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APPLYING TWO DIMENSIONAL KALMAN FILTERING FOR DIGITAL TERRAIN MODELLING

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ABSTRACT

Digital Elevation Models (DEMs) have been increasingly used to model the terrain surfaces to provide the 'physical bases' for environmental studies. However, DEM is subject to systematic errors, random noise and outliers. In this paper, a newly developed two dimensional (2-D) Kalman filtering approach to generating optimal estimates of terrain variables from a noisy grid DEM is introduced, which comprises a 2-D Kalman processor, a function for outlier detection and removal, and a two-step filtering procedure. The experiment of a simulated surface indicates that after applying the developed 2-D Kalman filtering technique the outliers of a DEM can be efficiently detected and well removed. The standard deviation of random noise of the DEM can be significantly reduced by approximately 70% for elevation and about 85% for the first partial derivatives of elevation, compared with their original values. The experiment of slope calculation shows that using this approach, the effect of random noise and outliers of slope can be efficiently reduced by more than 60% in terms of standard deviation, of the results derived from Arc/Info software using the same simulated DEM data.

1 INTRODUCTION

Digital Elevation Models (DEMs) have been increasingly used to model terrain surfaces in order to provide the 'physical bases' for environmental studies. Elevations together with gradient, slope and aspect derived from a DEM, are often used as the basic terrain variables to implement such terrain monitoring (Martz et al 1992, Moore et al 1993, Zhang et al 1994, Lee 1994, Meisels et al 1995, Pilotti 1996, Gysai-Agyei et al 1996, Tarboton 1997, Doytsher et al 1997). However, DEM are subject to systematic errors, random noise and outliers. The accuracy of a DEM and its derivative terrain variables are of crucial importance for DEM applications, since the errors in the DEM will be propagated through the spatial analysis based on the data (Bolstad et al 1994). It has been estimated that many of today's terrain modelling algorithms have not adequately dealt with the effects of elevation errors (Hunter et al 1997), or they have only adopted a simplified approach for DEM error treatment. Some algorithms produce poor results or they cause significant artefaces in the interpretation of the data (Zhuo et al 1997).

The issue of quality control and accuracy assessment of DEM has been extensively studied, but it is more recently that some researchers have investigated the quality of the derivatives of DEM. Bolstad et al (1994) attempted to compare the accuracy of elevation, slope and aspect of SPOT-based DEM and USGS 7.5-minute DEM with field measurements. Kraus (1994) confirmed that the random noise in a DEM affects the quality of representation of contours and slope lines. Hodgson (1995) determined the best cell size for the computation of slope aspect angle in a DEM. Rieger (1996) linked the slope errors with the non-independent height errors. Giles et al (1996) warned that caution must be taken before using a DEM and its derivative of topographic surfaces as estimates of the true landscape configuration. Hunter et al (1997) showed that errors in slope and aspect depend on the spatial structure of DEM errors, and attempt to model the propagation of such errors through GIS operations. However, this work does not comprise a general approach to systematically process the random errors, and detect and remove the outliers of a DEM in order to generate reliable and accurate terrain variables for better terrain interpretation. Such an approach is required in current applications of DEM, especially for the GIS industry.

In this paper, a new approach to applying the technology of digital signal processing for terrain modelling, ie. 2-D Kalman filtering technique, is presented. It comprises a newly developed 2-D Kalman vector processor, a function for outlier detection and removal and a two-step filtering procedure. This approach is developed to reduce the effect of the random errors, as well as detect and correct outliers in a DEM, while producing optimal estimates of terrain variables from noisy DEM data, which can be used for various terrain modelling applications.

2 2-D KALMAN FILTERING FOR DIGITAL TERRAIN MODELLING

Kalman filtering technique was firstly introduced in 1960 (Kalman 1960). But it was not until the late 1970's that serious attempts were made to extend Kalman filtering to two dimensions (Woods et al 1977, Woods et al 1981, Azimi-Sadjadi et al 1987, Tekalp et al 1989, Wu et al 1992). However, most existing 2-D Kalman processors have been developed for image processing, especially image restoration. They do not fully suit applications for terrain modelling using grid DEM for two reasons. Firstly, Kalman filtering is a model-based filter (Brown et al 1992). Therefore, if a 2-D Kalman filtering recursive algorithm is suitable for image restoration, it does not guarantee that its established model is suitable for other 2-D applications, such as DEMs. Secondly, in contrast to the applications of image restoration, terrain modelling using grid DEM has its own characteristics. Elevation is not the only terrain variable of interest for terrain modelling. Estimates of other terrain variables are also required to be derived from a noisy DEM for terrain modelling purposes. For these reasons the existing 2-D Kalman filtering techniques have not been adopted, but a new 2-D Kalman filtering algorithm for terrain modelling application has been developed.

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2.1 2-D Kalman Processor

An important concept of Kalman filtering is the 'State'. The state of a system is defined as the minimum information about the past and present, needed to determine all future responses of a system given the future input (Padulo et al 1974). In a certain random case, it can be considered as the minimum amount of information about past and present estimates, needed to determine an optimal casual estimate of future responses, given future noisy observations (Wood et al 1977). The concept of state actually forms the stochastic difference equations, ie. dynamic model, to govern Kalman filtering.

Elevation $H(i, j)$, the first partial derivative of the elevation along the X direction $H_x(i, j)$, and the first partial derivatives of the elevation along the Y direction $H_y(i, j)$ have been chosen to form our state vector. We use the term 'the first partial derivatives of the elevation' to represent the slopes along X and Y directions

$$\mathbf{S}(i, j) = \begin{bmatrix} H(i, j) \\ H_x(i, j) \\ H_y(i, j) \end{bmatrix} \quad (1)$$

We assume that the processor computes the DEM data in a certain order, ie. from left to right, from top to bottom, as in raster scanning. Therefore, at any processing point, some DEM points will be considered as the 'past' (processed points), some will be the 'future' (un-processed points), and the currently processing point is the 'present' point.

According to terrain geometry, we assume that the state vector of point (i, j) can be modelled by its neighbouring points $(i-1, j)$ and $(i, j-1)$ to some degree

$$\begin{aligned} H(i, j) &= b(i, j)\left[H(i-1, j) + H_x(i-1, j)dx\right] + \\ &\quad c(i, j)\left[H(i, j-1) + H_y(i, j-1)dy\right] + v_H(i, j) \\ H_x(i, j) &= H_x(i-1, j) + v_{H_x}(i, j) \\ H_y(i, j) &= H_y(i, j-1) + v_{H_y}(i, j) \end{aligned} \quad (2)$$

Where v_H, v_{H_x} and v_{H_y} are assumed as white sequences, dx and dy are the sampling intervals of the DEM, and $b(i, j)$ and $c(i, j)$ are blending factors, with a summation of 1.

Re-writing the **Eq.2**, a dynamic model of this random process is

$$\mathbf{S}(i, j) = \mathbf{B}(\mathrm{i, j}) \cdot \mathbf{S}(i-1, j) + \mathbf{C}(\mathrm{i, j}) \cdot \mathbf{S}(i, j-1) + \mathbf{v}_\mathrm{S}(i, j) \quad (3)$$

where, $\mathbf{B}(i, j) = \begin{bmatrix} b(i, j) & b(i, j)dx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$\mathbf{C}(i, j) = \begin{bmatrix} c(i, j) & 0 & c(i, j)dy \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{v}_\mathrm{S}(i, j) = \begin{bmatrix} \mathrm{v_H}(i, j) \\ \mathrm{v_{H_x}}(i, j) \\ \mathrm{v_{H_y}}(i, j) \end{bmatrix}$,

which is a white sequence with known covariance structure $\mathbf{Q}(i, j)$.

The observation of elevation is assumed to have a linear relationship with the state vector at the relevant position:

$$Z(\mathrm{i, j}) = \boldsymbol{D} \cdot \boldsymbol{S}(\mathrm{i, j}) + \mathrm{v_z}(\mathrm{i, j}) \quad (4)$$

where $\boldsymbol{D} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, and $\mathrm{v_z}(\mathrm{i, j})$ is a white sequence with known covariance structure $R(i, j)$ and having zero cross-correlation with the $\boldsymbol{v}_S(i, j)$ sequence.

Eq.4 is also known as the functional model of 2-D Kalman processor.

Based on these assumptions, according to Wang et al (1998), the following equations can be derived to control the recursive Kalman filtering process:

$$\mathbf{S}^{-}(\mathrm{i, j}) = \mathbf{B}(\mathrm{i, j}) \cdot \mathbf{S}^{+}(\mathrm{i-1, j}) + \mathbf{C}(\mathrm{i, j}) \cdot \mathbf{S}^{+}(\mathrm{i, j-1}) \quad (5)$$

$$\begin{aligned} \mathbf{P}^{-}(i, j) &= \mathbf{B}(i, j) \cdot \mathbf{P}^{+}(i-1, j) \cdot \mathbf{B}(i, j)^\mathrm{T} + \\ &\quad \mathbf{C}(i, j) \cdot \mathbf{P}^{+}(i, j-1) \cdot \mathbf{C}(i, j)^T + \mathbf{Q}(i, j) \end{aligned} \quad (6)$$

$$\mathbf{K}(i, j) = \mathbf{P}^{-}(i, j)\mathbf{D}^T(\mathbf{D}\mathbf{P}^{-}(i, j)\mathbf{D}^T + R(i, j))^{-1} \quad (7)$$

$$\mathbf{S}^{+}(i, j) = \mathbf{S}^{-}(i, j) + \mathbf{K}(i, j)(Z(i, j) - \mathbf{D}\mathbf{S}^{-}(i, j)) \quad (8)$$

$$\mathbf{P}^{+}(i, j) = \left[\mathbf{I} - \mathbf{K}(i, j) \cdot \mathbf{D}\right] \cdot \mathbf{P}^{-}(i, j) \quad (9)$$

where

$\mathbf{S}^{-}(i, j)$ predict estimate of state vector,

$\mathbf{P}^{-}(i, j)$ covariance matrix associated with $\mathbf{S}^{-}(i, j)$,

$\mathbf{K}(i, j)$ Kalman gain,

$\mathbf{S}^{+}(i, j)$ updated estimate of state vector, and

$\mathbf{P}^{+}(i, j)$ covariance matrix associated with $\mathbf{S}^{+}(i, j)$.

The matrices of $\mathbf{B}(i, j)$ and $\mathbf{C}(i, j)$ should be determined according to the variance and covariance factors included in $\mathbf{P}^{+}(i-1, j)$ and $\mathbf{P}^{+}(i, j-1)$. In practice, they can be approximated as a constant value of 0.5, which means that the contributions from X and Y directions to any current processing point are of equal weights.

2.2 Outlier Detection and Removal

To detect the outliers from the valid observations of a DEM, an innovation series $L(i, j)$ is formed to represent the difference between the predict estimate of elevation $H^{-}(i, j)$ and its observation $Z(i, j)$ at any arbitrary DEM position (i, j)

$$L(i, j) = z(i, j) - H^{-}(i, j) \quad (10)$$

Based on the assumption of the Kalman dynamic model and functional model, $L(i, j)$ can be proved as a normal distribution statistic variable with a zero mean and variance $\sigma^2_{L(i,j)}$, which can be determined by

$$\sigma_{L(i,j)} = \sqrt{\sigma^2_{H^{-}(i,j)} + R(i, j)} \quad (11)$$

where $\sigma^2_{H^-(i,j)}$ is the variance associated with $H^-(i,j)$, and can be calculated through **Eq.6**. $R(i,j)$ is the observation variance known in **Eq.4**.

Based on relevant probability theory, if a measurement is quantitatively close to the mean value 0, there will be a great chance of it to take place. In other wards, if an observation of $L(i,j)$ is too different from 0, it will be reasonable to suspect its validity and reject it, since this kind of event may unlikely to take place at all.

Therefore, $L(i,j)$ can be used as a test statistic to detect outliers in elevation measurements using criterion

$$|L(i,j)| > \xi_\alpha \cdot \sigma_{L(i,j)} \quad (12)$$

where ξ_α is a critical value with a risk level α.

If **Eq.12** is true, the measurement $Z(i,j)$ will be considered to contain an outlier. Otherwise, the elevation measurement is valid. The critical value can be mathematically determined by the risk level. A risk level is typically chosen as $\alpha = 5\%$, $\alpha = 1\%$, or $\alpha = 0.1\%$, while the subsequent critical values will be $\xi_{0.05} = 1.96$, $\xi_{0.01} = 2.58$, or $\xi_{0.001} = 3.89$, respectively.

Once an outlier is detected, it will be removed through the derivation of the Kalman gain in **Eq.7** by amplifying the observation error $R(i,j)$ into a large value. In this way, the effect of the outlier to the current updated estimate of elevation $H^+(i,j)$ will be eliminated. The updated estimate of state vector $\mathbf{S}^+(i,j)$ **(Eq.8)** and its covariance matrix $\mathbf{P}^+(i,j)$ **(Eq.9)** will remain stable by following their predictions given in **Eqs.5-6**.

2.3 A Two-Step Filtering Procedure

According to Kalman filtering theory, the quality of the state vector will vary according to its position within a DEM, since an estimate of the state vector is optimally derived using all available information of the processed points. While the filtering progresses, the numbers of the processed points will increase, so the state vector of the points close to the end of a processing record will be generally better than those close to the starting positions.

To overcome this problem, we introduce the two-step filtering procedure, by applying the suggested 2-D Kalman processor twice over the same DEM data with different orientations. Firstly, we apply the processor from the left to right and from top to bottom. Then, the same process is applied in reverse, ie. from the right to left and bottom to top. For any DEM point at position (i,j), the first process will generate an optimal estimate of its state vector denoted as $\mathbf{S}_\mathrm{F}{}^+(i,j)$ and its covariance matrix $\mathbf{P}_\mathrm{F}{}^+(i,j)$ based on a half plane of the DEM. The second process will generate another optimal estimate $\mathbf{S}_\mathrm{B}{}^+(N-i,M-j)$ and its $\mathbf{P}_\mathrm{B}{}^+(N-i,M-j)$ using the other half plane of the DEM, which has not been used for the first round process for that point. The 'final' estimate of the state vector of point (i,j) and its covariance are derived as

$$\mathbf{S}_{final}(i,j) = \left[\left(\mathbf{P}_B^+(N-i,M-j)\right)^{-1} + \left(\mathbf{P}_F^+(i,j)\right)^{-1}\right]^{-1} \cdot$$
$$\left[\left(\mathbf{P}_F^+(i,j)\right)^{-1}\mathbf{S}_F^+(i,j) + \left(\mathbf{P}_B^+(N-i,M-j)\right)^{-1}\mathbf{S}_B^+(N-i,M-j)\right]$$
$$\mathbf{P}_{final}(i,j) = \left[\left(\mathbf{P}_B^+(N-i,M-j)\right)^{-1} + (\mathbf{P}_F^+(i,j))^{-1}\right]^{-1} \quad (13)$$

where N, M are the numbers of rows and columns of a DEM.

3 EXPERIMENTAL RESULTS

A mathematical surface was simulated to test the efficiency of the developed algorithm in terms of the efficiencies of detection and removal of outliers, and reduction of random noise. The elevation surface is a smooth curve surface with the linear first partial derivatives along the X and the Y directions:

$$Z(i,j) = (i-75)\cdot(50-j)/400 \quad (14)$$

The surface is sampled at 10 metre interval with a grid size of 150 X 150. It is subject to random noise, which varies from –0.43 to 0.38 metre with a standard deviation of 0.1 metre. Three outliers are added at the positions of $(50,40)$, $(118,80)$ and $(90,100)$. The smooth surface of the simulated DEM, its derivatives along the X and Y direction can be used as ground truth shown in **Figs.1-3**. The noisy DEM is shown in **Fig.4**.

After the first filtering process, the estimates of elevation and the derivatives along the X and Y are generated at the same time shown in **Figs.5-7**. The results of the second filtering process are shown in **Figs.8-10**. The final estimates of the elevation, the first partial derivatives along the X and Y directions are shown in **Figs.11-13**.

The estimates of elevations are considerable smooth for the first and second filtering process (see **Fig.5** and **Fig.8**). The summation result (see **Fig.11**) is a smooth surface too. All derivatives (**Figs.6-7** and **Figs.9-10**) commence with poor estimates, which is due to inaccurate initial estimates. However, the estimates have improved significantly along the X or Y direction. They become stable as the processing progresses along these two directions, since more points contribute to the estimates of these derivatives. Both estimates of the derivatives are much smoother after the two-step filtering procedure is applied (see **Figs.12-13**).

3.1 Efficiency for Outlier Detection and Removal

While the filtering processes, the function of outlier detection is applied at each processing point to identify the possible invalid observations. Using the algorithm over the noisy DEM, three outliers have all been efficiently

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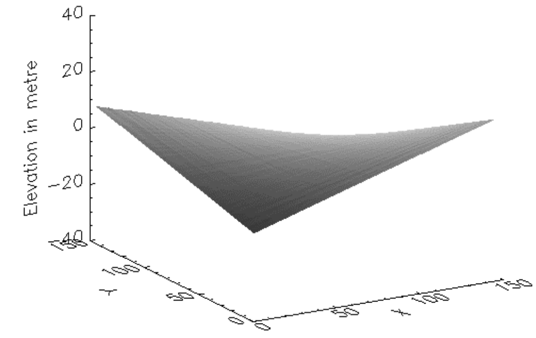

Fig.1 Simulated smooth DEM surface

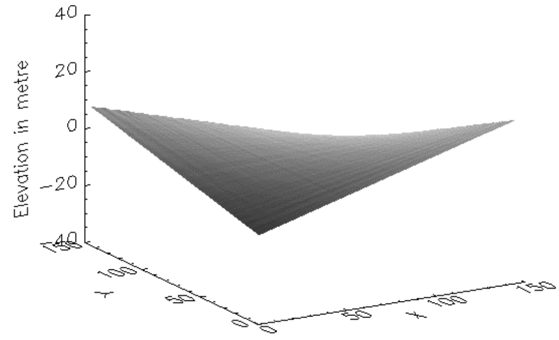

Fig.5 Estimates of elevations of 1st filtering process

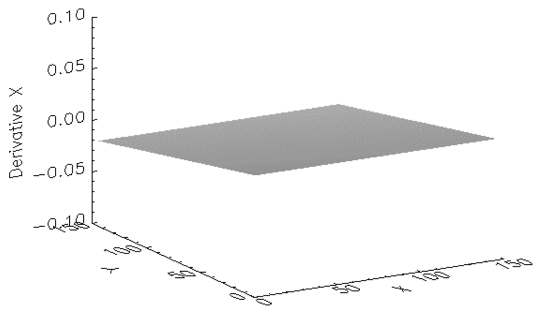

Fig.2 Smooth surface of derivatives along X direction

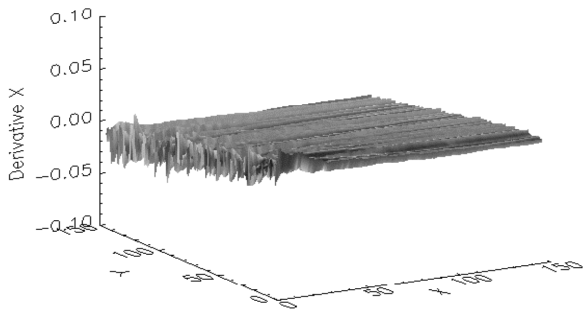

Fig.6 Estimates of derivatives X of 1st filtering process

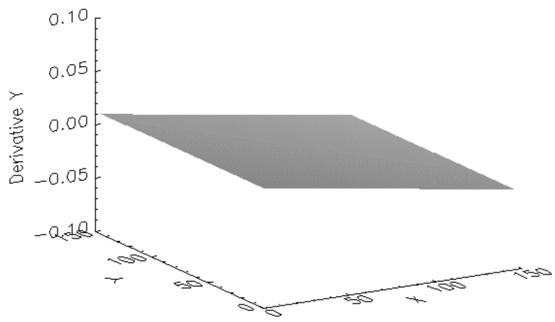

Fig.3 Smooth surface of derivatives along Y direction

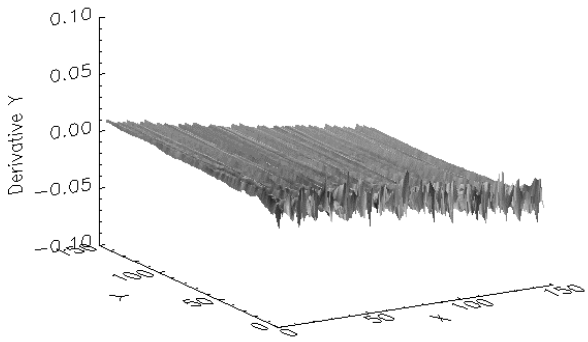

Fig.7 Estimates of derivatives Y of 1st filtering process

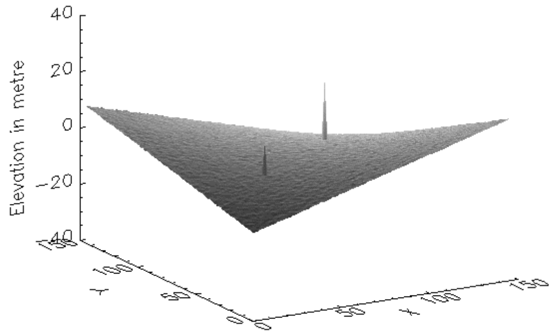

Fig.4 Noisy DEM with random noise and outliers

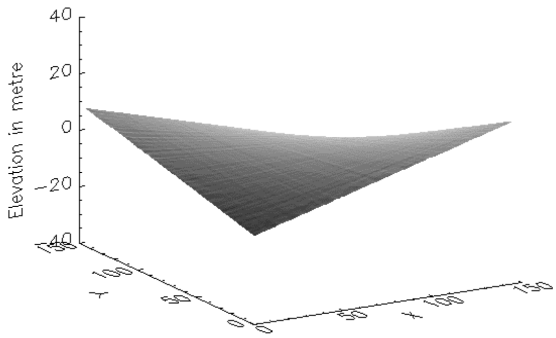

Fig.8 Estimates of elevations of 2nd filtering process

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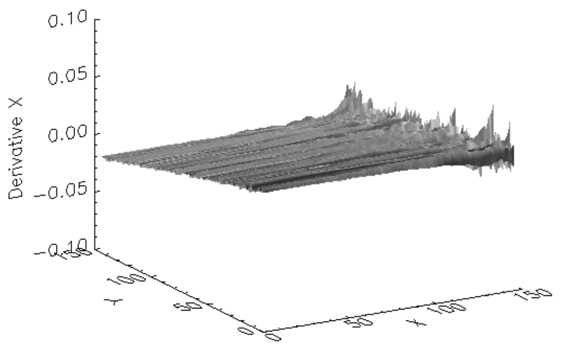

Fig.9 Estimates of derivatives X of 2nd filtering process

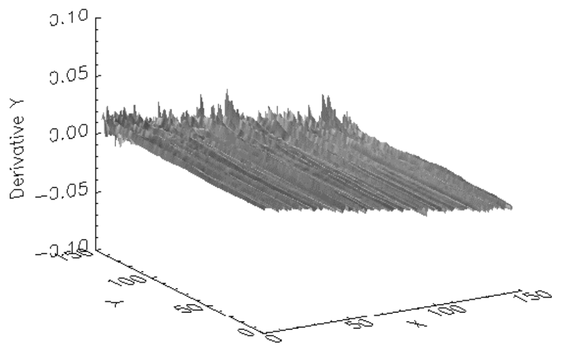

Fig.10 Estimates of derivatives Y of 2nd filtering process

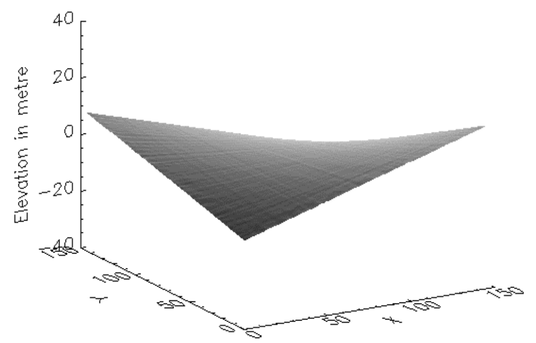

Fig.11 Summation results of elevations

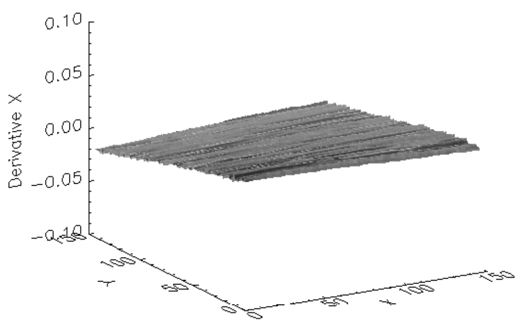

Fig.12 Summation results of derivatives X

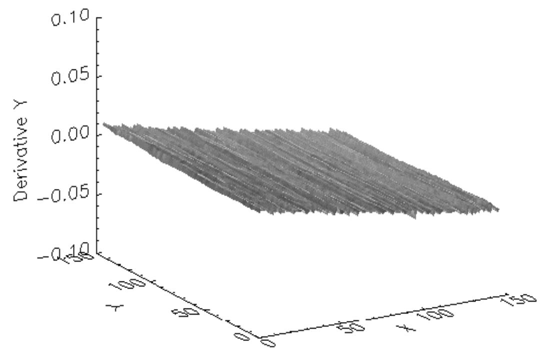

Fig.13 Summation results of derivatives Y

detected and removed from a large number of valid DEM points during the first round filtering process, as well as the second round filtering process. Compared with the original noisy DEM, the surfaces of estimates of the elevations (**Fig.5** and **Fig.8**) are all very smooth at the relevant positions where outliers occurred.

Table 1 Outlier Detection and Removal

(unit in metre)

	Outlier #1	Outlier #2	Outlier #3
Outlier	10.05	-15.10	18.75
True value	-0.72	-3.05	-1.72
Estimates 1	-0.78	-3.04	-1.74
Difference 1	**0.05**	**-0.01**	**0.02**
Estimates 2	-0.73	-3.06	-1.72
Difference 2	**0.01**	**0.01**	**0.00**
Estimates 3	-0.75	-3.05	-1.73
Difference 3	**0.03**	**0.00**	**0.01**

The true elevations of the outliers could be obtained (see the second row of **Table 1**), since the exact positions of the outliers are know. The Estimates 1 to 3 in **Table 1** represent the updated estimates of elevations of the first and second filtering process, and the summation process at the positions where outliers occur. The differences between Estimates 1 to 3 and the true elevations are denoted as Difference 1 to 3 shown in this table.

From this table, the updated estimates of elevations of the three outliers are all of high quality with as little as ± 0.05 metre difference compared with their true values for a single filtering process. The summation results are even more accurate than the estimates of the single process, which have further reduced the difference to ± 0.03 metre.

3.2 Efficiency for Random Noise Reduction

The smooth surfaces of elevation and the derivatives shown in **Figs.1-3,** can be used as ground truth to examine the efficiency of the reduction of the effect of a random noise.

The original random noise of elevation is obtained by subtracting the noisy DEM (without outliers) from the smooth DEM **Fig.1**. The subtracting result can be considered as the effect of the original random noise to

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the smooth elevations, which is evaluated into a mean and standard deviation shown in the first row of **Table 2**. The effect of the random noise remained in the estimates of elevations of the first filtering process, can also be obtained by subtracting the estimates of elevations from its ground truth, ie. **Fig.1**, and represented at the second row of the table. Similarly, the random noise remained in the second filtering process and the summation are derived. The results are shown in the third and fourth row in **Table 2**, respectively.

Following this way, the effect of random noise to the estimates of the derivatives X or Y are calculated by subtracting the relevant estimates from their ground truth **Fig.2** or **Fig.3**. The relevant results are shown in **Table 3** and **Table 4**.

Table 2 Statistical Analysis of Estimates of Elevations

(unit in metre)

	Mean	Standard Deviation	Std / Original Std
Original Random Noise	0.00	0.10	100%
1st filtering result	0.00	0.04	**40%**
2nd filtering result	0.01	0.04	**40%**
Summation result	0.01	0.03	**30%**

Table 3 Statistical Analysis of Estimates of Derivatives X

	Mean	Standard Deviation	Std / Original Std
Original Random Noise	----	0.014	100%
1st filtering result	0.006	0.004	**29%**
2nd filtering result	0.003	0.004	**29%**
Summation result	0.005	0.002	**14%**

Table 4 Statistical Analysis of Estimates of Derivatives Y

	Mean	Standard Deviation	Std / Original Std
Original Random Noise	----	0.014	100%
1st filtering result	-0.006	0.004	**29%**
2nd filtering result	-0.003	0.003	**21%**
Summation result	-0.005	0.002	**14%**

Form these tables, it is quite clear that after applying the 2-D Kalman filtering algorithm, the effect of the random noise for the three terrain variables has all been minimised in terms of mean and standard deviation. The standard deviation of the random noise in the elevation is reduced to 40% of its original level after the first filtering process. In the case of the derivatives, this number is reduced even further, to about 29% of their original level. The situation is similar in the second filtering process, which shows that the suggested 2-D Kalman processor produces similar result despite the order of the processing. The summation of the results of the two round processes significantly reduces the standard deviation of random noise, to approximately 30% of its original level for the elevation, and about 14% of its original values for the both derivatives.

3.3 Comparison of the Suggested Algorithm with Arc/Info Software for Slope Computation

The three terrain variables can be used as the basic attributes for digital terrain modelling. Here an example is shown using the optimal estimates of the derivatives derived by 2-D Kalman filtering algorithm to generate the accurate slope of the same surface. This result is compared with other terrain modelling algorithms for slope calculation.

There exist different algorithms to calculate slope from a grid DEM taken into account the effect of DEM errors (Skidmore 1989, Srinivasan et al 1991). Among them, the following algorithm (Burrough 1986) has a strong 'smoothing' effect for the local DEM errors, and is believed amenable to a mathematical analysis of error propagation (Hunter et al 1997). The algorithm has also been implemented into the GIS software Arc/Info for slope computation:

$$slope_{\max} = \sqrt{(\partial Z / \partial x)^2 + (\partial Z / \partial y)^2} \quad (15)$$

where $slope_{\max}$ the maximum slope

$\partial Z / \partial x, \partial Z / \partial y$ gradients

A maximum weighted algorithm is adopted to derived the gradients using the following equations:

$$\partial Z / \partial x = [z(1,1) + 2z(1,0) + z(1,-1) - z(-1,1) - 2z(-1,0) - z(-1,-1)] / (8 \times dx)$$
$$\partial Z / \partial y = [z(1,1) + 2z(0,1) + z(-1,1) - z(1,-1) - 2z(0,-1) - z(-1,-1)] / (8 \times dy) \quad (16)$$

where $Z(i, j)$ is the elevation of point (i, j) at the relevant position within the 3 X 3 scanning window, and dx, dy are sampling intervals of the DEM.

The result using Arc/Info software on the simulated noisy surface is shown in **Fig.14**. Another slope surface **Fig.15** is generated using the estimates of the derivatives H_x and H_x derived from the 2-D Kalman filtering algorithm and **Eq.17**.

$$slope_{\max} = \sqrt{(H_x)^2 + (H_y)^2} \quad (17)$$

The true slope surface using the smooth DEM data is also generated shown in **Fig.16**, which varies from 0° to 1.78° with 0.85° as its mean value.

Arc/Info software has failed to detect three outliers and produced a biased slope surface with three dominating

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

peaks where the outliers occurred. The effect of the random noise and outliers remained in **Fig.14** is as much as 0.62° in terms of the standard deviation.

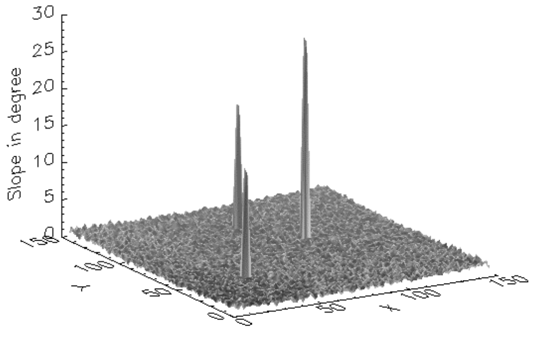

Fig.14 Slope surface using Arc/Info software and noisy DEM

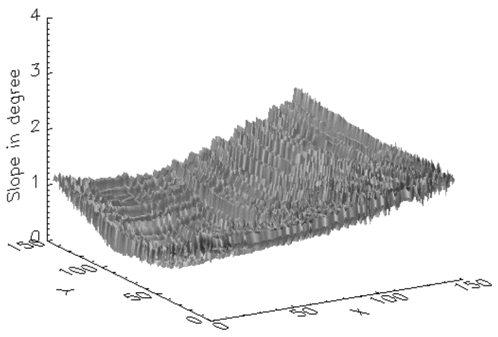

Fig.15 Slope surface using 2-D Kalman filtering method

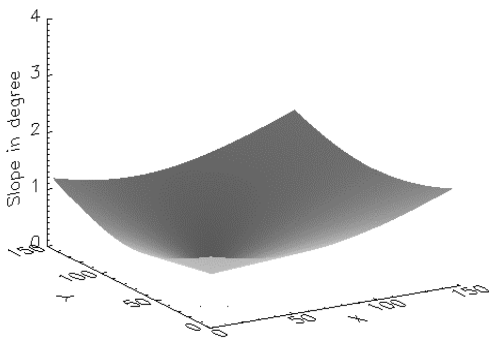

Fig.16 Smooth slope surface

Using the suggested 2-D Kalman filtering algorithm and **Eq.17**, the slope surface **Fig.15** clearly monitored the trends of the variation of the slope. The difference between the **Fig.15** and **Fig.16** is as little as 0.22° in terms of standard deviation (see **Table 5**).

Table 5 Statistical Analysis of Slope Calculation

(unit in degree)

	2-D Kalman Filtering	Arc/Info
Std	0.22	0.62

The algorithm adopted by Arc/Info is considerably powerful for smooth DEMs for calculating slope, but better slope calculation is achieved using the suggested algorithm and the relevant formula. This is due to its strong capability of detecting and removing outliers, as well as processing random noise. The estimates of terrain variables are generated using all the information of a DEM. Therefore, it is much more reliable and efficient to process the random noise and outliers, compared with those which can only use a limited number of local neighbouring points, such as 3 X 3 window, to produce the estimates of terrain variables.

4 CONCULSIONS

A newly developed 2-D Kalman filtering approach to generating estimates of terrain variables from a noisy grid DEM is introduced, which comprises a 2-D Kalman processor, a function for outlier detection and removal and a two-step filtering procedure. The state vector developed in this algorithm generates not only the optimal estimate of elevation from a noisy DEM, but also the estimates of the two first partial derivatives of elevation at the same time. The statistical analysis of the filtering over a simulated surface shows that the outliers can be well detected from a large number of valid DEM observations, and then accurately removed. The standard deviation of random noise can be reduced by approximately 70% for elevation, and 85% for its derivatives, compared with their original values after applying the two-step filtering process. The derived optimal estimates of terrain variables can be used for accurate digital terrain modelling. An example shows that estimates of the derivatives of elevations derived from the suggested algorithm, produces more accurate estimates of slope surface than those derived from Arc/Info software. In addition, this approach is especially useful as an analysis tool in further error analysis studies of the derived terrain variables, since the filtering covariance at each point is provided.

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