MULTIPATH REDUCTION ON REPETITION IN TIME SERIES FROM THE PERMANENT GPS PHASE RESIDUALS

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ABSTRACT:

In this research, the newly developed Hilbert Huang Transformation method is applied to decompose the time-shifted post-fit GPS phase signal residuals into a range of fluctuating components from high- to low-frequencies. Global Positioning System (GPS) satellite constellation returns to the same configuration in a sidereal day (23 h 56 m 4 s); however, daily GPS position estimates are usually based on the solar day. After processing GPS data of several days, the steady time shifts of the satellite constellation with respect to the station appear in the patterns of the post-fit phase residuals with time shift by 3 m 56 s per day. The effects of environmental multipath delays may be filtered out by the HHT method. We can, however, observe some multipath effects in the data from a static GPS antenna which occupies the same site over several days (or permanently), because the geometry between the GPS satellites and any (permanent) reflectors at that site repeats every sidereal day. In this paper, it was found that the time-shifted post-fit double-difference phase residuals are highly dependent on the GPS antenna site. The correlation coefficients are more than 0.94 .

1 INTRODUCTION

Multipath effects are due to the reflection and diffraction of satellite signals off nearby objects, such as buildings, antennas, or ground horizontal reflectors. They introduce significant errors in code and carrier measurements. In recent years, multipath effect testing and mitigation for GPS carrier phase observation have been an active area of research. The most test results were obtained under static conditions. All basic method is to place the GPS antenna in a low-multipath environment, away from any potential reflectors and with a good ground plane or choke ring. Another common method for reducing code multipath is to smooth the code measurements with the carrier phase measurements.

Phase multipath errors typically display sinusoidal characteristics. The error depends on four major factors: the reflecting environment, the satellite - antenna geometry, the antenna, and the receiver hardware/firmware types. However, daily GPS position estimates are usually based on the solar day. Because the GPS satellite constellation returns to the same configuration in a sidereal day (23 h 56min 4 s), the multipath repeats on this period.

In this paper, the objective is to show that Hilbert Huang Transformation analyses procedures can be used to identify multipath frequencies from high rate permanent GPS observation data records. By applying the procedure, mid-frequency multipath is detected and separated out the GPS receiver noise and other patterns in time series.

2 GENETIC ALGORITHMS

The procedures for the analysis are as follow steps :

(a) Analysing a time series data from the permanent GPS observation. to obtain DD phase residuals.
(b) Apply the EMD to decompose the DD phase residuals data into several IMFs. Then separated out the GPS receiver noise, possible multipath signals and other patterns.
(c) Obtained the GPS constellation orbit periods from the broadcast ephemeris.
(d) If the multipath signal of a certain model is dominant, the data are using a reduction model for user.

2.1 Double-difference Observation Model

The basic four double-difference (DD) observation equations of the carrier phase Φ_{1ij}^{gh}, Φ_{2ij}^{gh} (m) and the pseudorange ρ_{1ij}^{gh}, ρ_{2ij}^{gh} (m) can be written as:

$$
\begin{aligned}
\Phi_{1ij}^{gh} &= R_{ij}^{gh} + T_{\text{trop}ij}^{gh} - I_{\text{ion}ij}^{gh} - \lambda_1 N_{1ij}^{gh} + \varepsilon_{\phi_1 ij}^{gh} \\
\Phi_{2ij}^{gh} &= R_{ij}^{gh} + T_{\text{trop}ij}^{gh} - \alpha_f I_{\text{ion}ij}^{gh} - \lambda_2 N_{2ij}^{gh} + \varepsilon_{\phi_2 ij}^{gh} \\
\rho_{1ij}^{gh} &= R_{ij}^{gh} + T_{\text{trop}ij}^{gh} + I_{\text{ion}ij}^{gh} + \varepsilon_{\rho_1 ij}^{gh} \\
\rho_{2ij}^{gh} &= R_{ij}^{gh} + T_{\text{trop}ij}^{gh} + \alpha_f I_{\text{ion}ij}^{gh} + \varepsilon_{\rho_2 ij}^{gh}
\end{aligned}
\tag{1}
$$

where $\Phi_{ij}^{gh} = \Phi_j^h - \Phi_i^h - \Phi_j^g + \Phi_i^g$ and $\rho_{ij}^{gh} = \rho_j^h - \rho_i^h - \rho_j^g + \rho_i^g$, etc. The superscript *i* and *j* stand for the permanent GPS receiver. The reference satellite is denoted by the subscript *g*, and another satellite denoted by *h*. The *R* (m) represents the Euclidean satellite-receiver distance. The N_1 and N_2 (cycle) denote the integer ambiguity. The $_1$ and $_2$ (m) stand for the L1 and L2 carrier wavelength. T_{trop} and I_{ion} denote the tropospheric delay and ionospheric delay. The pseudorange error and phase error are indicated by ε_ρ (m) and ε_ϕ (m) . Let α_f = 77/60.

For over short distances (<10 km, depending on the ionospheric conditions), the ionospheric effects are neglected for common practice. An accuracy of centimeter level (standard deviation) or even better resolution can be reached. However, while in longer distances, the differential ionospheric residuals become larger and may hamper the phase integer ambiguity resolution process, or even make it impossible. Therefore, this paper presents the analyses of modeling algorithms with an additional virtual double-difference ionospheric observation equation which can estimate the ionospheric parameters of the stochastic measurement (Goad and Yang, 1997). The virtual double-difference ionospheric observation equation can be written as:

$$m = (\Delta\nabla I)_0 + \Delta\nabla\varepsilon_m \tag{2}$$

$$\sigma_m^2 = \lim_{\tau\to 0}\sigma_{ij}^{2gh}(\tau,s) = \sigma_\infty^2\cdot(1-e^{-2s/D}) \tag{3}$$

where m is a priori observation of the double-difference ionospheric that equals to 0. σ_m^2 is the variance of the ionosphere, a priori observation that the strength of the stochastic measurement is dependent on the instantaneous baseline length, and τ is the time interval , and s is the instantaneous baseline length. Assuming that σ_∞^2 equals to 2.0 m^2 to the upper limit of all practical kinematics GPS operations, the variance of the double-difference ionospheric effect at a distance approximately D equals to 1500 km. σ_m has the values of 0.012 m and 0.353 m in the distances of 0.115 km and 100.0 km, respectively.

2.2 Mixed Least-Squares Model and Ambiguity Decorrelation

A linear system of error equations can be expressed as follow:

$$\underset{c\times n}{\mathbf{B}}\ \underset{n\times 1}{\mathbf{v}} + \underset{c\times u}{\mathbf{A}}\ \underset{u\times 1}{\mathbf{x}} = \underset{c\times 1}{\boldsymbol{l}} \quad \text{with} \quad \underset{n\times n}{\boldsymbol{\Sigma}} \tag{4}$$

$$\mathbf{v} = \boldsymbol{\Sigma}_v\mathbf{B}^T\boldsymbol{\Sigma}_l^{-1}\boldsymbol{l} \tag{5}$$

where the **v**-vector is the measurement residual vector; the matrix **B** stands for the linear double difference combination matrix. The **x**-vector consists of the ambiguity and ionospheric parameters; the matrix **A** is the corresponding design matrix. The ***l***-vector denotes the reduced observation vector. The symmetrical matrix **Σ**represents the prior error covariance matrix. Real-valued ambiguity parameters and their covariance matrix are first estimated in a floating solution. By trial and error, the correct set of integer-valued ambiguities are statistically identified from a large number of candidate integer ambiguity sets. A whitening filter technique is employed for correctness of the fixed ambiguities resolution (Mohamed and Schwarz, 1998). When the ambiguity determination, then obtained DD residual solutions from Eqs. (5) .

2.3 GPS Phase Residuals Analysis

The GPS DD phase residuals analysis mainly consists of an application of the Hilbert- Huang method to decompose an empirical time series into a number of intrinsic mode functions (IMFs), calculation of the instantaneous phase of the resultant IMFs, and the statistics of the instantaneous phase for each IMF.

The Hilbert-Huang transform (HHT) is an algorithm designed for nonlinear and non-stationary time series analysis. The HHT consists of empirical mode decomposition (EMD) and the Hilbert transform analysis. The EMD method is developed by the assumption that any time series consists of simple intrinsic modes of oscillation. The essence of the method is empirically to identify the intrinsic oscillatory modes by their characteristic time scales in the data and then decompose the data into these modes. This is achieved by sifting data to generate IMFs.

The IMFs introduced by the EMD are a set of well-behaved intrinsic modes, and these functions satisfy the conditions that they are symmetric with respect to the local zero mean and have the same numbers of zero crossings and extremes. Therefore, the Hilbert transform can be directly used to calculate the instantaneous phase after the decomposition processes.

The algorithm to create IMFs in EMD has two main steps.
Step 1: The local extremes in the return time series data $x(t)$ are identified. Then, all the local maxima are connected by a cubic spline line $U(t)$ forming the upper envelope of the time series, and another cubic spline line $L(t)$ forming the lower envelope. Both envelopes will cover all the original time series, and the mean of upper envelope and lower envelope $m_1(t)$ given by

$$m(t) = \frac{U(t)+L(t)}{2} \tag{6}$$

is a running mean. The running mean $m_1(t)$ is then subtracted from the original time series $R(t)$ to yield the first component, $h_1(t)$,

$$x(t) - m_1(t) = h_1(t) \tag{7}$$

The resulting component $h_1(t)$ is an IMF if it satisfies the conditions: (i) $h_1(t)$ is free of riding waves. (ii) It displays symmetry of the upper and lower envelopes with respect to zero. (iii) The numbers of zero crossing and extremes are the same, or only differ by 1. If $h_1(t)$ is not an IMF, the sifting process has to be repeated as many times as it is required to reduce the extracted signal to an IMF. In the subsequent steps of sifting process, $h_1(t)$ is treated as the data,

$$h_1(t) - m_{11}(t) = h_{11}(t) \tag{8}$$

Again, if the function $h_{11}(t)$ does not yet satisfy requested conditions (i)–(iii), the first sifting process continues up to k times until some acceptable tolerance is reached,

$$h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t) \tag{9}$$

Step 2: If the resulting time series is the first IMF, then it is designated as $c_1 = h_{11}(t)$. Subsequently, the first IMF is subtracted from the original data, and the difference r_1 given by

$$x(t) - c_1(t) = r_1(t) \tag{10}$$

is a residue. The residue $r_1(t)$ is taken as if it were the original data, and we apply it again to the sifting process. Following above procedures, the process of finding more intrinsic modes, c_i, continues until the last mode is found. The final residue will be a constant or a monotonic function which represents the general trend of the time series data.

After decomposition, a time series $x(t)$ is decomposed into n IMFs c_i's and a residue r_n, *i.e.*

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t) \tag{11}$$

$$c_i(t) = r_{i-1}(t) - r_i(t)\ . \qquad (12)$$

The details of procedures of the EMD can be found in Huang et al. (1998) or http://perso.ens-lyon.fr/patrick.flandrin/ emd.html. The Figure 1 shows a typical EMD for a DD phase residuals time series.

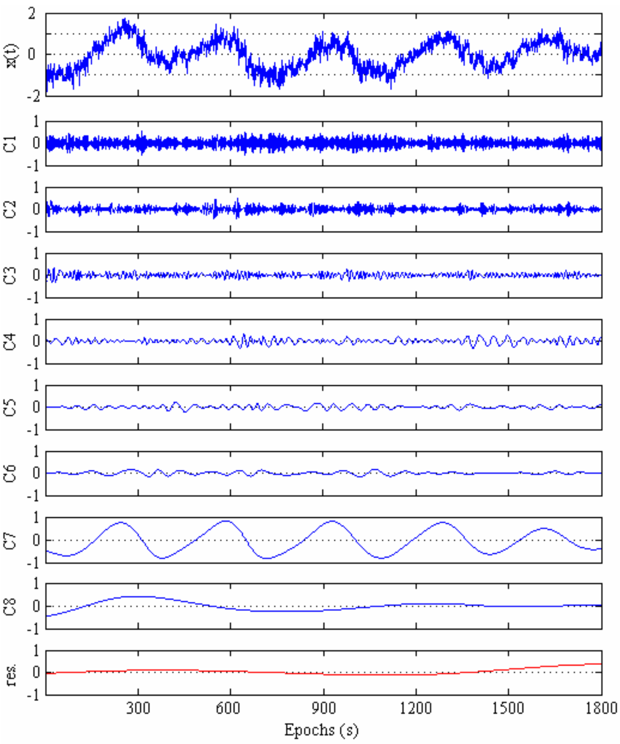

Figure 1. Example of EMD for a typical time series data from the permanent GPS DD phase residuals (PRN pair 18 and 14). Signal $x(t)$ is decomposed into 9 components including 8 IMFs and 1 residue.

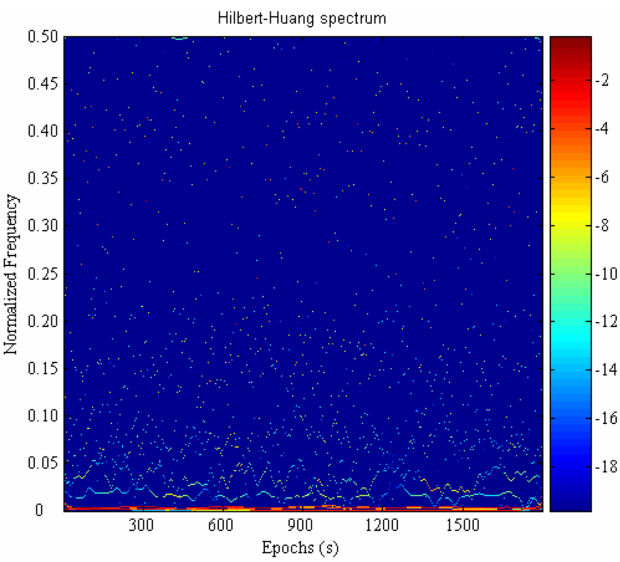

Figure 2. Hilbert-Huang spectrum of the IMFs time series data from Figure 1.

The instantaneous phase of the resultant IMFs can then be calculated by using the Hilbert transform. For the kth mode, this can be done by first calculating the conjugate pair of $c_k(t)$,

$$y_k(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c_k(t')}{t - t'} dt' \qquad (13)$$

where P indicates the Cauchy principal value. With this definition, the two functions $c_k(t)$ and $y_k(t)$ forming a complex conjugate pair define an analytic signal. Accordingly, we define

$$c_k(t) + iy_k(t) = A_k(t) e^{i\phi_k(t)} \qquad (14)$$

with the amplitude $A_k(t)$ and the phase $\phi_k(t)$ defined by

$$A_k(t) = \left[c_k^2(t) + y_k^2(t)\right]^{1/2} \qquad (15)$$

$$\phi_k(t) = \tan^{-1}\left(\frac{y_k(t)}{c_k(t)}\right) \qquad (16)$$

Then, we can calculate the instantaneous phase according to Eqs. (13) and (16).

Application of the phase statistics approach to investigate the IMFs time series is systematic error or white noises that show in Figure 3. It is important to distinguish between noises and any systematic patterns. The criteria for multipath signals in the sifting process is comparing the patterns of the IMFs periods. The C6 add C7 time series are possible signal of Multipath as shown in Figure 4.

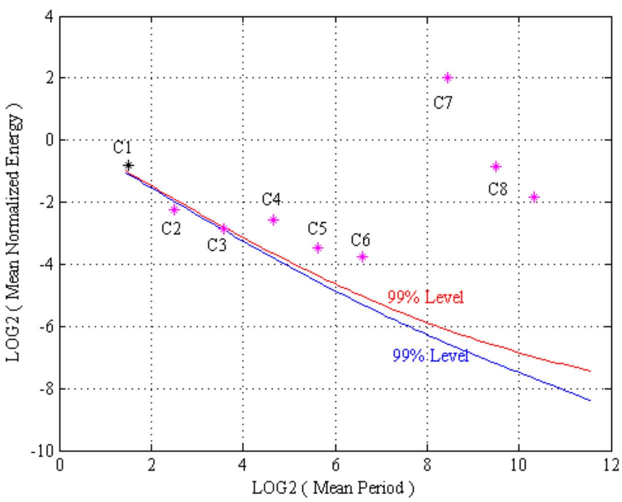

Figure 3. Significance test of IMFs of white noise.

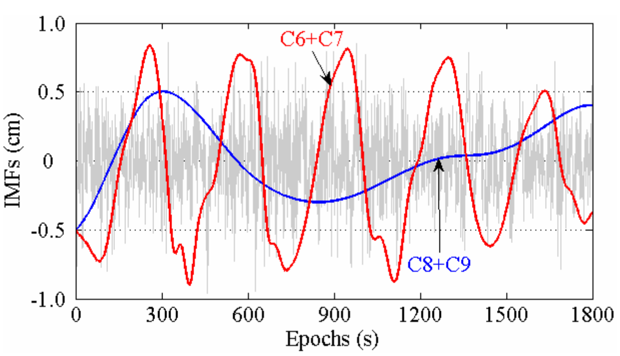

Figure 4. The trends of receiver noise (gray), multipath signals (red) and any systematic patterns (blue) divide from Figure 1.

2.4 GPS Constellation Orbit Periods Analysis

The GPS satellites constellation would repeat. The same satellites would appear in the same part of the sky with a period slightly less than 1 day. The usual rule of thumb was that the repeat time was 236 s (one solar day) earlier each day. A period the GPS satellites would complete exactly two orbits in inertial space, and one revolution of the Earth bringing everything back to the same geometry. Single-epoch estimates of GPS position are improved by removing multipath signals, which repeat when the GPS constellation does.

The orbit periods of the GPS satellites requires two parameters provided in the broadcast ephemeris (Agnew and Larson, 2007): a_s, the square root of the semimajor axis, and n_c, the correction to the mean motion that would be deduced from Kepler's third law. The repeat time T_0 is twice the orbital period by:

$$T_0 = 4\pi/n\ ,\quad n = \sqrt{GM}\, a_s^{-3} + n_c \tag{17}$$

where the $\sqrt{GM} = 1.996498 \times 10^7\ (\mathrm{m}^3/\mathrm{s}^2)$ is for the earth.

The Figure 5. shows the resulting periods, computed at 1-day intervals, from DOY 001 to DOY 100 in 2006. For most of the time span shown, the repeat time is between 238 s and 252 s (mean is 246 s)earlier each day.

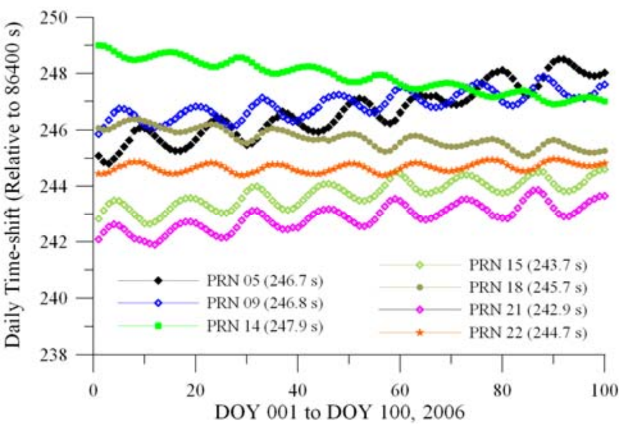

Figure 5. The GPS satellites constellation orbit periods results.

2.5 Multipath Reduction Model

The user multipath reduction equations consists of DD observation (between two permanent receiver with user receiver) Φ^{gh}_{1iu}, Φ^{gh}_{1ju}, Φ^{gh}_{2iu}, Φ^{gh}_{2ju} and multipath signals MP^{gh}_{1ij}, MP^{gh}_{2ij} can be written as:

$$\begin{aligned}
\Phi^{gh}_{1iu} &= R^{gh}_{iu} + T_{\mathrm{trop}\,iu}^{\ gh} - I_{\mathrm{ion}\,iu}^{\ gh} - \lambda_1 N^{gh}_{1iu} + MP^{gh}_{1iu} + \varepsilon_{\Phi_1,iu} \\
\Phi^{gh}_{1ju} &= R^{gh}_{ju} + T_{\mathrm{trop}\,ju}^{\ gh} - I_{\mathrm{ion}\,ju}^{\ gh} - \lambda_1 N^{gh}_{1ju} + MP^{gh}_{1ju} + \varepsilon_{\Phi_1,ju} \\
\Phi^{gh}_{2iu} &= R^{gh}_{iu} + T_{\mathrm{trop}\,iu}^{\ gh} - \alpha_f I_{\mathrm{ion}\,iu}^{\ gh} - \lambda_2 N^{gh}_{2iu} + MP^{gh}_{2iu} + \varepsilon_{\Phi_2,iu} \\
\Phi^{gh}_{2ju} &= R^{gh}_{ju} + T_{\mathrm{trop}\,ju}^{\ gh} - \alpha_f I_{\mathrm{ion}\,ju}^{\ gh} - \lambda_2 N^{gh}_{2ju} + MP^{gh}_{2ju} + \varepsilon_{\Phi_2,ju}\ .
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
MP^{gh}_{1ij} &= MP^{gh}_{1iu} - MP^{gh}_{1ju} + \varepsilon_{MP_1} \\
MP^{gh}_{2ij} &= MP^{gh}_{2iu} - MP^{gh}_{2ju} + \varepsilon_{MP_2}
\end{aligned} \tag{19}$$

where $MP^{gh}_{ij} = MP^h_j - MP^h_i - MP^g_j + MP^g_i$ etc. The subscript i and j stand for the permanent GPS receiver, and the user receiver denoted by u. The reference satellite is denoted by the superscript g, and another satellite denoted by h.

3. NUMERICAL EXPERIMENTS AND RESULT

The computation of DD residuals and multipath reduction program, called ManGo, has been developed by using the previous sections which were tested on two sets of GPS observations. The computation of HHT is performed in MATLAB. The details of HHT program can be found in http://rcada.ncu.edu.tw/ research1.htm.

The testing data of observations were from the north of Taiwan permanent GPS network as shown in Figure 6 at UT 12:00~13:00, DOY 066 to DOY 072 of 2006. The Figure 7 show the sites of SPP0 and SPP1 are on the top of building in Center for Space and Remote Sensing Reach. The GPS receiver are all using a dual-frequency Leica SR530 type.

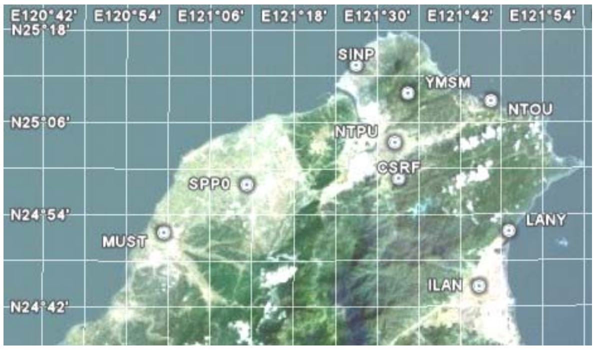

Figure 6. Taiwan Permanent GPS Network

Figure 7. The GPS antenna sites of SPP0 and SPP1 (western view)

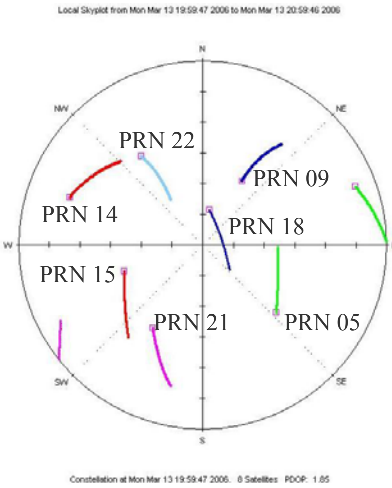

Figure 8 The GPS satellites constellation sky plot in SPP0 site

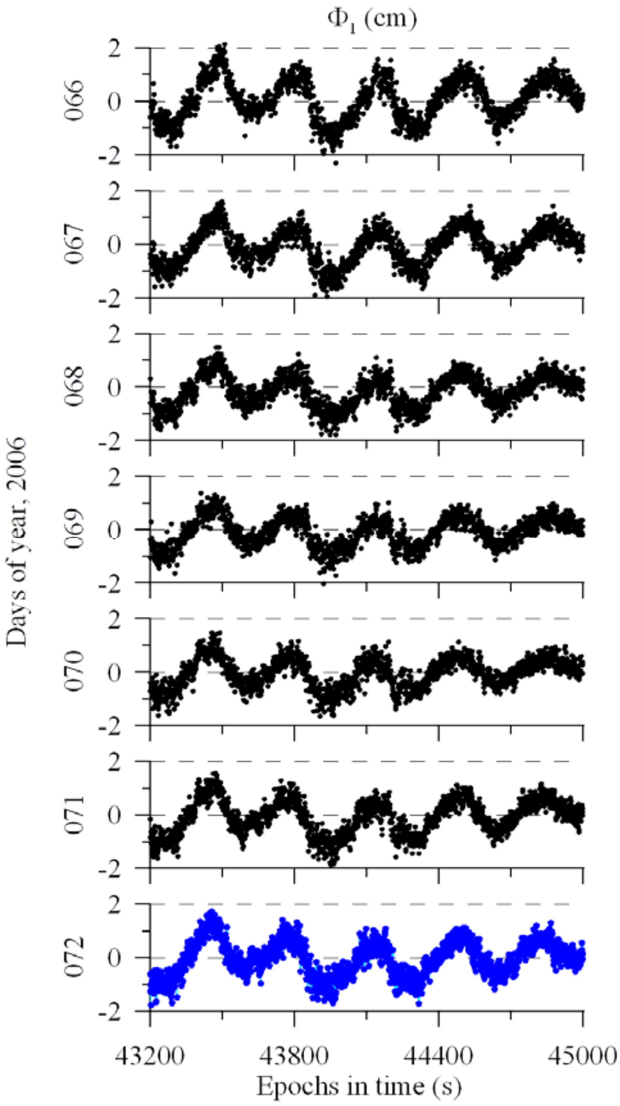

Figure 9. The double-difference phase Φ_1 residuals (PRN-18 and PRN-14, sampled at 1.0 Hz) are in a 13.5 m baseline of SPP0 – SPP1. The time shift of day-to-day is 246 s. An average correlation is 0.82 .

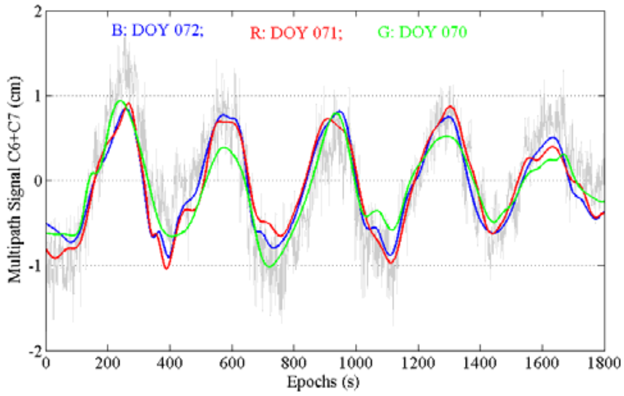

Figure 10. The trends of different timescale multipath signal are the baseline of SPP0 – SPP1. An average correlation is 0.94 .

The SPP0 antenna is LEIAT 504 and SPP1 antenna is LEIAT 502 which are at Taoyuan. The MUST is using LEIAT 502 antenna at Hsinshu. The GPS receivers are setting indicated a 1.0 Hz sampling rate, a 15º tracking mask angle, and carrier-smoothed ranging. The Figure 8. show that seven GPS satellites are in view. The satellite orbit has been the IGS final orbit data (igs13651 to igs13661).

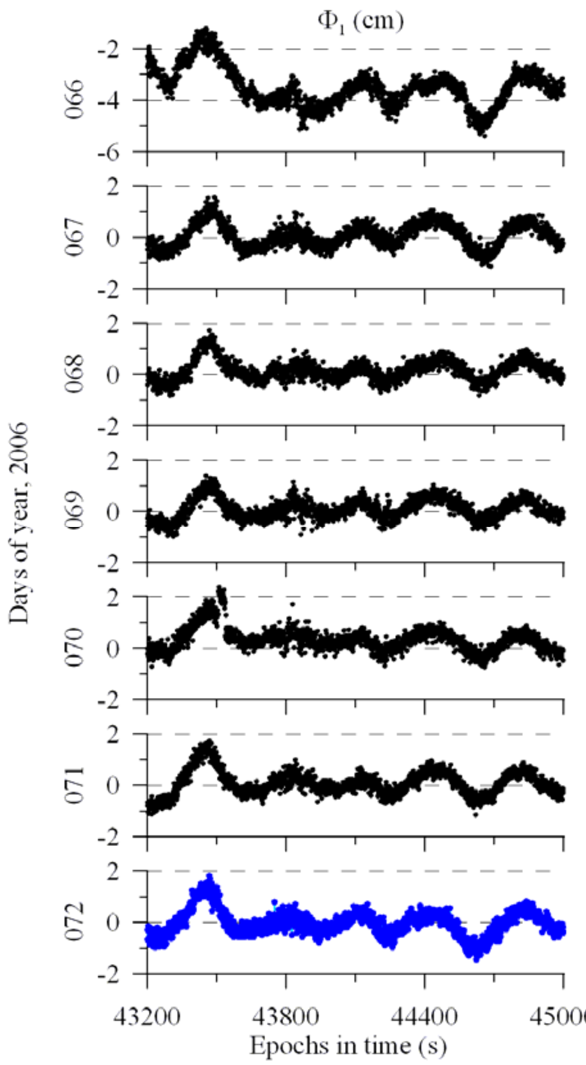

Figure 11. The double-difference phase Φ_1 residuals (PRN-18 and PRN-14, sampled at 1.0 Hz) are in a 23122.72 m baseline of SPP0 – MUST. The time shift of day-to-day is 246 s. An average correlation is 0.72 .

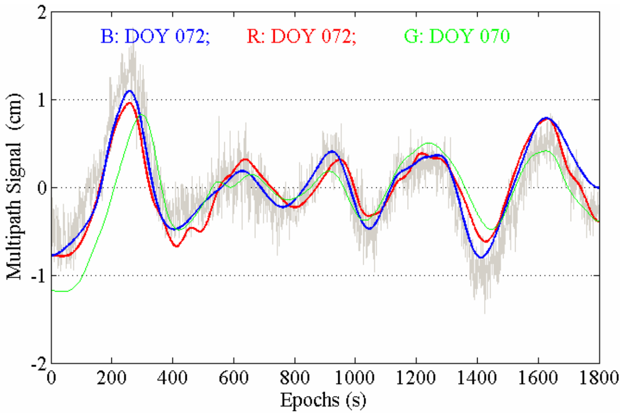

Figure 12. The trends of different timescale multipath signal are in the baseline of SPP0 – MUST. An average correlation is 0.88 .

Testing data sets are using SPP0-SPP1 and SPP0-MUST baseline. The SPP0-SPP1 is short baseline about 13.50 m. The SPP0-MUST is mid-length baseline about 23122.72 m. For instance in day 072 of 2006 year, the signal occurs in the time window 43,200–45,000 s from 0 h UTC compared to the same time window in days 071, 070 and 070. Each day time shift of orbit repeat is 246 s.

The DD phase residuals Φ_1 for the same satellite pair from the data collected in time shift 246 s are shown in Figure 9. This baseline is SPP0-SPP1. The satellites pair is PRN 18 and PRN 14. The day-to-day repeatability of the residual errors indicates that it is due to multipath. An average correlation is 0.82 that was observed between data collected. The amplitude are between + 2 cm and - 2 cm.

By applying the HHT procedure, middle frequency multipath is detected and separated out the GPS receiver noise and other patterns. The trends of different timescale multipath signals are shown in Figure 10. An average correlation is raised to 0.94 . The multipath amplitude are between + 1 cm and - 1 cm. An average period is equal to 347 s

The phase Φ_1 residuals of baseline SPP0-MUST for the same satellite pair are shown in Figure 11. The day-to-day repeatability of the residual errors indicates that it is due to multipath which the same as short baseline SPP0-SPP1. An average correlation is 0.72 . By applying the HHT procedure, middle frequency multipath is detected and An average correlation is raised to 0.88 as shown in Figure 12. The multipath amplitude are between + 1 cm and - 1 cm. An average period is equal to 350 s.

The above steps are applied to the time series of single epoch solutions from carrier phase data.

4. CONCLUSIONS AND DISCUSSION

We have briefly explored the scheme of the Hilbert-Huang method and its application to the study multipath signal in time series. The remarkable advantage of the EMD method is that it can catch primary structures of intrinsic rhythms from empirical data based on its adaptive feature. This property is especially suitable for performing phase multipath signal process on empirical time series. The phase multipath distributions corresponding to abruptly change behaviors indicate non-predictable and stochastic features of the DD residuals.

In this paper, the objective is to show that when 1.0 Hz GPS data records are available from permanent GPS network, the HHT analyses procedures can be used to convincingly identify multipath signals.

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