
2. Reconstructed contour
They are the contours which are interpolated on the surface of Grid DEM to evaluate.
3. Reference projected plane
It is the surface of TIN, which corrects the planar triangles through adding the terrain feature points. Supposed the surface is no error.
4. Reference triangle plane
They are the spatial triangles with different shapes which form the reference projected plane.
5. Projected contour
They are the spatial lines which are generated by projecting the reconstructed contours to the reference projected plane vertically.
6. Projecting elevation point
They are the discrete elevation points on the reconstructed contours.

2.1 The definition of the three-dimensional error model of grid DEM

In Figure 1, the line $ABCD$ is a section of one original contour line, the line $A'B'C'$ is the section of reconstructed contour line corresponding to the section of line $ABCD$, and the line $A''B''C''$ is the section of corresponding projected contour line. Hereinto, the three-dimensional error of reconstructed contour is the volume of the spatial object, which is constructed by the original contour line, the reconstructed contour line and the corresponding projected contour line, marking $\Delta_{Contour}$. Namely:

$$\Delta_{contour} = \Delta_{B-A'A''B''B'} + \Delta_{B''-BB'C} + \Delta_{C-B'B''C''C'} \quad (1)$$
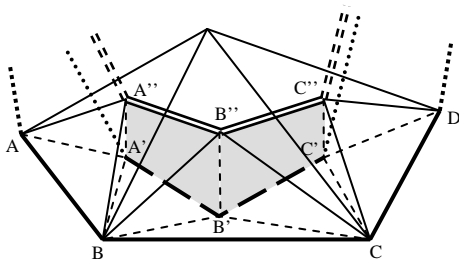

Figure 1. The 3-D error model of DEM

For a DEM built by n contours, we define the three-dimensional error of DEM using formula (1) as follows.

$$\delta = \sqrt{\frac{\sum_{i=1}^{n} \Delta_i}{n}} \quad (2)$$

Where Δ_i = the three-dimensional error of the ith contour
n = the total of the contours

From formula (1), it can be seen that the three-dimensional error not only reflects the matching degree horizontally between a reconstructed contour and its corresponding original contour, but also represents the vertical error. For all contours, the result of formula (2) reflects the matching degree between the whole DEM with the actual terrain. So this model can evaluate the DEM accuracy entirely, and overcome the weakness of RMSE which is that there are different results for different checking groups.

2.2 The calculation of the three-dimensional error of the grid DEM

The reference projected plane is formed by the spatial triangles with different shapes. The difference of the number of the elevation points, which are projected in each triangle, leads to not only the shape of projected contour lines different but also the calculation of three-dimensional error of DEM very complex. By analysis, the relationship between the reference triangle planes and the projecting elevation points can fall into three types.
1. There is only one projecting elevation point in a reference triangle plane;
2. There are more than one projecting elevation points in a reference triangle plane;
3. The adjacent projecting elevation points are projected in the adjacent reference triangle planes.
Now, we analyze the three types of the spatial error of DEM:

Case 1: There is only one projecting elevation point in a reference triangle plane.

In Figure 2, the line $ABCD$ is a section of one original contour line, the line $A'B'C'$ is the section of reconstructed contour line corresponding to the line $ABCD$, and the line $A''B''C''$ is the section of corresponding projected contour line. And then, the three-dimensional error of DEM can be expressed by following formula:

$$\Delta_{DEM} = V_{B-A'A''B''B'} + V_{B''-BB'C} + V_{C-B'B''C''C'} \quad (3)$$

Where $V_{B''-BB'C}$ = the volume of the triangular pyramid, in which the point B'' is the vertex, and the triangle $BB'C$ is the undersurface.

$V_{B-A'A''B''B'}$ = the volume of the quadrangular pyramid, in which the point B is the vertex, and the quadrangle $A'A''B''B'$ is the undersurface.

$V_{C-B'B''C''C'}$ = the volume of the quadrangle pyramid, in which the point C is the vertex, and the quadrangle $B'B''C''C'$ is the undersurface.
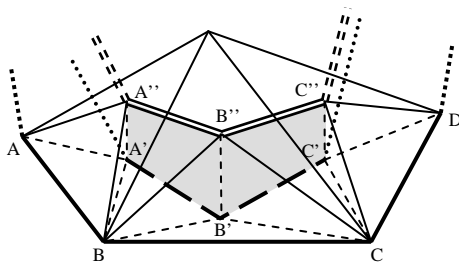

Figure 2. The 3-D error model of DEM in Case 1

Case 2: There are more than one projecting elevation points in a reference triangle plane.

In Figure 3, there is only two projecting elevation point in a reference triangle plane. The line $ABCD$ is a section of one original contour line, the line $A'B'C'D'$ is the section of reconstructed contour line corresponding to the line $ABCD$, and the line $A''B''C''D''$ is the section of corresponding projected contour line. And then, the three- dimensional error of DEM can be expressed by following formula:

$$\Delta_{DEM} = V_{B-A'A''B''B'} + V_{BC-B'B''C''C'} + V_{C-C'C''D''D'} \quad (4)$$

Where $V_{BC-B'B''C''C'}$ = the volume of the spatial object, in which the edge BC is the up-edge, and the quadrangle $B'B''C''C'$ is the undersurface.

$V_{B-A'A''B''B'}$ = the volume of the quadrangular pyramid, in which the point B is the vertex, and the quadrangle $A'A''B''B'$ is the undersurface.

$V_{C-C'C''D''D'}$ = the volume of the quadrangle pyramid, in which the point C is the vertex, and the quadrangle $C'C''D''D'$ is the undersurface.
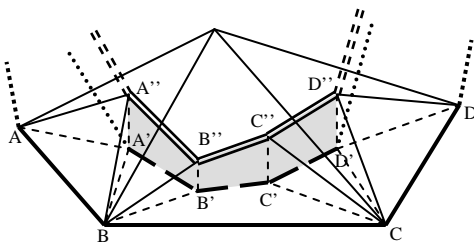

Figure 3. The 3-D error of DEM in Case 2

If there are more than two projecting elevation points in a reference triangle plane, the three- dimensional error of DEM can be expressed by following formula:

$$\Delta_{DEM} = V_{AB-A'B'C'D'D''C''B''A''} \quad (5)$$

Where $V_{AB-A'B'C'D'D''C''B''A''}$ = the volume of the special spatial object, in which the edge AB is the up-edge, and the quadrangle $A'B'C'D'D''C''B''A''$ is the undersurface.
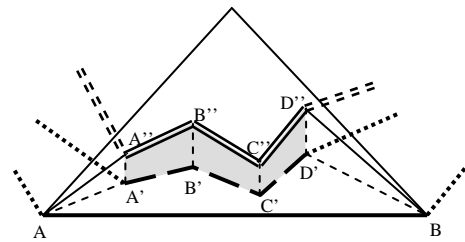

Figure 4. The 3-D error of DEM in Case 2

Because the object $O_{AB-A'B'C'D'D''C''B''A''}$ is too complicated to calculate its volume, it is superseded by a new object $O_{AB-A'A''D''D'}$, which is generated after connecting the starting and end points of the object $O_{AB-A'B'C'D'D''C''B''A''}$, as Figure 5. And then, the three- dimensional error of DEM can be expressed approximately by following formula:

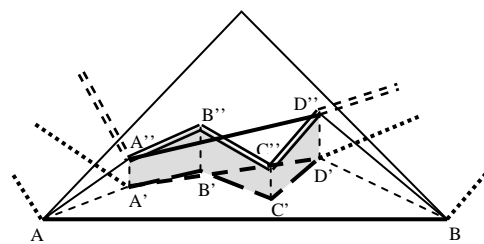$$\Delta_{DEM} = V_{AB-A'A''D''D'} \quad (6)$$

Figure 5. The 3-D error of DEM in Case 2

Case 3: The adjacent projecting elevation points are projected in the adjacent reference triangle planes.

In Figure 6, the line $ABCD$ is a section of one original contour line, the line $EFGH$ is a section of another original contour line adjacent to the contour $ABCD$, the line $A'B'C'$ is the section of reconstructed contour line corresponding to the line $ABCD$, and the line $A''B''C''$ is the section of projected contour line corresponding to the line $A'B'C'D'$. And then, the three- dimensional error of DEM can be expressed by following formula:

$$\Delta_{DEM} = V_{B-A'A''B''B'} + V_{B''-BB'K} + V_{C''-CC'K} + V_{C-C'C''D''D'} \quad (7)$$

Where $V_{B''-BB'K}$ = the volume of the triangular pyramid, in which the point B'' is the vertex, and the triangle $BB'K$ is the undersurface.

$V_{C''-CC'K}$ = the volume of the triangular pyramid, in which the point C'' is the vertex, and the triangle $CC'K$ is the undersurface.

$V_{B-A'A''B''B'}$ = the volume of the quadrangular pyramid, in which the point B is the vertex, and the quadrangle $A'A''B''B'$ is the undersurface.

$V_{C-C'C''D''D'}$ = the volume of the quadrangle pyramid, in which the point C is the vertex, and the quadrangle $C'C''D''D'$ is the undersurface.
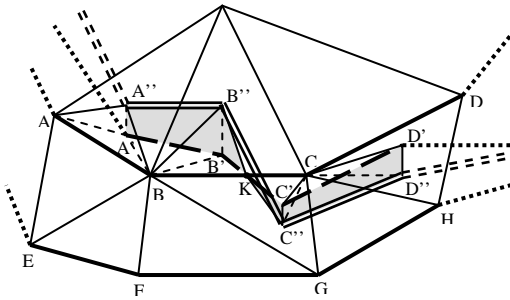

Figure 6. The 3-D error of DEM in Case 3

According to the analysis in the above paragraphs, we can conclude that the calculation of the three-dimensional error of grid DEM could be generalized to the volume calculation of three kinds of spatial objects, which are the triangular pyramid, the quadrangular pyramid and the special spatial object, as Figure 7. And the special spatial object can be disassembled to the triangular pyramid and the quadrangular pyramid, as Figure 8.

The triangular pyramid   The quadrangular pyramid   The special spatial object
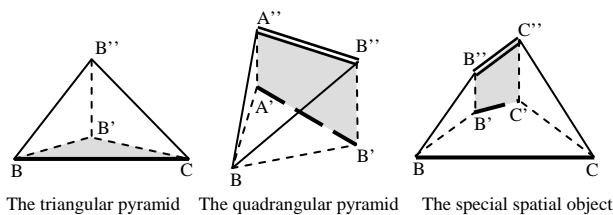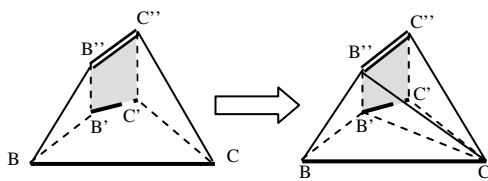
Figure 7. Three kinds of spatial object

Figure 8. The disassembling process of the special spatial object

Therefore, the three-dimensional error of grid DEM can be got by calculating the volume of the triangular pyramid and the quadrangular pyramid.

2.3 The evaluation process of grid DEM accuracy

Based on the proposed idea and method above, the basic steps to evaluate the DEM by the three-dimensional error are as follows:
1. Building the triangulated irregular network (TIN) based on the method in the reference (Renxi Chen, Yi Long, 2003);
2. Interpolating to generate the Grid DEM based on the above TIN and the resample technology;

3. Reconstructing the contour lines on the surface of Grid DEM based on the method in the reference (C. Q. ZHU, Z. W. WANG, 2008);
4. Confirming the one-to-one relationship between the original contours and the reconstructed contours based on the method in the reference (C. Q. ZHU, Z. W. WANG, 2008);
5. Calculating the three-dimensional error of Grid DEM based on the method in the section 2.2;
6. Calculating the accuracy indicator of Grid DEM by the formula in the section 2.1.

3. THE RESULTS AND ANALYSIS OF THE EXPERIMENTS

This paper selects the contour data of a 1:50000 map, in which the contour interval is 20 meters, as the experimental data to construct TIN. Then grids DEMs with 40m, 80m and 120m intervals respectively are interpolated based on the constructed TIN, as Figure 9.
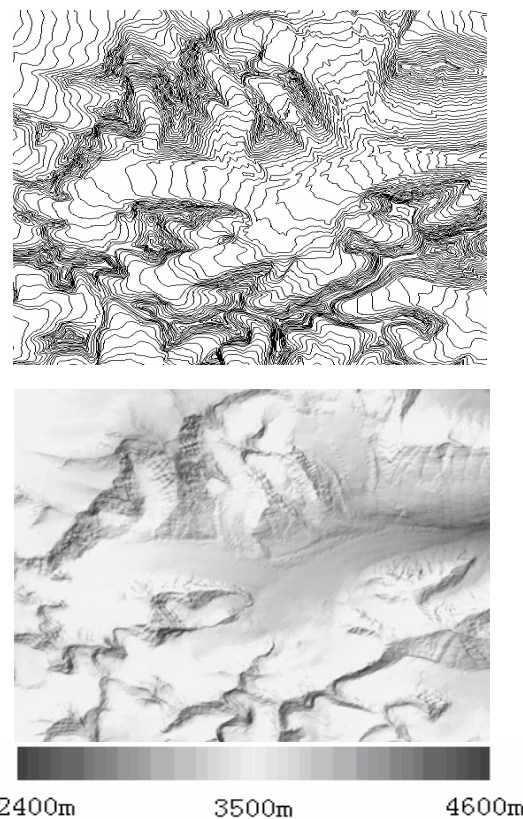

2400m   3500m   4600m

Figure 9. The contour lines and the DEM

With the proposed three-dimensional error model and mean square error model, the accuracies of grid DEMs with different intervals are evaluated. The evaluation results are as following Table 1.

Grid Interval	40 m	80 m	120 m
3-D Error Model (m^3)	1254.36	3654.68	10649.31
RMSE (m)	7.01	11.53	21.29

Table 1. The evaluation results of DEM accuracy

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In order to intuitively display the spatial distribution of the three-dimensional errors of grid DEM in the regions with different slopes, this paper selects the contour data of a local map as the experimental data and prorates the thickness degree of the different subsections of the original contours according to the errors, as Figure 10. The bigger is the error, the thicker is the subsection of contour. And contrarily, the smaller is the error, the thinker is the subsection of contour. At the same time, in order to compare the three-dimensional error with the MSE, the elevation errors, which are more two times than the MSE, are displayed by the circles with different radiuses, as Figure 11, where the centers of the circles are the elevation checking points, and the size of the radius expresses the size of the error. The bigger is the elevation error, the lengthier is the radius.
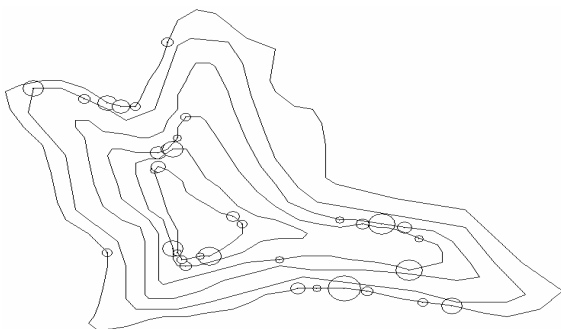

Figure 10. The 3-D error distribution of DEM

Figure 11. The MSE error distribution of DEM

The results of the Table 1 show that the three-dimensional error of DEM is increasing with the increasing of grid intervals, which is consistent with the MSE. This indicates that the proposed three-dimensional error model can evaluate the accuracy of DEMs with different grid intervals effectively. In Figure 10, the three-dimensional errors of DEM in the regions, where the contours are dense, are bigger than that in other regions, where the contours are sparse. In other words, the three-dimensional errors are bigger in the regions with bigger slopes, and they are smaller in other regions with smaller slopes. This indicates that the three-dimensional error accords with the law that the DEM error is increasing with the increasing of slope.

4. CONCLUSION

This paper develops a new approach, a three-dimensional error model, to evaluate the accuracy of DEM, which is based on the original contours, the reconstructed contours and the projected contours. Though analyzing the construction of three-dimensional error model, the paper disassembles the three-dimensional error to a series of triangular pyramids and quadrangular pyramids, and it can make the calculation simpler.

Then this model is tested by some experiments, and the results indicate that this model can evaluate the accuracy of DME exactly.

The proposed three-dimensional error is unique for a DEM, which can overcome the weakness of RMSE where different checking groups have different results. On the other hand, the proposed approach is more reasonable and more effective to evaluate the accuracy of DEM totally than to evaluate by RMSE.

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