A ALGORITHM ON BRANCHES NUMBER OF A TREE BASED ON EXTENDED FRACTAL SQUARE ROOT LAW

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ABSTRACT:

It is an important question to calculate number of grown branches in a tree in visualization of trees growth. The number of growth branches is determined by visual effect or developer′s experience. This paper overcomes disadvantages of branches calculation by man-made factors, and proposes an algorithm to calculate number of branches belonged to a tree based on extend square root principle of fractal. It is a result of cartography combination with math, forestry and computer graphics. This method can calculate number of a tree's branches after growing based on the tree heights before and after growing period of time, the number of a tree's branches before growing period of time, and the tree branches fractal. The paper offers deducing procession and application example of the expressions. It can be used in spatio-temporal GIS and in visualization of plant's growth especially tree's growth.

1. INTRODUCTION

Growth of a tree includes growth in diameter, height, section area, shape, volume, weight etc. Trees grow a lot of branches when they are growing in height or section area. Similarly， The growth in height and diameter, number of growing branches of a tree have to be considered when virtual growth of a tree. Without doubt, the height of a tree must have something to do with its branches number. The existed technique of fractal trees is that the trunk of a tree produces the first class branch, and keep the first class branch as "the trunk" to produce the second class branch, and so do iterative loop until to finish iterative number. SUN etc. (1998) simulated plants growth applying L system of fractal theory. According to fractal theory, every kind of plant has its elementary string. The growth procession is realized by every variable in the elementary string to be used definite ramification model. With this method, a new string is generated from a variable in an elementary. We can replace the variable in the new string with a ramification model. We can do so iteratively until to reach the iterative number. As soon as the iterative number is decided, the number of branches is decided also. The decision of the iterative number has not been mentioned from existed reference. However it is based on following three points:(1) experiences of program developer (2) visual effect of the trees beauty (3) memory limitation of the computer to do the work. The first two points are not scientific and difficult to operation. The third point is decided by the computer characteristic completely. Some user whose computer is of smaller memory may not realize virtual growth on trees with complicated crown shape.

This paper gets rid of the disadvantages that the decision of branches number (or the iterative number to produce branches) is by experience or visual effect. It provides a kind of scientific and quantitative method.

2. REALIZATION OF ALGORITHM

Against the disadvantage to calculate the branches number in a growing tree unscientifically on visualization of tree growth with computer, this paper propose a kind of method to calculate number of branches on a growing tree scientifically and quantificationally. It includes contents as follow: (1) to calculate the fractal of a tree (2) to measure the height of the tree at first of its growth (3) to measure or calculate the height of the tree after its growth with its growth equation (4) to calculate the branches number after growing with extend square root principle of fractal. The flowing graph of the algorithm is as Fig.1.

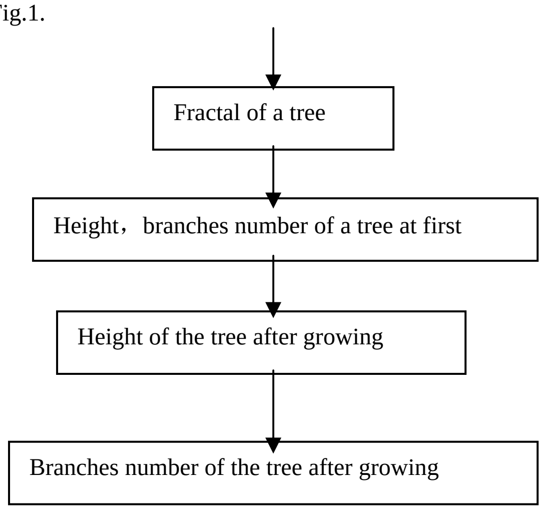

Figure1. Flow chart of algorithm to calculate number of branches belonged to a tree based on extend square root principle of fractal

（1） Calculation of a tree's fractal

Acquiring a tree's image without its leafs, we overlay the image with a square grid with both row spacing and col width as ε, and calculate the grid number N(ε) where a branch in the tree crosses. According to fractal geometry principle, we have following formula:

$$N(\varepsilon) = C_2 \varepsilon^{-D_B} \quad (1)$$

where D_B = fractal

It describes N(ε) increasing ratio according to ε becoming smaller from formula (1). It describes the much or less degree of the tree including details and its ability to fill in its plane too. C2 is a variable without dimension。 If we make a logarithm to formula（1）, we will have following formula：

$$\log N(\varepsilon) = \log C_2 - D_B \log \varepsilon \quad (2)$$

We will have formula (3) based on formula (2) and principle of linear regression analysis：

$$D_B = \frac{M \sum_{i=1}^{M} [(\log \varepsilon_i)(\log N(\varepsilon_i))] - (\sum_{i=1}^{M} \log \varepsilon_i)(\sum_{i=1}^{M} \log N(\varepsilon_i))}{M \sum_{i=1}^{M} (\log \varepsilon_i)^2 - (\sum_{i=1}^{M} \log \varepsilon_i)^2} \quad (3)$$

where M = the class number of grid size.

Based on it we can calculate the fractal of the tree.

（2） Survey the height of the tree at its early days, make a statistic on its branch number.

We survey the height of the tree at its early days with a theodolite or other surveying instrument. Branch number of the tree must be counted. It is possible to count its branch number by photographing the tree's plane photo for a tree with complicated structure and difficult to count its branch number.

（3） Survey or calculate the height of the tree with the growth equation of the tree after growing

We can survey the height of the tree after growing with a theodolite or other surveying instrument. We also can to calculate the height of the tree after growing in several years with the growth equation of the same kind of tree in same place area according the age, height at its early days.

（4） Calculate the branch number after growing in several years using extended fractal square root law.

Square root law is a kind of mathematics model used in cartography based on experience law. The law formula is as equation (4)

$$N_2 = N_1 \sqrt{M_1 / M_2} \quad (4)$$

where N_1, N_2 = the feature counts in original map and compiled map

M_1, M_2 = the scale denominator in original map and compiled map

Formula (4) is called elementary formula of square model. According to the model, we can make some selection control on some features counts, such as the selection of the number of residential areas or rivers in different scale relief maps.

WANG (1996) amended the way to use the law. He amended square root in formula (4) to flowing: square root are replaced by D/2 power to features of planar point and distributing along a curve, by D power to polygon features, by -D/2 power to river features. Here D is the fractal of the above features. The whole selection law formula on number of river features is:

$$N_2 = N_1 (M_1 / M_2)^{-D/2} \quad (5)$$

A tree grows trunk first, and grows main branches in the trunk. Main branches grows secondary branches, and the secondary branches grows more secondary branches etc. A tree's structure is much similar with the rivers structure with branches shape. The river generation in cartography is just reverse to a tree's growth in forestry: with zooming out of the map scale, smaller rivers in the drainage basin are discarded by generation in cartography, and main rivers to keep the structure of the drainage basin are selected. Procession to generate in cartography is the procession to discard smaller rivers. Procession for a tree to grow in forestry is the procession to grow new and more secondary branches in main branches. We can use square root law to generalize rivers in cartography to control the rivers number after generalization, by contraries, we can use square root law to control the number of the tree growing branches. When we represent a tree's growth in visual with computer graph principle, the branches number of a grown up tree is very big. We can't simulate all branches in a big tree for limited memory of a computer. We have to discard or compress the branches number. Square root law in cartography is very revelatory to select features。 We can use it to determine the branches number after a tree growing in several years. We use scale denominator as its dimension to generalize in cartography. Similarly, we can use the height of a tree as dimension to calculate the number of a tree's branches with square root law.

Branches number of the tree after growing in several years is determined. The formula (5) are improved: M_1 replaced by the tree height early days (H1), N_1 indicates the branches number of the tree early days. $M_2$is replaced by the tree height after growing (H2), N_2 indicates the branches number of the tree after growing. Accordingly, formula (5) is modified to flowing:

$$N_2 = N_1 (H_1 / H_2)^{-D/2} \quad (6)$$

According to formula（6）, the fractal of the tree, the tree height and branches number early , the tree height after growing, the branches number of the tree can be calculated.

Comparing with existed technique, the algorithm is of convenient, safe and reliable. In spit of anyone does visualization, he will get the same branches number with the algorithm as long as the given condition is same (tree height early days, branches number early days, tree age or tree height after growing). It get rid of the disadvantage of existed technique to reach different branches numbers based on experience or visual effect. It is necessary to point out that the algorithm is fit for all plants with branches and self-similar character to be calculated growth branches number. In addition, this algorithm is fit for whole life procession of the plants form early days to death. Combining with fractal algorithm such as L system, the Law can determine the times of iterating. Furthermore, we can display a tree growing in visualization. In the same way, we can display other plants growing in visualization with self- similar character such as cotton, legume, flowers with branches. We can extend the growing from a plant to colony of the plants for example

3. EXAMPLES

Chinese white poplar sample whose fractal will be calculated is as fig. 2. When this tree was 3 years old, it was 5.6 meters tall, its branches number was 156. When it was 5 years old, it was 7.2 meters tall. Please calculate the fractal of the tree and its branches number when it was 5 years old.

Figure2. Branch stylebook to calculate fractal

Firstly we must get the fractal of the Chinese white poplar with grid overlaying method. Draw grids on the photo of the Chinese white poplar. The number of grids where branches cross is 1,2,4,8,130,310 for grids structure as 1X1、2X2、4X4、8X8、16X16、32X32. Using formula (3), we calculate its fractal as 1.22.

H_1=5.6，H_2=7.2，N_1=156，According to extended fractal square root law (formula (6))，when the tree was 5 years old, its branches number is：

$$N_2 = N_1(H_1/H_2)^{-D/2} = 156 \times (5.6/7.2)^{-1.2/2} = 181$$

4. CONCLUSION

This paper proposes an algorithm to calculate number of branches belonged to a tree based on extend square root principle of fractal. It is a result of cartography combination with math, forestry and computer graphics. This method can calculate number of a tree's branches after growing based on the tree heights before and after growing period of time, the number of a tree's branches before growing period of time, and the tree branches fractal. Its calculating formula is:

$$N_2 = N_1(H_1/H_2)^{-D/2}$$

N1, N2 = the branches number of the tree early days and after growing H1 and H2 = the height of the tree early days and after growing D = the fractal of the tree）

The branches number of the tree after growing can be calculated uniquely when the first condition and tree height after growing. It can be used to calculate branches number of trees and other plats with self-similar character for whole life and realize the visualization of the plants growing. This paper overcomes disadvantages of man-made factors when do visualization of trees growth.

REFERENCES

HU Bao-gang，ZHAO Xing，YANG Hong-ping etc., 2001. Plant growth Modeling and Visualization –Review and Perspective，*Acta Automatica Sinica*， 27（6）, pp.816-835

KANG Meng-zhen，Philippe de Reffye，HU Bao-gang，ZHAO Xing, 2004. Fast construction of Geometrical Structure of Plant with Substructures Algorithm，*Journal of Image and Graphics*， 9（3）, pp.79-86

Norm Roller, 2000. Intermediate Multi-spectral Satellite Sensors[J]. *Journal of Forestry*, 98(6):, pp.32-35.

SONG Tie-ying, 1998. A Visual Stand Model Based on Image of tree.*Journal of Beijing Forestry university* , 20(4), pp.93-97.

Simona E. Grigeorescu, Nicolai Petkov, and Peter Kruizinga, 2002. Comparison of Texture Features Based on Gabor Filters. *IEEE Trans On Image Processing*, 11(10), pp.1160-1167.

SUN Jun-bai, LIU Gen-quan, ZHANG Jun, 1998. Application of fractal in simulating the Growth of Plants .*Mini-Micro Systems*, 19(2), pp.54-59.

WANG Qiao, 1996.*Research on Some Extensions and Applications of the Fractal Theory in the Automatic Handling of Map Graphical Data*，Wuhan Technical University of surveying and mapping，pp.89-128.

YANG Bo-gang, 2006. *Extract and analysis on Multi-scale Forest Cover information in Beijing*，Beijing Forestry university， pp.64-93.

YANG Cun-jian, LIU Ji-yuan, 2001. The decision analysis of return forestland and grassland in Yunnan province by using RS and GIS. *Geographic Science*. 56(2), pp.181-187.

ZHANG Bao-gang， ZHU Chong-guang， WANG Run-sheng, 2005. An Improved Method of Base state with Amendments on Spatio-temporal Data. *Acta Geodaetica et Cartographica Sinica*， 34(3), pp.252-256.

ZHAO Xing，de Reffvye P,XIONG Fan-lun，HU Bao-gang，ZHAN Zhi-gang, 2001. Dual-scale Automaton model for Virtual Plant Development ，*Chinese Journal Computers*. 24(6), pp.608-615

Pouliot D A, King D J, Bell F W, et al. 2002. Automated Tree Crown Detection and Delineation in High-resolution Digital Camera Imagery of Coniferous Forest Regeneration. *Remote Sensing of Environment*, (82), pp.322-334.

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