# MULTI-RESOLUTION REPRESENTATION OF DIGITAL TERRAIN MODELS WITH TOPOGRAPHICAL FEATURES PRESERVATION 

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#### Abstract

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Multi-resolution TIN model is a promising solution for achieving rapid visualization speed and interactive frame rates in the contexts of visualization, virtual reality (VR), and geographic information systems (GIS). Most of previous automatic algorithms are not able to identify topographic features, such as peak, pit, ridge, channel and pass, so as to produce poor approximations when a model is simplified to a low level of detail. This paper proposes a new method for constructing multi-resolution TIN models with multi-scale topographic feature preservation. The proposed method is driven by a half-edge collapse operation in a greedy framework and employs a new quadric error metric to efficiently measure geometric errors. We define topographic features in a multi-scale manner using a center-surround operator on Gaussian-weighted mean curvatures. Then we employ an adaptive weight based on topographic features for the control of simplification process. Experimental results identify that proposed method performs better than previous methods in terms of topographic feature preservation, and can achieve multi-resolution TIN models with a higher accuracy.


## 1. INTRODUCTION

High-resolution terrain model leads to a dilemma between the rendering time, interactive frame rates, and data processing. A model with a fixed resolution is not good for all applications and all users because of different requirements, available computer resources, and affordable time. Therefore, it would be ideal for users to have the ability to control the size of the model, the processing time needed, and the accuracy of the model. The representation of digital terrain models at different levels of accuracy and resolution has an impact on applications such as Geographic Information Systems (GISs), Virtual Reality (VR), progressive transmission of spatial data, mobile visualizations, and Web-GIS. Multi-resolution terrain models allow for representation, analysis and manipulation of terrain data at variable resolutions, decreasing the burden of rendering and achieving interactive frame rates, and provide a promising solution for the progressive transmission of spatial data, spatial data compression, mobile visualizations, and so on. However, the existing methods and algorithms mainly focus on the accuracy and running times of generating the levels-of-details (LoDs) of terrains. Less attention has been paid to topographic features preservation of terrains, particularly at a low resolution model. Suppose that the original topographic features are lost at a low resolution terrain model. Poor visualization effects and spatial analysis results will be generated.

In this paper, we propose an algorithm for generating multiresolution terrain models with a good performance in terms of good preservation of topographic features and rapid running time. Two key issues are encompassed in the proposed method, namely, measuring deviations between the original terrain
model and its approximations based on a new error metric, detecting and adaptively ranking topographic features based on the Gaussian-weighted of surface curvatures. The former one aims to achieve rapid running time; the latter one aims to preserve topographic features during the generation of multiresolution TIN models hence improve the accuracy of multiresolution models in terms of the RMSEs and Hausdorff distances (Hausdorff distance is defined as the maximal Euclidean distance between any point of original model and the closest point of its approximation.).

## 2. PREVIOUS WORK

There has been extensive research work on generating multiresolution models. In this section, we review previous work, focusing on only those contributions most relevant to this paper. Readers can refer to Luebke D. et al. (2003) for the surveying of these simplification schemes. The local operators including vertex removal, edge collapse, and triangle collapse, are commonly used for generating multi-resolution models. Among these local operators, edge collapse and triangle collapse operators can be conceptualized as gradually shrinking the appropriate geometric primitive such as edge and triangle to a single vertex. Therefore, they are well suited for implementing geomorphing between successive LoDs. Moreover, the advantage of the iterative edge collapse operator is its hierarchical structure which is essential to retaining the topological relationship of the model. The main difference among these edge collapse algorithms is in the selection of the candidate edges and determination of new vertices. Garland and Heckbert (1997) proposed quadric error metrics (QEM) and,

[^0]based on this, developed QSlim software (QSlim 1997). Yang et al. (2005) proposed a simplification method which extends the full-edge collapse and vertex split algorithms to dynamically generate multi-resolution TIN models. However, previous simplification methods have difficulties in preserving topographic features at a low resolution terrain model. Lee et al. (2003) developed a mesh saliency model for capturing visually interesting regions on a mesh. They modified the quadricsbased simplification method (QSlim) of Garland (1999) by weighting the quadrics with mesh saliency. Their work provides a good start in merging perceptual criteria with mathematical measures based on discrete differential geometry for terrains. However, their method is quite time consuming. Therefore, their method is difficult in practice application of the terrain models which are in the great volume of data.

## 3. PROPOSED ALGORITHM

### 3.1 The new quadric error metric

The edge collapse simplification method involves two main issues: one is choosing a measure that specifies the cost of collapsing an edge $e$ (simplification error measure); the other is determining the position of the vertex $V$ that replaces the edge (vertex placement). Kobbelt et al. (1998) has proved that the topological operator to vertex placement does not have a significant impact on the simplification results. The quality of the simplification result is much more sensitive to the criteria which decides where to execute the reduction operation. Therefore, Kobbelt et al. (1998) recommended to apply the topological operator itself as simple as possible. Following their considerations, in this paper, we use half-edge collapse operator (as illuminated in Figure 3) to simplify the input TIN models. We apply the half-edge collapse operator to simplify the topology of original TIN models because this operator does not contain any unset degrees of freedom. Moreover, this reduction operation does not create new geometry, i.e., the new vertex. Therefore, the vertices of the decimated TIN generated by halfedge collapse are always a proper subset of the original vertices.


Figure 1: Half-Edge collapse.
In this paper, we are concerned with rapid producing approximations which remain faithful to the original topographic features. Therefore, the geometric errors should reflect how much that half-edge collapse changes the surface and the error measures also should be cheap to evaluate. Previous QEM-based methods estimate the simplification errors by accumulating the distance of the new vertex from the original surfaces. In order to achieve rapid simplification, we adopted a more efficient measure of these errors.

In our approach, the simplification error is measured by the distance of vertex from the "new" planes which undergo a transformation following a half-edge collapse. As shown in

Figure 1, a half-edge collapse $\left(v_{i}, v_{j}\right) \rightarrow v_{j}$ causes the triangular mesh $T_{n}$ to degenerate and the remaining triangles $T_{n-1}=T_{n}-T_{s}$ to undergo a transformation. A straight solution to estimate the current error during the reduction process is to compute the deviation of the submesh $T_{n-1}$ that replaces the mesh $T_{n}$ in the $n$th step of the simplification. After singular triangles $T_{s}$ are removed, the transformation of $\left(v_{i}, v_{j}\right) \rightarrow v_{j}$ can be interpreted to be pull the vertex $V_{i}$ into $V_{j}$ and its adjacent triangles $T_{t}(i)$ are transformed to $T_{t}^{\prime}(i)$ by replacing $v_{i}$ with $v_{j}$, and vice versa. These triangles $T_{t}(i)$ (red triangles in Figure 1) are defined as the triangles which surround the vertex $V_{i}$ but are not adjacent to $V_{j}$. In our approach, a big difference from the QEM-based method is that the new quadric error metric is defined over the triangles ( $T_{t}^{\prime}(i)$ or $T_{t}^{\prime}(j)$ ) that have been transformed in a half-edge collapse $\left(\left(v_{i}, v_{j}\right) \rightarrow v_{j}\right.$ or $\left.\left(v_{i}, v_{j}\right) \rightarrow v_{i}\right)$ rather than all of the neighbour triangles. As shown in Figure 2, we make an error quadric bounding the region by accumulating both the squared distance $E_{Q\left(v_{i}\right)}$ and $E_{Q\left(v_{j}\right)}$. Where

- $E_{Q\left(v_{i}\right)}$ is the squared distance of the vertex $v_{i}$ from the "new" planes $T_{t}^{\prime}(i)$;
- $\quad E_{Q\left(v_{j}\right)}$ is the squared distance of the vertex $v_{j}$ from the "new" planes $T_{t}^{\prime}(j)$.


Figure 2: Measuring the cost of half-edge collapse.
Garland (1999) applied "Quadric" to assess the cost of edge collapses because it provided a very convenient representation for the squared distance. Readers can refer to (Garland, 1999) for the detail of "quadric" representation. In our approach, we apply this "quadric" to represent the squared distance for its computationally efficient. For example, as shown in Figure 2,
given one of the "new" planes $T_{t}^{\prime}(i)$ which is the set of all points $n \times v+d=0$ with a unit normal $n=\left[\begin{array}{lll}a & b & c\end{array}\right]^{T}$ and a scalar constant $d$, for point $V_{i}$, the quadric $Q$ is defined

$$
\begin{equation*}
Q=(A, b, c)=\left(n n^{T}, d n, d^{2}\right) \tag{1}
\end{equation*}
$$

where $A$ is a $3 \times 3$ matrix, $A=n n^{T}, b$ is a 3 -vector, $b=d n$, and $C$ is a scalar. The squared distance $Q\left(v_{i}\right)$ of point $V_{i}$ to the plane $T_{t}^{\prime}(i)$ can be calculated using the second order equation

$$
\begin{equation*}
Q\left(v_{i}\right)=v^{T} A v+2 b^{T} v+c \tag{2}
\end{equation*}
$$

Therefore, given a set of fundamental quadrics $Q_{i}\left(v_{i}\right)$ determined by a set of planes $T_{t}^{\prime}(i)$, the quadric error $E_{Q\left(v_{i}\right)}$ is computed by the sum of the quadrics $Q_{i}\left(v_{i}\right)$ :

$$
\begin{equation*}
E_{Q\left(v_{i}\right)}=\sum_{i}^{n} Q_{i}\left(v_{i}\right)=\left(\sum_{i}^{n} Q_{i}\right)\left(v_{i}\right) \tag{3}
\end{equation*}
$$

In light of the greedy framework, the collapsed half-edge is selected in an increasing order of cost, which is calculated according to the new quadric error metric. In general, a good simplification algorithm should preserve important regions, such as the regions with salient topographic features. Consequently, the edges in the regions with salient topographic features should have heavier weights (we will introduce the topographic-feature weights in next section). Therefore, these two distances $E_{Q\left(v_{i}\right)}$ and $E_{Q\left(v_{j}\right)}$ should be weighted by the topographic importance of $v_{i}$ and $v_{j}$, respectively. Then, we take the minimum as the cost of this half-edge collapse $C_{e_{i, j}}$ and implement half edge collapse $\left(v_{i}, v_{j}\right) \rightarrow v_{j}$ or ( $v_{i}, v_{j}$ ) $\rightarrow v_{i}$ according to the minimum cost. Therefore, the new quadric error metric can be written as

$$
\begin{equation*}
C_{e_{i, j},}=\min \left\{\omega_{i} E_{Q\left(v_{i}\right)}, \omega_{j} E_{Q\left(v_{j}\right)}\right\} \tag{4}
\end{equation*}
$$

where $\omega_{i}$ and $\omega_{j}$ are the weights from the feature map $W$ for topographic salience of vertex $V_{i}$ and $V_{j}$, respectively. In the case of Figure 2, $\omega_{i} E_{Q\left(v_{i}\right)}<\omega_{j} E_{Q\left(v_{j}\right)}$, so $C_{e_{i, j}}=\omega_{i} E_{Q\left(v_{i}\right)}$, the half-edge collapse $\left(v_{i}, v_{j}\right) \rightarrow v_{j}$ should be implemented, and vice versa. Since the new quadric error metric does not involve the decimated triangles ( $T_{s}$ ) or evaluating the optimal position of new vertex for error measurement and cost determination, it is more computationally efficient than the previous QEM.

### 3.2 Detecting topographic features

Wood (1996) subdivided of all points on a terrain surface into one of plane, peak, pit, ridge, channel, and pass (as illustrated in Figure 3), which are defined as topographic features. The
proposed algorithm firstly detects the topographic features according to the curvature of terrain surface.

(a)

Figure 3: The category of topographic features: (a) plane, (b) peak, (c) pit, (d) ridge, (e) channel, and (f) pass.

The geometric curvature of terrain surface indicates the property of a terrain surface. The mean curvatures of vertices of a surface identify whether they are feature vertex or zerofeature vertex. Extensive research work has been done for estimating discrete mean curvature of TIN based surfaces. Unfortunately, no one is widely accepted as the most accurate method or the best method for curvature estimation. Surazhsky et al. (2003) showed that the paraboloid fitting method (Hamann, 1993) is the best one for estimating the mean curvature of meshes. The paraboloid fitting method approximates a small neighborhood of the TIN model around a vertex $V$ by an osculating paraboloid. The mean curvature of the vertex on the terrain surface is considered to be identical to the mean curvature of the osculating paraboloid. Therefore, in this paper, assume a direction $X$ and $y=Z \times X$, the osculating paraboloid can be represented by following equals

$$
\begin{equation*}
z=a x^{2}+b x y+y^{2} \tag{5}
\end{equation*}
$$

The coefficients $a, b$ and $c$ are found by solving a least square fit to each vertex $V_{i}$ and its neighboring vertices. Then, the mean curvature is computed as $H=a+c$.

In our method, we firstly compute mean curvature for each vertex. Then, we compute the Gaussian-weighted average of the mean curvatures for each vertex within a radius $2 \sigma$, where $\sigma$ is Gaussian's standard deviation. The topographic features are determined at different scales by varying $\sigma$.The multi-scale model is used to ignore local perturbations that go against the overall trend of the linear feature. Let the mean curvature map $M$ define a mapping from each vertex of a TIN model to its mean curvature, i.e. let $M(v)$ denote the mean curvature of vertex $V$. Let the neighborhood $N(v, \sigma)$ for a vertex $V$, be the set of points within a distance $\sigma$. In our scheme, we use the Euclidean distance $N(v, \sigma)=\left\{v_{i}:\left\|v_{i}-v\right\|<\sigma, v_{i}\right.$ is a vertex on the TIN surface $\}$. Let $G(M(v), \sigma)$ denote the

Gaussian-weighted average of the mean curvature. $G(M(v), \sigma)$ can be computed by using underlying formula:

$$
\begin{equation*}
G(M(v), \sigma)=\frac{\sum_{v, \in N(v, 2 \sigma)} M(x) \exp \left[-\left\|v_{i}-v\right\|^{2} /\left(2 \sigma^{2}\right)\right]}{\sum_{v_{i} \in N(v, 2 \sigma)} \exp \left[-\left\|v_{i}-v\right\|^{2} /\left(2 \sigma^{2}\right)\right]} \tag{6}
\end{equation*}
$$

where $\sigma$ is the standard deviation of the Gaussian filter. For all the results in this paper we have used seven scales $\sigma \in\{1 \varepsilon, 2 \varepsilon, 3 \varepsilon, 4 \varepsilon, 5 \varepsilon, 6 \varepsilon, 7 \varepsilon\}$, where $\varepsilon$ is defined as $0.3 \%$ of the length of the diagonal of the bounding box of the model (Lee et al., 2005).

In our method, we assigned a weight to each vertex according to the relationship between it and the global topographic features. Let the topographic feature map $W$ define a mapping from each vertex of a TIN model to its feature. As shown in Figure 4 (b), the mean curvature map may have far too many "bumpy" being flagged as features. However, we can promote salience maps with a small number of high values by calculating Gaussian-weighted mean curvature in large scale. One can see that the topographic features are more coherent in the large-scales. Figure 4(c)-(f) gives an overview of topographic feature map such as peak, pit, ridge, channel and pass in different scales. We use pseudo-colours to texture the surface according to the feature weights: warmer colours (reds and yellows) show high weights, cooler colours (greens) show low weights, and blues show zero-feature. We guide the order of iterative half-edge collapses using a weight map $\omega$ derived from the topographic feature map $W$. In our algorithm, we use values of Gaussian-weighted mean curvature to evaluate the point to the extent of topographic feature. In order to improve the speed of processing, we don't classify different features, such as peak, pit, ridge, channel and pass in our algorithm. However, the feature classification can easily be achieved according to the rules of Wood (1996).


Figure 4: Topographic feature detection: Image (a) shows the "Crater" model, image (b) shows its mean curvature distribution. Images (c) - (f) show the salient features at scales of $1{ }^{\varepsilon},{ }_{3} \varepsilon$,

$$
5^{\mathcal{E}}, \text { and } 7 \mathcal{E}
$$

## 4. RESULTS AND DISCUSSION

Among previous simplification methods, the QEM-based method holds much promise in terms of its time efficiency and relatively high quality of approximations. Garland and Zhou (2005) extended the QEM-based algorithm to simplify simplicial complexes of any type embedded in Euclidean spaces of any dimension and based on this, developed new GSlim software. However, the performance of their newer GSlim system on triangulated models is essentially identical to that of the earlier QSlim 2.0.

Surazhsky and Gotsman (2005) have tested nine softwares for mesh simplification, including both commercial (Geomagic Studio 5.0, Rapidform 2004, 3ds max 7, Maya 5.0 ,
Action3D Reducer 1.1, SIM Rational Reducer 3.1 and VizUp Professional 1.5) and academic offerings (QSlim 2.0 and Memoryless Simplification). They examined these software packages on the seven models of different sizes, properties and acquisition sources. According to their experiment results, they concluded the Hausdorff distance reflects visual fidelity better than the average distance. The possible reason is that a large deviation from the original surface even at just a small localized feature of the mesh can significantly affect the visual perception of the model, and this will be reflected in the Hausdorff distance even if the rest of the simplified mesh is very close to the original. In their experiments, Geomagic Studio was the leader with respect to the Hausdorff distance.

Therefore, in the experiments, we use "Crater" model to compare our scheme with QSlim 2.0, which use area-based weights and optimizes vertex locations, and Geomagic Studio 8, which is the latest version of Geomagic Studio, for generating multi-resolution models in terms of visual performance, geometric errors (RMS), Hausdorff distance and time performance. Our approach was implemented in C++ language on Windows XP operation system platform. The experiment was undertaken in a 3.0 GHz Intel Pentium IV machine with 512 MB of main memory. Figure 5 shows the "peak" in multiresolution "Crater" model generated by QSlim 2.0, Geomagic Studio 8 and our scheme ( $\delta=3 \varepsilon$ ) from 199,114 triangular faces to $4,000,2000$, and 300 triangular faces, respectively. One can see that our new algorithm has better performance in terms of the preservation the topographic features.

(a) The "peak" of the original "Carter" model and the results simplified by QSlim 2.0.

(b)The results simplified by Geomagic Studio 8.

(c) Topographic feature map ( $\sigma=3 \varepsilon$ ) and the results simplified by our scheme.

Figure 5: The visual comparison of "peak" by QSlim (Garland, 1999), Geomagic Studio 8 and our new scheme (4000, 2000 and 300 triangles, respectively).

Figure 6 shows the multi-resolution "Gargano" model generated by QSlim, Geomagic Studio 8 and our new scheme, respectively. It is seen that the regions with important topographic features were simplified less than other regions and topographic features were better preserved. Although in this case, our method preserves the desirable high curvature regions with topographic features, it can also selectively ignore the undesirable high curvature regions, such as small perturbations in flat areas. Figure 7 shows the multi-resolution "Shasta" model from our new scheme by varying $\sigma$. Notice how our method retains more triangles around the regions with topographic features than other regions.

(c) Results from Geomagic with 7800 and 3900 triangles

(d) Results from our scheme with 7800 and 3900 triangles

Figure 6: Multi-resolution "Gargano" model from QSlim (Garland, 1999), Geomagic Studio 8 and our new scheme: image (a) shows the original model with 780,992 triangles, image (b), (c) and (d) show results from QSlim 2.0, Geomagic Studio 8 and our new scheme, respectively.

(a) An original "Shasta"model
(b) $\sigma=1 \varepsilon$

(c) $\sigma=3 \varepsilon$
(d) $\sigma=5 \varepsilon$

Figure 7: Multi-resolution "Shasta" model from our new scheme: image (a) shows the original model with 935,712 triangles, image (b), (c), (d) show results from our new scheme with 3,000 triangles by $\sigma=1 \varepsilon, \sigma=3 \varepsilon, \sigma=5 \varepsilon$, respectively.


Figure 8: The comparison of geometric error and execution time of multi-resolution "Gargano" and "Shasta" models, generated by QSlim 2.0, Goemagic Studio 8 and our new schemes. These errors were measured using Metro 4.06 (Cignoni et al., 1998).

It is seen from Figure 8 that our new method improves the accuracy of multi-resolution models. The relative errors are
measured by Metro 4.06 (Cignoni et al., 1998) which is based on sampling and point-to-surface distance computation. The geometric error results show that our new scheme has a better performance in terms of RMS geometric error and Hausdorff distance. While our algorithm with adaptive weights from topographic feature map improves the accuracy greatly in terms of Hausdorff distance. We believe that the reason behind this is that by leaving high LoD at the regions with important features we are ensuring these topographic shapes, which usually become deformed in previous simplification, are well preserved by our scheme. Therefore, the comparison demonstrates that our new scheme is able to achieve a higher accuracy multiresolution model and improve the visual fidelity of multiresolution model.

Execution time shows that our scheme with new error metrics is the fastest of the methods, followed by QSlim 2.0 and Geomagic Studio 8. As shown in Figure 8, the time to compute topographic feature map depends on the scale at which it is computed. Larger scales ensure greater continuity of topographic features, but also require identification and processing of a larger number of neighborhood vertices and therefore are more time consuming. Spatial data-structures such as a grid, while a wedge or an octree can greatly improve the running time for establishing the neighborhood at a given scale.

## 5. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed a half-edge collapse method in the greedy framework for constructing multi-resolution model with topographic features preservation. We developed a model of topographic feature detection using center-surround filters with Gaussian-weighted mean curvatures in a multi-scale manner. We elaborated the calculation of the new quadric error metric and the weights from topographic feature map. We have shown how incorporating feature weights can visually enhance the results. For a number of examples we have shown in this paper, one can see that our method is able to preserve regions with topographic features.

We use various models to evaluate the performance of our proposed scheme in terms of feature preservation, geometric errors and execution times. Moreover, the multi-resolution models generated by our scheme were compared with those did by QSlim and Geomagic. The comparisons show that our scheme is able to generate visually superior and more accurate multi-resolution models, and preserve important topographic features particularly at a low LoD. Moreover, the new error metric evaluates more quickly. It should be equally easy to integrate our weight map with any other mesh simplification scheme, such as QSlim.

The model of topographic feature detection promises to be a rich area for further research. We are currently defining topographic features using mean curvature. It should be possible to improve this by using better measures of shape, such as principal curvatures and normals. Our current method of computing topographic feature map in large scale takes a long time. It should be possible to significantly speed it up by using a multi-resolution mesh hierarchy to accelerate filtering at coarser scales. We foresee the computation and use of topographic feature detection as an increasingly important area in TIN data processing. A number of tasks can benefit from a computational model of topographic feature detection, such as feature line extraction and shape analysis of TIN model.

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