An alternative formulation for the integration of GPS and INS measurements

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Abstract

GPS/INS systems are increasingly used for the direct geocoding of airborne remote sensing measurements. Such systems allow for the efficient and economical determination of the exterior orientation elements of the sensors (e.g. Laser scanner, photogrammetric camera, SAR-antenna). Typically in most operational systems the GPS and INS measurement integration and error control is done using a Kalman filter; the sensor trajectory determined from the primary sensor (INS) is frequently updated and controlled by the secondary sensor output (GPS positions). In this contribution we provide an alternative formulation of the integration methodology for the aforementioned applications based on the following observations:

- Under benign flying conditions, differential GPS carrier phase positioning determines the translational components of the sensor trajectory at a moderate data rate (1Hz – 10Hz) with a very high accuracy (5cm – 10 cm). Typically the INS output cannot improve this position data.
- The high data rate of INS data primarily serves to determine the rotational components of the sensor trajectory (attitude of the sensor), and to a lesser degree is useful for the interpolation of the translational components between the DGPS determined positions.
- In the above mentioned applications, there is no requirement for real-time data processing algorithms; final data processing typically takes place post mission in the office.
- Mission duration typically extends no longer than a few hours.

Based on these observations we have formulated an algorithm for the integration of DGPS data (positions) and INS data (linear and rotational velocity increments) based on classical least-squares estimation methodology.

1. Introduction

Satellite-based navigation systems play an important role in modern precise navigation, positioning and attitude determination applications. One of the large advantages of such a navigation system is its consistent precision during the entire mission duration. Accuracies of less than 10cm can be reached under good measurement conditions in kinematic applications. The main disadvantage of such systems is that availability is not always guaranteed. The required free line-of-sight between the satellite and the receiving antenna can be briefly interrupted by a number of disturbing influences. Additionally, satellite based navigation systems cannot supply the orientation parameters with the required temporal resolution and the necessary accuracy.

Contrary to satellite positioning systems, INS based systems are self contained and provide position and orientation autonomously. The advantages of such INS systems are high data rates and their large short term stability. Unfavorably there is a time dependent growth of systematic errors when working in stand alone application.

Due to the opposite error behavior of these two measuring systems the shortcomings of either system can be eliminated by system integration. In the last few years there were two different main approaches for GPS/INS integration: the loosely coupled and the tightly coupled solution, both of which have advantages and disadvantages.

1.1. Loosely coupled Kalman Filter

In the loosely coupled integration mode a decentralized filtering strategy is used. GPS raw measurements are pre-analyzed in a local Kalman filter to determine the GPS positions and velocities in a geographic coordinate system, [1]. In a second Kalman filter these positions are combined with the INS raw measurements which calculates more reliable positions and velocities, and also orientation parameters and INS sensor errors. For high precision applications in a post processing mode the Kalman filter runs in two directions and the both solutions are combined afterwards (Figure1). Due to this fact gaps in GPS data can be better bridged.

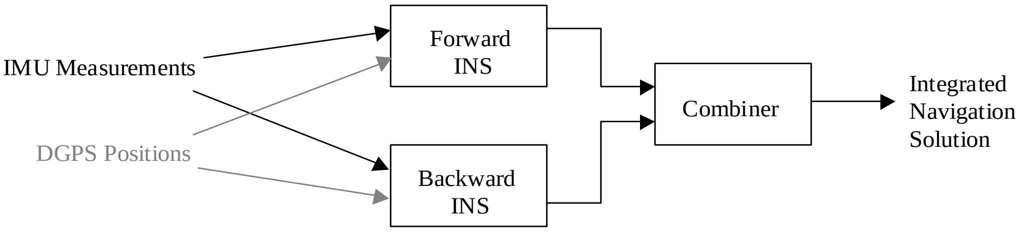

Figure 1: Combination of forward and backward INS to a combined integration solution (in [3]).

1.2. Tightly Coupled Kalman Filter

In tightly coupled Kalman filters there is only one central filter which combines the raw data of both systems. This approach has the advantage also to take into account GPS data from less than 4 satellites, [1]. A disadvantage of this concept consists in a rather complicated integration of additional sensors.

Both technologies have the disadvantage to depend on the flight dynamics and not to take into account the whole data at one time. Reliable Kalman filter results rely heavily on the correct definition of the stochastic model used in the filter process.

1.3. Integration Strategy for our Application

Instead of using a Kalman filter we propose a solution based on classical least squares algorithm. This approach is possible since we handle the data only post mission and not in real time. Since typically the position accuracy must be better than 10 cm for Laser scanner missions, post mission processing of GPS carrier phase data is required.

As can be seen in Figure 1 laser scanner flights or photogrammetric flights in general are characterized by straight line flights above the survey area, where the lines are typically not longer than 100 km. In the turns the pilots are advised to restrict the roll angles to less than 20° to avoid shading effects of the GPS satellite signals.

For calculating the GPS positions we evaluate GPS phase measurements utilising reference data from a base station, which is mostly located close to the survey area. In the case of long flight lines, more than one base station or a network of base station is used. Base station data are typically provided at a rate of 1Hz.

Because of these rather favorable flight conditions with low dynamics we can also ensure a post mission GPS position accuracy of better than 10 cm.

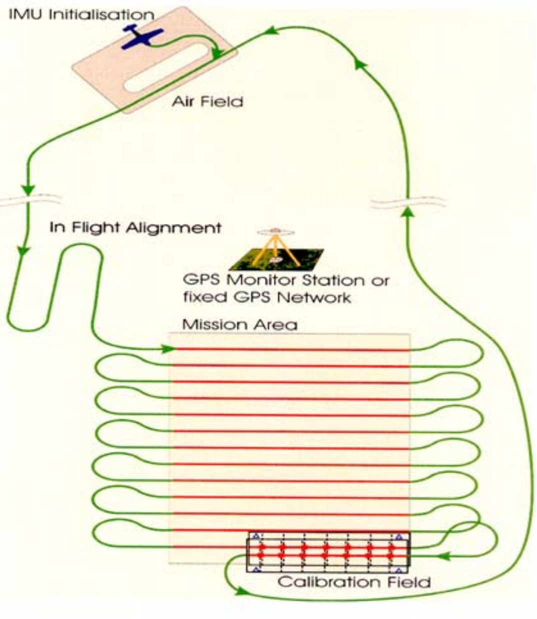

Figure 2 : example for a laser scanner mission (from [4]).

For our integration we assume that the GPS positions and velocities contained in the vector $\boldsymbol{x}_i^{GPS}$ derived from a commercial analysis software are known, as well as the positions, velocities and attitude of the INS calculated with approximate initial values of sufficient accuracy in a separate algorithm.

2. Inertial Navigation Solution

In this treatment an Inertial Navigation System is a strapdown unit of at least 3 accelerometers and 3 gyros. Therein the gyros measure the rotational velocity $\omega_{ib}{}^{b}$ of the body-fixed system b in relation to the inertial system i and output these as angle increments $(\Delta\alpha)$ per sampling interval. The accelerometers supply accelerations a^b within the body-fixed coordinate system in the form of velocity increments (Δv^b) per sampling interval.

$$\boldsymbol{\Delta\alpha} = \int \boldsymbol{\omega}_{ib}^{b}(t)dt \qquad\qquad \boldsymbol{\Delta v}^{b} = \int \boldsymbol{a}^{b}(t)dt \tag{1}$$

The definitions of the coordinate systems are listed down in Table 1, wherein an quasi inertial system is used instead of an inertial system. Based on [5] we can draw the following system of first order differential equations in the n-system:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\lambda} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} v_n/(M+h) \\ v_e/((N+h)\cos\phi) \\ -v_d \end{pmatrix} \tag{2a}$$

$$\dot{\boldsymbol{v}}^{n} = \boldsymbol{C}_b^n \underline{\boldsymbol{a}^b} - \left(2\boldsymbol{\Omega}_{ie}^n + \boldsymbol{\Omega}_{en}^n\right)\boldsymbol{v}^n + \boldsymbol{g}^n - \boldsymbol{C}_e^n \boldsymbol{\Omega}_{ie}^e \boldsymbol{\Omega}_{ie}^e \boldsymbol{x}^e(\phi,\lambda,h) \tag{2b}$$

$$\dot{\boldsymbol{C}}_b^n = \boldsymbol{C}_b^n \cdot \left(\underline{\boldsymbol{\Omega}_{ib}^b} - \boldsymbol{\Omega}_{in}^b\right) \tag{2c}$$

with

ϕ,λ,h	position components in the earth centered earth fixed system
$\boldsymbol{v}^n = (v_n\, v_e\, v_d)^T$	velocity displayed in the n-system
$\boldsymbol{C}^l_m$	transformation matrix from the m-system to the l-system
$\boldsymbol{\Omega}_{km}^l = \boldsymbol{\Omega}(\boldsymbol{\omega}_{km}^l) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$ with $\boldsymbol{\omega}_{km}^l = (\omega_x\ \omega_y\ \omega_z)^T$	rotational velocity of the m-system with respect to the k-system expressed in the l-system
$\boldsymbol{g}^n$	gravitational acceleration in the n-system

In equation (2) the output from the gyros and accelerometers is underlined and the primary unknowns are the three position coordinates and the three velocity components as well as three independent parameters describing the rotation between the n- and the b-system, concealing in the $\boldsymbol{C}_b^n$ matrix (compare equation (3): unknowns at time i).

$$\boldsymbol{x}_i^{INS} = \left(\phi_i, \lambda_i, h_i, v_{n\,i}, v_{e\,i}, v_{d\,i}, yaw_i, pitch_i, roll_i\right) \tag{3}$$

By solving the above differential equations we get an initial value problem, in which the inertial solution at time i presents itself as:

$$\boldsymbol{x}_i^{INS} = f\left(\boldsymbol{x}_0, \boldsymbol{\Delta v}_{0\ldots i}, \boldsymbol{\Delta\alpha}_{0\ldots i}\right) \tag{4}$$

This relationship shows the dependency of the inertial solution at time i from the initial values $\boldsymbol{x}_0$ and all measurements until time i.

In figure 3 you will see the principle of an inertial navigation solution. On the left hand side of this figure the initialization process is displayed in which the initial values are needed to start the inertial navigation process. After this initialization is done a self running process of mechanization and numerical integration of the navigation equations follows whereby in the mechanization the attitude information is computed from the gyro measurements. Therewith the accelerometer specific force vector measurements can be transformed in the local level frame n. Afterwards a numerical integration algorithm integrates the specific force accelerations to obtain velocities and positions, see figure 3.

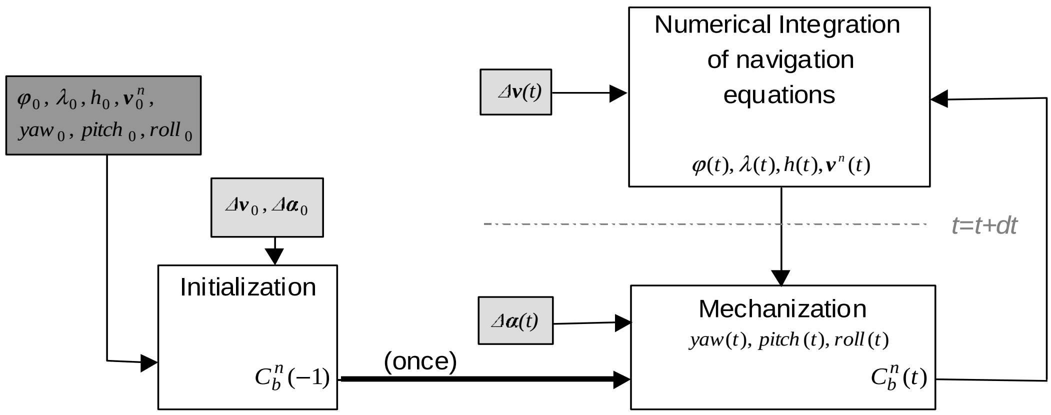

Figure 3: functionality of the inertial navigation algorithm.

For the sake of completeness the solved navigation equations are listed below and can also be taken from [5]:

$$\boldsymbol{v}_i^n = \boldsymbol{v}_{i-2}^n + \frac{1}{6}\left(\boldsymbol{C}_b^n(i-2)\left(3\boldsymbol{\Delta v}_{i-1}^b - \boldsymbol{\Delta v}_i^b\right) + 4\boldsymbol{C}_b^n(i-1)\left(\boldsymbol{\Delta v}_{i-1}^b + \boldsymbol{\Delta v}_i^b\right) + \boldsymbol{C}_b^n(i)\left(3\boldsymbol{\Delta v}_i^b - \boldsymbol{\Delta v}_{i-1}^b\right)\right) + f\left(\boldsymbol{x}^n, \boldsymbol{v}^n, \boldsymbol{\Omega}_{in}^n, \dot{\boldsymbol{\Omega}}_{in}^n, \boldsymbol{g}^n\right)_{i-2}\Delta t \tag{5}$$

$$\phi_i = \phi_{i-2} + \frac{(v_n)_{i-1}\cdot\Delta t}{M_{i-1} + h_{i-1}} \tag{6a}$$

$$\lambda_i = \lambda_{i-2} + \frac{(v_e)_{i-1}\cdot\Delta t}{(N_{i-1} + h_{i-1})\cdot\cos\phi_{i-1}} \tag{6b}$$

$$h_i = h_{i-2} - (v_d)_{i-1}\cdot\Delta t \tag{6c}$$

3. Integration Strategy

As already expressed in equation (4), the inertial solutions depend largely on the initial values of inertial navigation. If we proceed from error free inertial measurements, then these are the only influencing variables on the final result. So if we want to improve the inertial result per least squares estimation, the only possibility to do so is to modify the initial values suitably, see Figure 4.

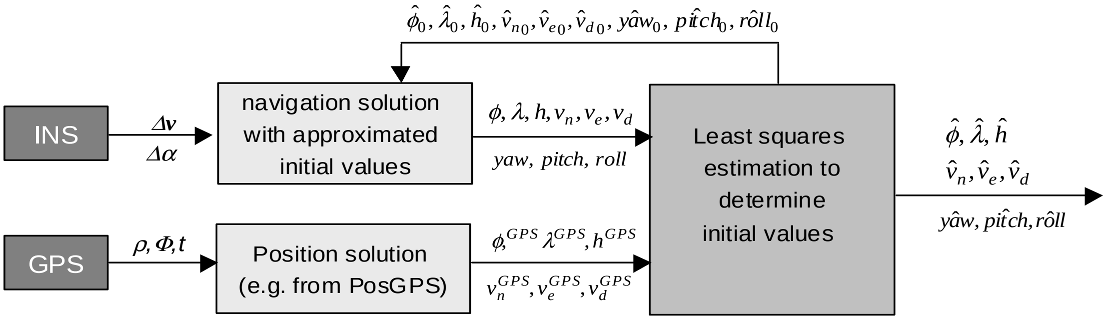

Figure 4: Integration model.

3.1. Model description

On the basis of a functional model

$$\boldsymbol{L} + \boldsymbol{v} = \hat{\boldsymbol{L}} = f(\hat{\boldsymbol{y}}) \qquad \wedge \text{ adjusted values} \tag{7}$$

in which ***L*** represents the as ***v*** to improving observations and ***y*** are the unknowns, we assign the initial values of the inertial navigation the role of the unknowns and the GPS solutions the place of the observations. This leads to:

$$\boldsymbol{x}_i^{GPS} + \boldsymbol{v}_{xi}^{GPS} = \hat{\boldsymbol{x}}_i^{INS} \quad \text{with} \quad \hat{\boldsymbol{x}}_i^{INS} = f(\phi_0, \lambda_0, h_0, vn_0, ve_0, vd_0, yaw_0, pitch_0, roll_0) \tag{8}$$

By selecting the method of the least squares as integration beginning, the improvement of the observations is done under the condition to determine the adjusted values that the sum of these square improvements is a minimum:

$$\boldsymbol{v}^T \boldsymbol{P v} \Rightarrow \min \tag{9}$$

So it can be achieved that the adjusted measured values and the unknowns are in the sense of the probability calculation most probable values and all adjusted variables possess the smallest possible standard deviation.
As for the solution of such a system a linear relation between measurements and unknowns is required, approximate values for the unknowns are needed and the error equations have to be linearized by a Taylor series expansion. Derived from equation (7) the linearized error equations are:

$$v_k = f_k(\boldsymbol{y}_0) + \sum_{j=1}^{u} \frac{\partial f_k}{\partial y_j} dy_j - L_k \qquad \hat{\boldsymbol{y}} = \boldsymbol{y}_0 + \boldsymbol{dy} \tag{10a}$$

$$\boldsymbol{v} = \boldsymbol{A dy} - \boldsymbol{l} \quad \text{with} \quad -\boldsymbol{l} = -(\boldsymbol{L} - f(\boldsymbol{y_0})) \tag{10b}$$

n	number of measurements $k = 1 \ldots n$
u	number of unknowns
$\boldsymbol{y_0}$(u,1)	approximated values for the vector of unknowns
dy(u,1)	supplement to unknowns (to estimate)
A (n,u)	design matrix of the functional model containing the derivatives

Applied to our practical application we get:

$$v_{x_{ij}}^{GPS} = \frac{\partial f_{ij}}{\partial \phi_0} d\phi_0 + \frac{\partial f_{ij}}{\partial \lambda_0} d\lambda_0 + \frac{\partial f_{ij}}{\partial h_0} dh_0 + \frac{\partial f_{ij}}{\partial vn_0} dvn_0 + \frac{\partial f_{ij}}{\partial ve_0} dve_0 + \frac{\partial f_{ij}}{\partial vd_0} dvd_0 + \frac{\partial f_{ij}}{\partial yaw_0} dyaw_0$$
$$+ \frac{\partial f_{ij}}{\partial pitch_0} dpitch_0 + \frac{\partial f_{ij}}{\partial roll_0} droll_0 + \left(x_{ij}^{INS} - x_{ij}^{GPS}\right) \quad \text{for the j-th component of } x_i \text{ and time i} \tag{11}$$

The minimum criterion leads to the normal equations:

$$\hat{\boldsymbol{y}} = \left(\boldsymbol{A^T PA}\right)^{\boldsymbol{-1}} \boldsymbol{A^T Pl} \tag{12}$$

3.2. Generation of the Design Matrix

An analytic formulation of the connection between the momentary result of inertial navigation and the associated initial values quickly turned out to be impossible to solve. This circumstance led to the development of a numeric beginning.

Comparing equation (11) with equation (10b) implicates, that the design matrix A has to contain the first derivatives of the functions to the unknowns $\partial \boldsymbol{f} / \partial \boldsymbol{x}_0$, which itself are not easily to build. The $\partial \boldsymbol{f} / \partial \boldsymbol{x}_0$ are similar to $\delta\boldsymbol{f}/\delta\boldsymbol{x}_0$ for small $\delta\boldsymbol{x}_0$. In praxis we can alter the approximated initial values $\boldsymbol{x}_0$ with a $\delta\boldsymbol{x}_0$ and consequently the navigation solution at time i will change to $\boldsymbol{x}_i^{INS} + \delta\boldsymbol{x}_i^{INS}$. If we now attach $\delta\boldsymbol{x}_0$ only to one component of the initial value vector

$$\text{e.g.} \quad \delta\boldsymbol{x}_0 = \left(\delta\phi_0 \quad 0 \quad 0 \quad 0 \quad \cdots \quad 0\right)^T \Rightarrow \quad \delta\phi(1)_i = \phi(\boldsymbol{x}_0) - \phi(\boldsymbol{x}_0 + \delta\boldsymbol{x}_0)$$
$$\text{or} \quad \delta\boldsymbol{x}_0 = \left(0 \quad \delta\lambda_0 \quad 0 \quad 0 \quad \cdots \quad 0\right)^T \Rightarrow \quad \delta\phi(2)_i = \phi(\boldsymbol{x}_0) - \phi(\boldsymbol{x}_0 + \delta\boldsymbol{x}_0) \tag{13}$$

The relation between the change in the initial value and the change in the result can be numerically derived and the design matrix can be set up as:

$$\boldsymbol{A}_i = \begin{pmatrix} \frac{\delta\phi(1)_i}{\delta\phi_0} & \frac{\delta\phi(2)_i}{\delta\lambda_0} & \frac{\delta\phi(3)_i}{\delta h_0} & \cdots & \frac{\delta\phi(9)_i}{\delta roll_0} \\ \frac{\delta\lambda(1)_i^I}{\delta\phi_0} & \frac{\delta\lambda(2)_i}{\delta\lambda_0} & \frac{\delta\lambda(3)_i}{\delta h_0} & & \vdots \\ \frac{\delta h(1)_i}{\delta\phi_0} & \frac{\delta h(2)_i}{\delta\lambda_0} & \frac{\delta h(3)_i}{\delta h_0} & & \vdots \\ \vdots & & & \ddots & \vdots \\ \frac{\delta roll(1)_i}{\delta\phi_0} & \cdots & \cdots & \cdots & \frac{\delta roll(9)_i}{\delta roll_0} \end{pmatrix} \tag{14}$$

4. INS Sensor Error Model Implementation

Based on [5] the general model for the error behavior of a single gyro $\delta\omega_{ib}^{\ b}$ or accelerometer axis $\delta\boldsymbol{a}^b$ can be drawn as follows:

$$\delta a^b = B + \kappa_A \cdot a^b + C_A \tag{15}$$

$$\delta\omega_{ib}^b = D + \kappa_G \cdot \left(\omega_{ib}^b\right) + C_G \tag{16}$$

where:	B	= accelerometer bias	D	= gyro drift
	κ_A	= accelerometer scale factor	κ_G	= gyro scale factor
	C_A	= random noise of the accelerometer	C_G	= random noise of the gyro

Depending on the powers of the angular rate the scale factor error itself may contain linear and non linear terms. Supplementary there are at least three accelerometer and gyro axis. So we have to extend the models for the total drifts to:

$$D = D_0 + c_1 a_1 + c_2 a_2 + c_3 a_3 + c_4 a_1 a_3 + c_5 a_1 a_2 + c_6 a_2 a_3 + c_7 B_1 + c_8 B_2 + c_9 B_3 \tag{17}$$

$$B = B_0 + c_2 a_2 + c_3 a_3 \tag{18}$$

where D_0 and B_0 are the constant drift biases, a_j is the acceleration along the j-th case axis, and B_j is a component of the magnetic field.

To take the INS errors into account in the integration process we have to extend the vector of unknowns by these error components and to estimate them in the least squares algorithm. Similarly to the setting up of the design matrix for the initial values we also can consider the influence of the sensor errors on inertial navigation numerically. Besides first the INS raw data file must be duplicated. The raw data of the duplicated file will be subsequently modified with additional errors, according to the modeled sensor errors, and supplied to an inertial navigation. The differences between the in such way produced navigation results and the original navigation solution can be used to build the elements of the Design matrix.

5. Conclusion

This paper has presented the concept of an alternative post processing integration method for GPS and INS measurements. Up to now the considerations were restricted to simulated data. The error implementation and the extension to real data will be done once the simulation show sufficient results. Further on it is necessary to verify how correct the approximate initial values need to be for the algorithm to converge and how robust the error estimation is.

6. Appendix

coordinate system	index	origin	x-axis	y-axis	z-axis
quasi inertial system	i	center of mass of the earth	in the equatorial plane and points to the vernal equinox	in the equatorial plane (right handed system)	in direction to the CIO-Pole
earth centered earth fixed system	e	center of mass of the earth	intersection between the equatorial plane and the plane of greenwich meridian	in the equatorial plane (right handed system)	in direction to the CIO-Pole
local level system	n	center of the accelerometer triad	points to north	points to the east direction (right handed system)	along the negative ellipsoidal normal
body-fixed system	b	center of the accelerometer triad	along the longitudinal axis of the plane (positive in flight direction)	along the lateral axis of the plane (right handed system)	along the yaw-axis, positive is down direction

Table 1 definition of the coordinate systems

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