# AUTOMATIC GENERATION TRIANGULATED SURFACES WITH NORMAL FAULTS AND REVERSE FAULTS 

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#### Abstract

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In order to deal with the geological faults, a new algorithm of constrained Delaunay triangulation is proposed. The algorithm builds a generic Delaunay TIN from scatter points firstly. After defining coal field boundaries and fault lines as constrained lines, the algorithm embeds the constrained lines into the unconstrained Delaunay TIN and rebuilds triangular net locally according to the constrained rules, and finally implements a constrained Delaunay TIN. The algorithm, validated by various coal geological data with complex geology faults and fault combination, is simple, high-efficient, and useful for automatic creation of coal floor contour map.


## 1. INTRODUCTION

Structural discontinuties, especially normal faults and reverse faults, have been a key problem of automatic generation of coal floor contour map. Irregular triangulated surfaces (TIN), which often used to represent geological stratigraphy and structure, is a good solution for tracing and producing the coal floor contour map. However, the traditional TIN algorithm could not handle the problem of geological structural discontinuities effectively. Especially, as far as the reverse fault is concerned, its hanging wall and footwall have an overlapping region and the control points on the walls are mixed together in this region. The generic TIN algorithm is difficult to resolve the problems.

There are some TIN algorithms to represent stratigraphic surface with the faults. Some algorithms define faults as separate surfaces to the existing stratigraphic surface. For example, 3D modeller GOCAD creates the surface of the fault and geological bodies, and then creates the topological relation between the faults and geological bodies, therefore being able to develop a tool for querying those relationships (Marcus, 2001). A TRICUT program can build outcropping strata and faulted horizons polygonal meshes to intersect each other (Lindenbeck, 2002). Some algorithms divide the research region into several small regions by the faults lines and builds TIN separately with triangle growth method (Mao, 1999). However, the triangle growth method is low- efficient and timeconsuming.

To solve these problems, a new Constrained Delaunay Triangulation algorithm for automatically generating TIN is proposed in this paper.

## 2. RESEARCH APPROACH

There are many algorithms of generating Delaunay triangulation according to the scatter data. Most of them need to considerate the constrained condition. For example, configuration of ground surface contains ridge and valley lines, scarps and islands, and etc. Therefore, the research on Constrained Delaunay Triangulation (CDT) is a hotspot of Delaunay triangulation. (Lee and Lin, 1980; Lee and Schachter, 1980; Boissonnat, 1988; Piegl and Richard, 1993; SloaN, 1987, Floriani, Du Q., 2003) The CDT algorithm could be classified as five categories: Constraint Graph Method, Subdividing Merging Method, Encrypt Method, Shell Triangular Method and Two Steps Method. Two Steps Method, which can build DT and CDT both easily and flexibly, are widely used. The difficulties lie in the efficiency of DT algorithm and the validity of algorithm of embedding the constrained line.

The traditional CDT algorithm could not handle reverse fault effectively. In the new algorithm, we should consider faults, folds, subsiding pillars, erosion areas, and other complex geological conditions. During the process of building TIN, the algorithm deals with geological structure, mainly including normal faults, reverse faults, normal fault cutting, reverse fault cutting, normal and reverse fault cutting, erosion zones, efflorescent oxygenized belts and subsided column. The algorithm defines the structure line and the fault line as constrained line. Unlike the other algorithms that build faults as separate surfaces to cross stratigraphic TIN, the algorithm takes the Two Step Method as the basic method, generates unconstrained Delaunay TIN in the first step, then deletes triangles which intersect with faults, rebuilds local TIN according to several constrain rules in the second step, and finally realizes Constrained Delaunay Triangulation with complicated geological faults.

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## 3. THE NEW CDT ALGORITHM

Normal faults, reverse faults and fault combination are different from genesis and form. The combination type of faults includes graben and horst type, back thrust and ramp thrust type, half graben and domino-style, mono thrust and parallel style, echelon shear type and so on. We mainly study the geological types of coal seam. We found that no matter how complicated the faults and fault combination are, they can be divided into different fault wall area and form many child characteristic polygons. Then, triangles will be reproduced in these polygons with certain rules. This section poses the rational constrained rules, unconstrained TIN creation and local reconstruction TIN algorithm.

### 3.1 Definitions

Defination1: Line Segments can be linked from beginning to end to shape simple polygons. The normal vector of a polygon can distinguish the polygon as "positive polygon" and "negative polygon". If the vertexes are arranged anticlockwise, the polygon is positive; whereas if the vertexes are arranged clockwise, the polygon is negative polygon. In the same way, a triangle can be defined as positive or negative, shown as figure 1. The normal vector of a polygon can be computed from a convex polygon with topological isomorphism.

(a) Positive polygon (b) Positive triangle (c) Negative triangle Figure 1. The definition of a polygon and a triangle

Defination2: From the view of cartography, a fault is composed by two opposite fault lines, which calls the hanging wall line and footwall line. For normal fault, the two fault lines shape a negative polygon, shown as figure 2(a); whereas for reverse fault, two fault lines shape a positive polygon, shown as figure 2(b). As for fault combination, although the fault lines are cut into pieces, the rule is the same, the hanging wall line and footwall line is conjugated, and two lines can shape positive or negative polygon, shown as figure 2(c).


Figure 2. Sketch map of different fault type

### 3.2 Data pretreatment

Boundary line pretreatment: Firstly, the boundaries of research region are defined as constrained polygon. Secondly, the algorithm search the begin node and end node of every line
segment and connect them into polygons. Finally, the new polygon vertexes are rearranged to be anticlockwise to form positive polygons.

Fault line pretreatment: The foot wall and hanging wall are defined as constrain lines. Fault name, fault attribute and fault wall attribute are set to the constrain lines. According to the fault definition, the node order of the normal fault line is adjusted as clockwise direction and the reverse fault line is adjusted anticlockwise direction.

Scatter point extraction: Drill holes, exposed points and the other control points are regarded as the scatter points for Delaunay triangulation. Especially, the end-nodes of each fault line are extracted into the point set. The inclusion analysis judges the relationship of scatter point and boundary line. The topological relationship is recorded in the point attribute.

Topological relationship establishment: the topological relation of adjacency, intersection, overlapping and inclusion of the boundary is established and the inclusion relation between boundary polygon and constrain line is established.

### 3.3 Unconstrained Delaunay Triangulation

After data pretreatement, an unconstrained triangular net is established firstly. According to the separated data to generating the Delaunay TIN, there are many algorithms (Bowyer,1981; Lawson,1981; Watson,1981), the most classical algorithm is Incremental insertion algorithm and empty circumcircle test algorithm. Here, we take Waston's Incremental insertion algorithm to realize the triangulation. In this step, the constrained lines do not joint into TIN, and the scatter data only include the fault line's end-node.

The steps of the algorithm are presented as following:

1) Find the maximal and minimal $X, Y$ coordinate of the data set and define a bounding rectangle. The bounding rectangle is divided into two triangles to be the original Delaunay triangle;
2) Find arbitrary point $P$ from the dataset and insert into the exist Delaunay TIN;
3) Find the triangle which contain point $P$, connect the corresponding point to create the three new triangles;
4) Apply Lawson local optimal algorithm to make the triangle net to be Delaunay TIN.
5) Repeat 2nd-4th step until all of the points are inserted into Delaunay TIN;
6) Delete the triangles which associate with bounding rectangle.

### 3.4 Locally Constrained Delaunay Triangulation

After the unconstrained Delaunay triangulation, we embed the fault constrain lines gradually and reconstruct the local TIN to form the correct spatial relationship of faults. The steps of the algorithm are presented as following:

Step1: Embed the boundary lines into the general Delaunay TIN.

1) Search the triangles which intersect with the boundary lines. And delete the related triangles from the TIN dataset to shape an empty cavity.
2) Rebuild local TIN with constrained polygon. If the new triangle is positive triangle, keep the triangle; if the new triangle is negative triangle, abandon the triangle, as figure 3 shown. Triangle ACB is a lawful triangle for Delaunay TIN.
3) Add the new triangles into Delaunay TIN.


Figure 3. Sketch map of building boundary triangle
Step2: Embed the fault lines into the general Delaunay TIN.

1) Search the triangles which intersect with the fault lines. Configure outline of triangle set into a polygon. Delete the inside triangle and shape an empty cavity.
2) Choose fault lines as constrain lines and embed them into the cavity. The points on the fault line participate in triangulation.
3) Use triangle growing law to build TIN of cavity. Choose an edge with maximal length in the cavity as the beginning edge. Search the optimal extend point to build a new triangle. Searching will not finish until no triangle will be build any more.

During the process of growing the triangle, 2 principles will be taken into account.

Rule 1: The included angle of two growing edge of new triangle is maximized.

Rule 2: When the two vertex of growing triangle locate on a constrained line, only positive triangle is lawful. In figure 4, since the two vertexes of triangle A is on the hanging wall of normal fault and the triangle is positive, triangle A is lawful. However, two vertexes of triangle $B$ is on the footwall of normal fault and triangle is negative, triangle $B$ is unlawful.


Figure 4. The rules of triangulation
4) Add the new triangle net into the triangle chain and adjust the topological relationship.

According to this algorithm, the Delaunay triangulation with faults and fault combination can be easily implemented. Here, we present a detailed explanation on the algorithm. Figure 5 shows a process of a triangulation with a normal fault. Figure 6 shows a process of a triangulation with a reverse fault. And
figure 7 shows a process of a triangulation with fault combination.


Figure 5. The local triangulation of a normal fault


Figure 6. The local triangulation of a reverse fault


Figure 7. The local triangulation of fault combination

In reverse fault, the hanging wall and footwall are overlapping in the same region. The common triangulation algorithm cannot solve the problem efficiently. The new CDT algorithm can solve the TIN with fault, especially reverse fault and can judge and handle the arbitrary shape of boundary polygon and fault lines.

### 3.5 The Applications

Based on the above algorithm, we develop a test container for building the constrained TIN with Visual C++ 2008. Taking drill holes, faults, exposed points, mine field boundaries as basic data, we establish the TIN, shown as figure 8 and figure 9.


Figure 8. The test container for experiment


Figure 9. CDT result including various fault type
Table 1 shows the experiment statistic data using different point number. The configuration of PC is double core 1.8 CPU and 2G RAM. Form the table, we can see that the running time is short and algorithm efficiency is high.

| Point <br> number | Constrained <br> line number | Triangle <br> number | Running <br> time(s) |
| :---: | :---: | :---: | :---: |
| 363 | 20 | 1569 | 0.02 |
| 1382 | 43 | 4628 | 1.30 |
| 2635 | 107 | 9835 | 5.14 |

Table 1. Algorithm performance

## 4. CONCULUSION

The paper has put forward an algorithm of automatically generating TIN. Compared to other algorithm, it has the merits of :

1) Resolving the problem of fault, especially the problem of generating triangulation of reverse fault and fault combination. While reconstructing in a local region, the normal vector is uesd to validate the legalization of the triangle.
2) Resolving the problem of connecting net of any structure of boundary. The type of the boundary could be concavity or convex, even the adjacent boundary could be inlaid (many independently sealed boundaries.
3) The algorithm can deal with complex geology faults and fault combination. It is simple, high-efficient, and useful for automatic creation of coal floor contour map.

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