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A FLEXIBLE MULTI-MEDIA BUNDLE APPROACH

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ABSTRACT:

Multi-media photogrammetry has gained increased importance in many application fields over the last decades. Nowadays many measurement tasks have to be carried out under special conditions, where the light beam from an object to the imaging sensor passes different optical media. This paper describes the concept of a multi-media module and its integration into a bundle adjustment tool. The presented approach allows the simultaneous determination of refractive indices as well as the surface parameters of the interfaces. There is neither a restriction to the number of passed media nor the shape of the surfaces between them. Only a mathematical model of the surface has to be introduced. The contribution covers the basic concept as well as the algorithmic system. Special focus is given on the description of the implemented ray-tracing model. Further, the paper addresses strategies to decrease computational efforts and problems with extreme inclination angles between ray and interface. Finally, the multi-media bundle approach is tested in an experimental environment.

1. INTRODUCTION AND MOTIVATION

To perform photogrammetric measurements in static or dynamic environments, where the camera and the object of interest are not in the same optical media, requires the extension of standard photogrammetric imaging models. The ray between the perspective centre of the camera and an object point is no longer a straight line. It becomes a poly-line with the object point as start and the corresponding image point as end point. Assuming all passed media to be homogeneous and isotropic, the supporting points in between are the piercing points of the image ray through the interfaces of the different media – besides the principal point of the imaging system.

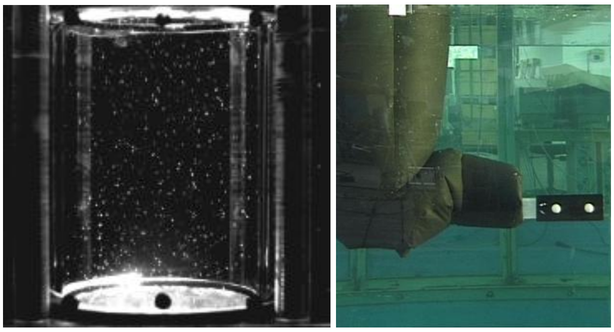

Figure 1. Measurement tasks in multi-media environments. Left: Particles in water (Putze, 2009). Right Positioning of a robotic arm in a water basin (Bachmann, 2000).

For special applications, many specialized solutions are given: The first suggestions came from (Rinner, 1948). His aim was the stepwise reduction of the problem down to known procedures of standard (single-media) photogrammetry on analogue instruments. One suggested method was the introduction of a radial correction term to correct the image coordinates for refraction effects. An application of this method for an underwater photogrammetric system is given in (Li et.al., 1997). (Rinner, 1948) also suggested the inclusion of a compensation term for the focal length or the orthonormal distance of the projection centre to the interface. (Maas, 1995) developed a simplified correction module, which computes a radial shift of an object point relative to the nadir point of the respective camera. This term can be introduced directly to the collinearity equation. The limitation of this method is that one coordinate system plane has to be parallel to the plane interface normal. A more general solution was given by (Kotowski, 1987). He developed a mathematical model for ray tracing through an arbitrary number of parameterized interfaces. In his model the coordinates of the piercing point nearest to the camera defines the image ray together with the principal point and the image point on the sensor. The main task in this approach is the complete reconstruction of the image ray path through the different media. The main advantages of this solution are its universality and flexibility as well as the possibility to implement it in a bundle program.

The photogrammetry section at TU Dresden is working on several projects in multi-media environments (e.g. Putze, 2009; Westfeld & Maas, 2010). These require a suitable multi-media bundle program. Because of the different experimental layouts, a flexible solution was needed. For this reason, the approach suggested by Kotowski was used as a basis for the presented bundle implementation.

2. MULTI-MEDIA MODULE

Many papers were published in the past dealing with different aspects of traditional bundle approaches. Because of that, the article focuses on the necessary alterations to make a conventional bundle model multi-media capable. The main idea behind many approaches, like (Maas, 1995) or (Kotowski, 1987), is the introduction of a multi-media module into the collinearity equations. Normally the 3D coordinates of an object point are used directly in the model. If an image ray passes media with different refractive indices, the collinearity of the image point, the projection centre and the object point is normally not given. So the object point coordinates have to be corrected by a multi-media module in order to fulfil the collinearity equations again.

(Kotowski, 1987) substitutes the object point coordinates in his model with the piercing point coordinates of the image ray with the interface nearest to the camera. This way, the main task of the multi-media module would be the calculation of the piercing point coordinates from given interface parameters, refractive indices and the principal point position. This can only be done with the complete reconstruction of the image ray path from the object point through the different media to the principal point. In computer graphics this problem is called ray tracing.

2.1 Ray tracing

In computer graphics ray tracing is used to create realistic 2-D images from virtual 3-D data (e.g. Glassner, 1989). To simulate the effects of light rays when encountering with virtual objects, it's essential to calculate their paths. This can be done in two different ways. The intuitive approach is tracing the light path from the object to the eye or sensor. In computer graphics this is called forward ray tracing. For computational reasons the more common method is the backward ray tracing (Glassner, 1989). Here the paths are traced from a point (e.g. a pixel) in the image plane, through the viewpoint until they hit some object in the virtual scene or go off to infinity.

The notation in computer graphics is contrary to the definitions in photogrammetry. Following the designations for (forward) intersection and (backward) resection, the backward ray tracing (BRT) is defined here as the imaging path of an object point through an arbitrary number of media onto the camera sensor. The projection of an image ray into the object space is noted as forward ray tracing (FRT). For a multi-media module in a photogrammetric bundle the BRT is the according approach. FRT can be used for example for spatial intersection based on image coordinates and a given image acquisition configuration.

2.1.1 Forward ray tracing (FRT): For better understanding, the forward ray tracing will be discussed first. The light resp. image ray is defined by the direction from the perspective centre P_0 to the image point p', which is transformed into the object space with given parameters of the exterior orientation (R, P_0) of the sensor. Mathematically, the projection of an image point to a corresponding object point can be described as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + m \cdot \begin{bmatrix} X_{L0} \\ Y_{L0} \\ Z_{L0} \end{bmatrix}$$

where

$$\begin{bmatrix} X_{L0} \\ Y_{L0} \\ Z_{L0} \end{bmatrix} = R \cdot \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (1)$$

in which: X_0, Y_0, Z_0 = coordinates of projection centre P_0
X, Y, Z = object point coordinates
X_{L0}, Y_{L0}, Z_{L0} = direction vector L_0 of image ray
R = rotation matrix
m = scale factor
x', y' = image coordinates of p'
z' = focal length

If the ray passes through an interface T_t, the outgoing ray is defined by the piercing point P_t and the refracted direction L_{t+1}.

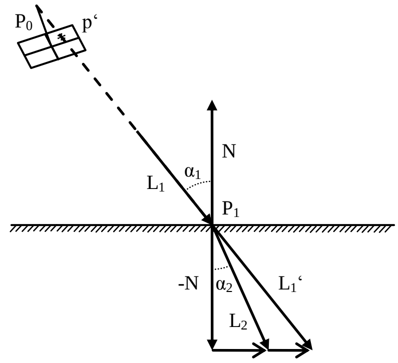

Figure 2. Geometry for forward ray tracing.

The determination of the outgoing ray can be divided into two steps. First, P_t hast to be computed by intersect the image ray with the interface. The problem can be described by the following two constraints: The piercing point is an element of the interface T_t and has to fulfil eq. (1). Given an implicit surface, defined by:

$$T_t(X_t, Y_t, Z_t) = 0 \quad (2)$$

the point (or points) of intersection can be computed by substituting the coordinates of P_t with eq. (1). Specific solution algorithms can be given for different types of surfaces. For example, when the first intersected interface is a plane, P_1 can be calculated by the line-plane intersection algorithm.

The formula for the refracted direction L_{t+1} of incoming vector L_t can be derived from Snell's law

$$\frac{\sin \alpha_t}{\sin \alpha_{t+1}} = \frac{v_t}{v_{t+1}} = \frac{n_{t+1}}{n_t} \quad (3)$$

where α_t = angle of incidence
α_{t+1} = angle of refraction
v_t = velocity of light in medium before interface
v_{t+1} = velocity of light in medium behind interface
n_t = refractive index of medium before interface
n_{t+1} = refractive index of medium behind interface

and the coplanarity of incoming ray, outgoing ray and surface normal. A possible mathematical description for L_{t+1} is given by (Glassner, 1989):

$$L_{t+1} = \frac{L_t}{\mathbf{n}_t} + \left(\frac{C}{\mathbf{n}_t} - \sqrt{1 + \frac{1}{\mathbf{n}_t^2}(C^2 - 1)} \right) N_t$$

where

$$C = -N_t \cdot L_t \quad , \quad \mathbf{n}_t = \frac{n_t}{n_{t+1}} \quad (4)$$

in which: L_t = normalized incoming direction vector
L_{t+1} = refracted direction vector (not normalized)
N_t = surface normal vector of T_t in P_t
$\mathbf{n}_t$ = relative refractive index

In case of multiple interfaces in the ray path, the refraction has to be computed sequentially. FRT can be used for spatial intersection or the calculation of start values for BRT (see 2.1.2).

2.1.2 Backward ray tracing (BRT): In contrast to FRT, the backward ray path can't be computed directly. When starting the ray tracing from the object point, no initial direction for the ray path can be given. Therefore, a direct determination of the ray path is impossible. Another consequence is that the reconstruction of the whole ray path has to be done simultaneously. Mathematically, the problem can be reduced to the unknown interface points P_t (see figure 3).

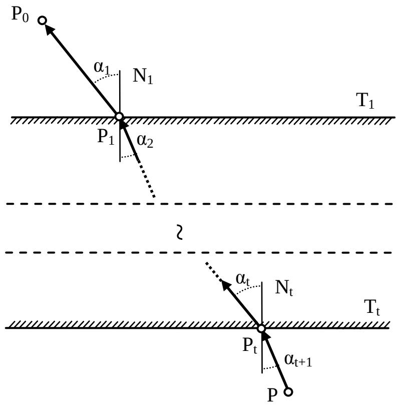

Figure 3. BRT through an arbitrary number of interfaces.

As already mentioned, the approach by (Kotowski, 1987) can be used for BRT. The model describes the problem with three constraints for each refracting point P_t:

1. P_t is located on the surface T_t.:

$$T_t(X_t, Y_t, Z_t) = 0 \qquad (5)$$

2. Fulfilling Snell's Law:

$$\sin\alpha_{t+1} \cdot n_{t+1} - \sin\alpha_t \cdot n_t = 0 \qquad (6)$$

The angles of incidence α_t and refraction α_{t+1} can be introduced as functions of the normal vector N_t in point P_t, the coordinates of the refraction point P_t as well as the coordinates from of the ray-defining points in front and behind the interface:

$$\cos\alpha_t = \frac{\begin{bmatrix} N_{tX} \\ N_{tY} \\ N_{tZ} \end{bmatrix}^T \cdot \begin{bmatrix} X_{t-1} - X_t \\ Y_{t-1} - Y_t \\ Z_{t-1} - Z_t \end{bmatrix}}{\left\| \begin{bmatrix} N_{tX} \\ N_{tY} \\ N_{tZ} \end{bmatrix} \right\| \left\| \begin{bmatrix} X_{t-1} - X_t \\ Y_{t-1} - Y_t \\ Z_{t-1} - Z_t \end{bmatrix} \right\|}$$

$$\cos\alpha_{t+1} = \frac{\begin{bmatrix} N_{tX} \\ N_{tY} \\ N_{tZ} \end{bmatrix}^T \cdot \begin{bmatrix} X_t - X_{t+1} \\ Y_t - Y_{t+1} \\ Z_t - Z_{t+1} \end{bmatrix}}{\left\| \begin{bmatrix} N_{tX} \\ N_{tY} \\ N_{tZ} \end{bmatrix} \right\| \left\| \begin{bmatrix} X_t - X_{t+1} \\ Y_t - Y_{t+1} \\ Z_t - Z_{t+1} \end{bmatrix} \right\|} \qquad (7)$$

3. The normal vector N_t in point P_t as well as the entering and leaving light rays lie in a geometric plane. Its coplanarity is described as follows:

$$\det \begin{bmatrix} X_{t-1} - X_t & Y_{t-1} - Y_t & Z_{t-1} - Z_t \\ X_t - X_{t+1} & Y_t - Y_{t+1} & Z_t - Z_{t+1} \\ N_{tX} & N_{tY} & N_{tZ} \end{bmatrix} = 0 \qquad (8)$$

Note that the system has to be set up for each additional interface. For example, the equation system for a configuration with two interfaces consists of six equations with six unknown coordinates (in X, Y and Z) for the two interface points P_1 and P_2. Due to the quadratic structure of the system and the nonlinear character, Newton's method can be used for solving.

The BRT approach by (Kotowski, 1987) has some advantages, like compactness and simplicity, but also some drawbacks:

1. Solving the equations system is defined as long as the refraction on all interfaces is non-zero. If the direction of the incident ray and the surface normal in P_t are collinear, the equation system will have a rank defect due to eq. (6) and eq. (8). To avoid this effect, the direction has to be tested on collinearity before solving the system. If a collinearity of the incident ray and the surface normal occurs, the BRT becomes a simple line-surface intersection problem. Nevertheless, this approach only works for imaging setups with one refracting interface. For more complex setups, the equations of the appropriate surface can be omitted and consequently, the set up of equations have to be rearranged.

2. Due to the ambiguity of the second constraint in the BRT model (eq. (6) and (7)), the determination of α_{t+1} is not unique. Figure 4 shows a situation at which the constraints of the model are fulfilled by two different ray paths. According to Fermat's principle, a ray of light takes the shortest path. Accordingly, P_1 is the true piercing point.

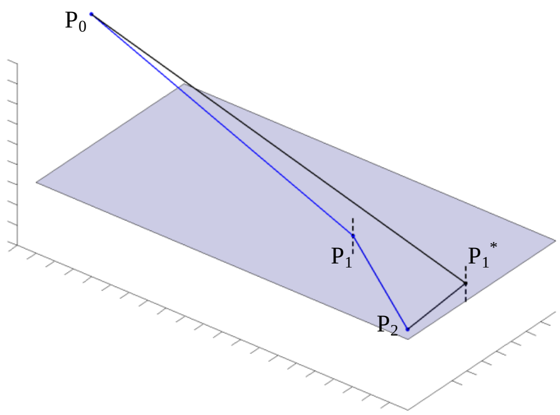

Figure 4. Ambiguity problem of BRT-model (one planar interface). The constraints of the BRT model are fulfilled by two different ray paths.

Avoiding such ambiguity problem, initial values for the coordinates of the piercing points sufficiently close to their true positions are necessary. (Kotowski, 1987) recommended for this task the line-surface intersection algorithm. The line is defined by the projection centre and the object point, the surface(s) by the interface(s). In most cases, the line-surface intersection delivers initial values, which are good enough for a convergence of the Newton's algorithm to the true interface point(s). However, if the angle of incidence is less than 45°, the algorithm tends to converge to the false point P_1^*. Modifying the BRT in a way that eq. 7 is tested against ambiguity is possible. However, this measure is only suited for configurations with one interface. Multiple interfaces cause problems in the BRT-model, as can be seen in figure 5. Here, when testing of eq. (7) against ambiguity in point P_2*, the inconsistency of eq. (6) reaches large values, which are numerically problematic. Then the equation system would become singular.

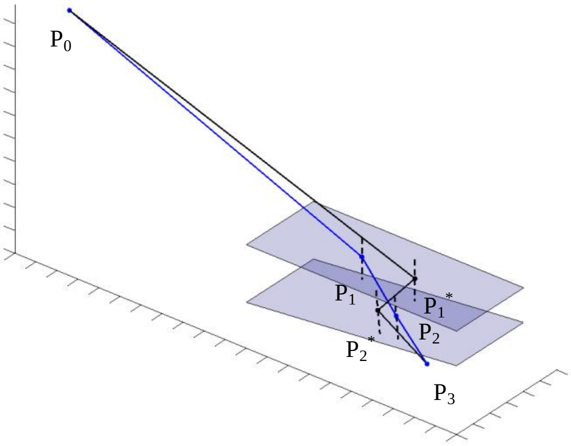

Figure 5. Ambiguity problems of BRT-model (two non-parallel planar interfaces).

3. The solving of the equations system causes problems if an interface with alternating surface inclination is part of the setup (e.g. a sinusoidal wave). After several iterations the system frequently becomes singular if the initial values for the interface point coordinates are not sufficiently close to the real position.

Due to the ambiguity and the singularity problems as described above, the provision of good initial interface points coordinates is essential. One possible way is FRT based on an approximated imaging configuration, which can be derived for example from a conventional bundle adjustment with additional "mono-media" points in the scene.

For imaging setups without initial information available, another solution had to be found. The alternating forward ray tracing (AFRT) was developed as an alternative BRT algorithm. Figure 6 shows the basic principle.

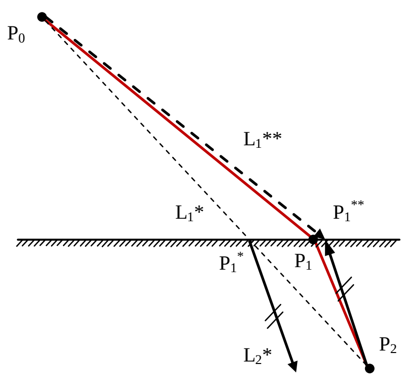

Figure 6. Principle of alternating forward raytracing (AFRT)

First, the algorithm starts with FRT from point P_0 with ray direction from P_0 to object point P_2 (L_1*). The calculated direction vector of the refracted ray (L_2*) is then attached to P_2 and the FRT is performed again. The obtained interface point P_1** defines together with P_0 the ray (L_1**) for the next iteration. The algorithm continues until the change of the position of interface point P_1 is smaller than a predefined value. In contrast to the model of (Kotowski, 1987), AFRT is much more robust and gives a unique solution. A further advantage is that the refraction can be zero without causing singularities. A main drawback is the slower convergence behavior. Because of that, AFRT is intended to be used for the computation of initial values for BRT model of (Kotowski, 1987) only, so that the algorithm can be stopped after the first or second iteration.

3. INTEGRATION OF THE MULTI-MEDIA MODULE IN THE BUNDLE ADJUSTMENT

The way of integration of the multi-media module into the collinearity equations model depends on the imaging configuration. (Kotowski, 1987) distinguishes between object-invariant and bundle-invariant interfaces. The first case occurs if the position and orientation of the interfaces relative to the measured object is constant. Bundle invariant interfaces are defined by its constant location and orientation relative to the imaging system. In this publication only the configuration with object-invariant interfaces will be discussed, because many measurement tasks, like the examples in figure 1, had to be carried out in this way. Furthermore, in case of bundle-invariant interfaces, for example a windowpane of an underwater camera, most of the effects can be compensated by additional parameters for an image point shift in the collinearity equations model.

3.1 Correction equation

From the central projection model (eq. (1)) and the considerations in section 2, the system of collinearity equations can be deviated:

$$x' = x'_0 + z' \frac{r_{11}(X_1 - X_0) + r_{21}(Y_1 - X_0) + r_{31}(Z_1 - X_0)}{r_{13}(X_1 - X_0) + r_{23}(Y_1 - X_0) + r_{33}(Z_1 - X_0)} + \Delta x'$$

$$y' = y'_0 + z' \frac{r_{12}(X_1 - X_0) + r_{22}(Y_1 - X_0) + r_{32}(Z_1 - X_0)}{r_{13}(X_1 - X_0) + r_{23}(Y_1 - X_0) + r_{33}(Z_1 - X_0)} + \Delta y' \tag{9}$$

With: X_1, Y_1, Z_1 = object coordinates of the first interface point in the ray path
$r_{11} - r_{33}$ = Elements of rotation matrix R
x'_0, y'_0 = coordinates of principle point
$\Delta x'$, $\Delta y'$ = correction values for imaging errors

Using the measured image coordinates of object points, the following extended system of correction equations can be derived from eq. (9):

$$x'_{ij} + vx'_{ij} = F(X_{0j}, Y_{0j}, Z_{0j}, \omega_j, \varphi_j, \kappa_j, x'_{0k}, z'_k, \Delta x'_k, X_1{}^l{}_{ij}, Y_1{}^l{}_{ij}, Z_1{}^l{}_{ij})$$

$$y'_{ij} + vy'_{ij} = F(X_{0j}, Y_{0j}, Z_{0j}, \omega_j, \varphi_j, \kappa_j, y'_{0k}, z'_k, \Delta y'_k, X_1{}^l{}_{ij}, Y_1{}^l{}_{ij}, Z_1{}^l{}_{ij}) \tag{10}$$

where the coordinates of the interface point P_1^l are provided by the FRT equation system:

$$(X_1{}^l{}_{ij}, Y_1{}^l{}_{ij}, Z_1{}^l{}_{ij}) = f_{P_1}(X_{0j}, Y_{0j}, Z_{0j}, X_i, Y_i, Z_i, \boldsymbol{a}^l, \boldsymbol{n}^l) \tag{11}$$

in which: i = point index
j = image index
k = camera index
t = interface index
l = set of interface indices t
$\mathbf{a}^l$ = set of interface parameters $\mathbf{a}_t$
$\mathbf{n}^l$ = set of relative refractive indices $\mathbf{n}_t$

The structure of correction equations allows the flexible application to various ray paths, such as none refracted, once refracted on interface A, twice refracted on interfaces A and B or other combinations.

Linearization

The collinearity equations are non-linear. The integrated multi-media module is a system of non-linear equations, too. For the least squares estimation in the Gauss-Markov model, it is essential to linearize the correction equations. Normally, the differential quotients for the design matrix (Jacobian matrix) are obtained by first derivative function of the correction equations with respect to the approximate values of parameters. The indirect approach of the multi-media module obviates an analytical differentiation. Hence, the differential quotients can best be calculated via numerical differentiation. Due to the iterative FRT implemented into the multi-media module, the computational effort for setting up the design matrix can be enormous. The number of necessary iterations in the BRT process can decreased significantly if the interface point coordinates of the previous bundle iteration are used as initial values for the actual iteration.

4. OPTIONS AND LIMITATIONS

The approach as described above and the structure of the implemented program are highly versatile. All model parameters (including e.g. refractive indices) can be treated as unknowns. Additionally, constraints between the unknowns, e.g. parallelism of planes or membership of an object point to a surface etc. can be defined. But it has to be taken into account that certain combinations of unknown parameters may lead to singularity of the bundle adjustment. For example, when defining all refractive indices as unknowns, the solution becomes singular.

As usual, the stability of the adjustment depends on the imaging configuration and the distribution of control points. Multi-media geometry requires a partly different view on what is known as an optimal image configuration. Empirical rules for solving conventional photogrammetric problems are not longer valid. For measurement task in a multi-media environment, it is strongly advised to test the imaging configuration prior by simulation.

5. EXPERIMENTAL PROOF

For the evaluation of the method and the implemented bundle adjustment, an experiment was carried out, where a reference field was placed in a water basin made from 4 mm sheet glass. The test field consist of a 30 x 20 cm aluminium base plate on which a 6 x 6 cm plate is mounted in a distance of 5 cm. Three rods with a length of 12 cm are fixed on the base plate to ensure a defined distance to the front glass sheet. Circular markers are fixed on the plates of the test field and on the front glass sheet (figure 7). Using this experimental setup, multiple-refracted as well as straight rays can be provided.

The images where captured in a configuration which should be suitable for a stable self calibration of the camera (figure 8). Eleven images were taken, two of them with the camera rolled by 90°/180°. The image coordinates of the targets where measured in a commercial photogrammetric software (AICON 3D Studio) by an ellipse-operator at a precision of 0.02 to 0.1 pixel. The image coordinate data were coded after its ray path (refracted, non-refracted) and then passed into the bundle block adjustment program.

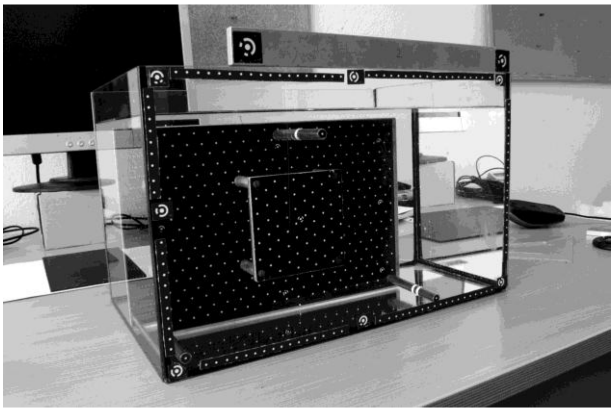

Figure 7. Experimental setup: Test field in water filled basin.

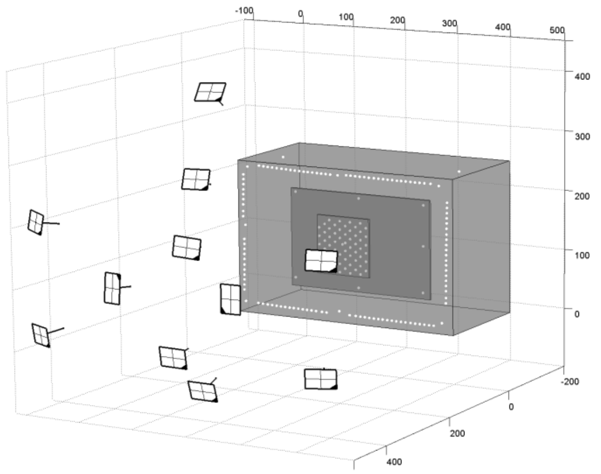

Figure 8. Image configuration.

The data was processed with different parameter settings. First, the image coordinates of the markers on the front glass sheet were used in the bundle adjustment exclusively (calculation example I, table 1). To avoid any effects from inconsistent reference points, the datum was defined as a free-network adjustment. The scaling information was derived from a separately measured distance between two object points. The RMS value of image point measurements after adjustment was 0.03 pixel.

For a rigorous validation of the implemented multi-media model, the interior as well as the exterior orientation were fixed and the adjustment was performed again with all image points (calculation example II, table 1). The plane parameters of the glass sheet and the refractive indices of water were introduced as unknowns. The thickness of the glass sheet was fixed to 4 mm. A comparison of the RMS values of the image measurements (0.024 pixel of points ‘in air’ vs. 0.054 pixel of points in water, table 1) indicates that the model is correct. The (small) difference in accuracy can be explained by a degraded imaging quality by dispersion effects caused by the media water and glass as well as chromatic aberration. A further possible reason are planarity deviations in the front glass plate.

It is noteworthy that the plane parameters of the front glass sheet could be determined with high precision. The same accuracy is reached when a plane is fitted in the object points on the glass. Also, the refractive index of water could be determined with high precision (table 1).

To evaluate the internal quality of the photogrammetric network, as many parameters as possible where introduced as unknowns: Exterior and interior orientation, object point coordinates, the plane parameters of glass sheet as well as the refractive index of water (calculation example III, table 1). Again, the datum was defined as a free-network adjustment and the scale by the distance between two object points.

	I	II	III
Sigma 0 [mm]	0.000256	0.000347	0.000344
RMS x',y' (air) [pixel]	0.024	0.024	0.029
RMS x',y' (water) [pixel]	-	0.054	0.043
std. dev. c_k [mm]	0.00046	-	0.00054
std. dev. x_0 [mm]	0.00110	-	0.00112
std. dev. y_0 [mm]	0.00107	-	0.00109
RMS X_0 [mm]	0.0095	-	0.0098
RMS Y_0 [mm]	0.0110	-	0.0106
RMS Z_0 [mm]	0.0091	-	0.0107
RMS Omega [gon]	0.0036	-	0.0034
RMS Phi [gon]	0.0036	-	0.0035
RMS Kappa [gon]	0.0011	-	0.0011
RMS X (air) [mm]	0.0024	0.0021	0.0036
RMS Y (air) [mm]	0.0023	0.0021	0.0033
RMS Z (air) [mm]	0.0046	0.0043	0.0078
RMS X (water) [mm]	-	0.0047	0.0031
RMS Y (water) [mm]	-	0.0052	0.0032
RMS Z (water) [mm]	-	0.0233	0.0131
std. dev. N_X (sheet)	-	7.54·E-5	9.11·E-5
std. dev. N_Y (sheet)	-	0.00010	0.00013
std. dev. N_Z (sheet)	-	1.23·E-7	9.79·E-7
std. dev. D (sheet) [mm]	-	0.115	0.058
std. dev. n (water)	-	0.0012	0.0012

Table 1. Bundle adjustment results.

When comparing the results of example II and III, it becomes clear, that the simultaneous determination of all unknowns results in an increase of RMS for the image measurements of mono-media points and a decrease of RMS for the multi-media image points, despite the adjusted weighting of the image measurements. The same can be observed for the accuracy of the object points. The comparison of I and III shows that the additional image observations does not affect the accuracy of interior and exterior orientation. Surprisingly, the accuracy of the plane normal in example II is higher than in III, while the accuracy of the distance from the origin D is lower. The fixation of the interior and exterior orientation in calculation II puts an additional geometrical constraint in the network of mono-media and multi-media image point observations. This results, as expected, in a higher Sigma 0 than in calculation III.

6. CONCLUSION AND FUTURE WORK

The presented concept of a flexible multi-media bundle meets the need for a universal photogrammetric tool, which is able to be used as a module in a wide range of measurement tasks under various environmental conditions.

The implemented software package provides the functionality of a conventional bundle program plus the capability of flexible refraction handling. All parameters in the model can be treated as unknowns and can be determined simultaneously. First experiments proved the quality of the implemented model and showed the accuracy potential. Further tests are necessary to derive new rules of thumb for suitable imaging configurations for different measurement tasks. Beside analytical optimisation methods, the Monte Carlo simulation can be used for this task.

The chromatic aberration as a disturbing effect has to be investigated, too. This effect was negligible in the first experiments, but has to be taken to account when extreme inclination angles between image ray and interface occurs.

The modular layout of the implemented program allows the easy integration of further implicit surfaces. The implemented multi-media module can for instance easily be modified for processing mirrored image rays by replacing the equation for Snell's law by the law of reflection.

A long-term research goal is the expansion of functionality of the method to facilitate the determination of the spatio-temporal behaviour of moving refracting surfaces such as waves.

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