Multiangular videopolarimetry of an underlying surface

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Abstract – The generalization of the small angle approximation in the form (Goudsmit, 1940) on the case of the vectorial radiative transfer equation (VRTE) is carried out. The obtained approximate solution contains all the features of the exact solution. Therefore the difference between the small angle approximation and the exact VRTE solution is a smooth function, the determination of which does not represent difficulties by any numerical method. On the basis of the analysis of the obtained solution the multiangular method of remote sensing of the underlying surface is offered.

Keywords: remote sensing, polarization, multiangular scheme, vectorial radiative transfer equation.

1. INTRODUCTION

All the information accessible for optical methods at the remote sensing of the underlying surface can be received only at measuring of the reflected radiation polarization. However the extreme sensitivity of the polarization performances not only to optical parameters of the underlying surface, but also to the geometry of the observation scheme, does not allow interpreting unequivocally the observed information. Such interpretation is possible only in the presence of the mathematical model of the polarized radiation reflection by the underlying surface.

Within the framework of the ray optics the development of the similar model is reduced to the solution of a boundary-value problem for the vectorial radiative transfer equation (VRTE). The distinctive feature of any natural formation is a light scattering by the particles with the size much greater than the wavelength that gives a strong anisotropic light scattering. The complexities of the VRTE solution for the anisotropic scattering essentially increase even in the scalar version. The boundary-value problem for the difference of the exact solution and the solution in the approach of the small angle modification of the spherical harmonics method (MSH) for the solution of the VRTE for the media with a strong anisotropic scattering is formulated. As MSH contains all the singularities of the exact solution, the indicated difference is a smooth function, which can be found by any known numerical method.

In this article the analytical solution of the problem for the case of a medium slab in the matrix form is obtained. It allowed developing the model of the polarized radiation reflection from the underlying surface, which includes, besides the multiple scattering in the medium slab, the diffusely-specularly reflection on slab borders. The obtained model allowed determining polarization parameters of radiation invariant to the change of geometry in the remote sensing scheme of the underlying surface. The determination of such parameters does not require measuring of the polarization state in the particular sighting angle, but measuring of the angular dependence of Stokes parameters of the reflected radiation - polarized phase curves (PPC). The analysis of PPC on the basis of the obtained mathematical model showed, that it could be recovered by 4 angular measuring. It gives a multiangular remote sensing scheme of the underlying surface, for which the hardware implementation is offered.

2. REMOTE POLARIMETRY OF UNDERLYING SURFACE

Maximum information about characteristics of the natural objects at the remote sensing, which is available for the optical methods can be obtained by measuring of the radiation polarization. Using the optical methods of the remote sensing an investigator obtains the information about subjects of investigation through the data on angular, spatial, spectral and polarization distributions of the radiation reflected and scattered by the objects under the known illumination conditions (Rozenberg, 1967):

$$\mathbf{L}(\mathbf{r},\hat{\mathbf{l}},\lambda) = \int \ddot{\mathrm{P}}(\mathbf{r}',\hat{\mathbf{l}}',\lambda' \to \mathbf{r},\hat{\mathbf{l}},\lambda)\mathbf{L}_0(\mathbf{r}',\hat{\mathbf{l}}',\lambda')d^2r'd\hat{\mathbf{l}}'d\lambda', \qquad (1)$$

where $\mathbf{L}_0(\mathbf{r},\hat{\mathbf{l}},\lambda)$ is Stokes vector-parameter of the incident radiation in the point $\mathbf{r}$ of the light field in the direction $\hat{\mathbf{l}}$;
λ is the wavelength;
$\ddot{\mathrm{P}}(\cdot)$ is the radiation transfer matrix.

Hereinafter a double arrow above the symbol designates a matrix.

According to (1) the experimentally measured matrix $\ddot{\mathrm{P}}(\cdot)$ depends purely on the substance properties of the media of the investigated subjects. This matrix contains all the information about characteristics of these media, which is available for the optical methods. The information about the substance structure is carried by the attenuation coefficient ε, the normalized phase function matrix $\ddot{x}(\hat{\mathbf{l}},\hat{\mathbf{l}}')$, the scattering coefficient σ and the single scattering albedo Λ. The feature of the up-to-date methods of the single-angular polarization remote sensing is their high sensitivity both to the optical parameters of substance and to the scheme of observation.

The experimentally obtained laws of the polarization behavior of the radiation reflected by objects cannot be explained only by the surface reflection. It is necessary to take into account the radiation penetration into the thickness of the substance as well as to analyze multiple interactions between the radiation and the volume of the substance.

The experimental studies of the polarization reflective properties of the underlying surfaces demonstrate a substantial difference of the dependence of the polarization degree p on the phase angle α (polarization-phase curves (PPC)) from Fresnel law for real objects with a complex structure (see fig. 1) (Lyot, 1924):

1) the location of the polarization maximum and the value of polarization at the maximum point significantly differ from those defined by Fresnel law;

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2) the substances with the complex structure demonstrate at the small phase angles the presence of «the negative branch of polarization» for the reflected radiation, that corresponds to the situation when the polarization plane coincides with the reflection plane;
3) the action of Umov low is typical of the positive branch of PPC: the higher the surface albedo is the lower the polarization degree of radiation is.

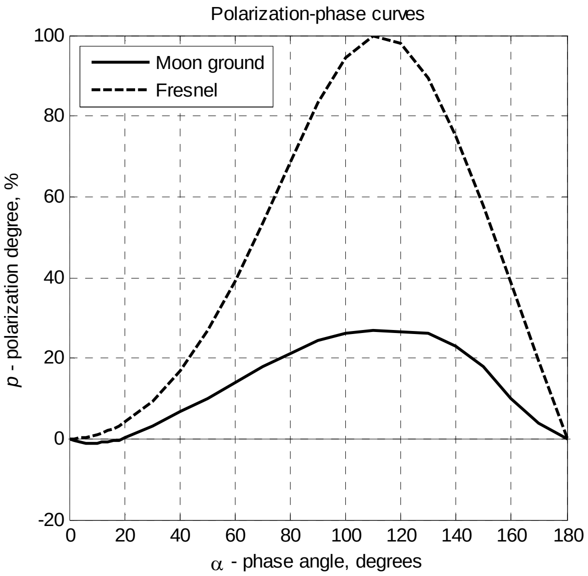

Figure 1. PPC for reflection by a dielectric (Fresnel) and by an underlying surface (moon ground).

We have proposed a mathematical model, which provides the most complete consideration of the mechanisms accompanying the polarization transformations of the radiation in the medium with anisotropic scattering. This model of the polarization characteristics of the solar radiation reflected by the natural objects can be constructed using the following assumptions:

1) The underlying surface is represented by the plane-parallel layer of the turbid medium. The substance of the medium has the refraction index m_0 and contains the suspended particles (assumed to be ball-shaped), which scatter light independently from each other and are characterized by a complex refraction index m and the function of the particle size distribution.
2) The layer is bounded from above and below by the substances having different values of the refraction index m_1 and m_2. Let us represent the upper interface of two media as a randomly rough surface, which can be defined as a set of random realizations of the small square of about an average level. Each element of the randomly rough surface is characterized by Fresnel reflection.
3) Within the ray approximation the transformation of the radiation inside the layer under the known irradiation is represented by the transfer process in the 5-dimensional phase space, described by VRTE.

Taking into consideration the linearity of VRTE and boundary conditions we can represent the reflection coefficient of the underlying surface as follows:

$$\overleftrightarrow{\rho} = \overleftrightarrow{\mathrm{R}}_2 \otimes \left\langle \overleftrightarrow{\mathcal{R}}_+ \right\rangle \otimes \overleftrightarrow{\mathrm{R}}_1 + \overleftrightarrow{\mathrm{R}}_2 \otimes \left\langle \overleftrightarrow{\mathcal{T}}_+ \right\rangle \otimes \overleftrightarrow{p}(z=0) \otimes \left\langle \overleftrightarrow{\mathcal{T}}_- \right\rangle \overleftrightarrow{\mathrm{R}}_1 , \qquad (2)$$

where symbol "⊗" means convolution over angle $\hat{\mathbf{l}}$;

$\overleftrightarrow{\mathcal{R}}_\pm, \overleftrightarrow{\mathcal{T}}_\pm$ is the Fresnel matrix of the reflection and the transmission of each elements of the randomly rough surface (± - Up or Down);

<> is the average operation by the realization of the randomly rough surface assembly;

$\overleftrightarrow{p}(z;\mathbf{r}_0,\hat{\mathbf{l}}_0 \to \mathbf{r},\hat{\mathbf{l}})$ is the transfer matrix (Green function for VRTE), which is defined only by the layer properties;

$\overleftrightarrow{\mathrm{R}}(\hat{\mathbf{l}} \times \hat{\mathbf{l}}' \to \hat{\mathbf{l}} \times \hat{\mathbf{l}}_0') = \overleftrightarrow{\mathrm{R}}(\chi)$ is the transformation matrix for the vector-parameter at the rotation of the reference plane (rotator), where χ is a dihedral angle between planes $(\hat{\mathbf{l}} \times \hat{\mathbf{l}}')$ and $(\hat{\mathbf{l}} \times \hat{\mathbf{l}}_0)$.

The boundary-value problem of VRTE for $\overleftrightarrow{p}(z;\mathbf{r}_0,\hat{\mathbf{l}}_0 \to \mathbf{r},\hat{\mathbf{l}})$ in the case of a turbid layer irradiated by a plane unidirectional source of the arbitrarily polarized light can be represented as follows:

$$\mu \frac{\partial}{\partial \tau} \overleftrightarrow{p}(\tau,\hat{\mathbf{l}}) + \overleftrightarrow{p}(\tau,\hat{\mathbf{l}}) = \frac{\Lambda}{4\pi} \oint \overleftrightarrow{\mathrm{S}}(\hat{\mathbf{l}},\hat{\mathbf{l}}') \overleftrightarrow{p}(\tau,\hat{\mathbf{l}}') d\hat{\mathbf{l}}', \qquad (3)$$

$$\begin{cases} \overleftrightarrow{p}(\tau,\hat{\mathbf{l}})\Big|_{\Gamma_1} = \overleftrightarrow{1}\delta\left(\hat{\mathbf{l}} - \hat{\mathbf{l}}_0\right), \\ \overleftrightarrow{p}(\tau,\hat{\mathbf{l}})\Big|_{\Gamma_2} = 0; \end{cases}$$

where τ is optical depth;

$\overleftrightarrow{\mathrm{S}}(\hat{\mathbf{l}},\hat{\mathbf{l}}') = \overleftrightarrow{\mathrm{R}}(\hat{\mathbf{l}} \times \hat{\mathbf{l}}' \to \hat{\mathbf{l}} \times \hat{\mathbf{l}}_0') \overleftrightarrow{x}(\hat{\mathbf{l}},\hat{\mathbf{l}}') \overleftrightarrow{\mathrm{R}}(\hat{\mathbf{l}}_0 \times \hat{\mathbf{l}}' \to \hat{\mathbf{l}} \times \hat{\mathbf{l}}')$ is the local transformation matrix;

$\Gamma_1 = \left\{ z = 0, \hat{\mathbf{l}} \in \Omega_- \right\}, \quad \Gamma_2 = \left\{ z = z_0, \hat{\mathbf{l}} \in \Omega_+ \right\}$;

z_0 is the thickness of the turbid layer;

$\Omega_\pm$ is the up/down hemisphere of the space directions.

3. METHOD OF VRTE SOLUTION

Unfortunately this boundary-value problem has no exact solution of VRTE in the case of the strong anisotropic scattering. In the paper (Boudak, 2003) the effective numerical method of the solution of the scalar radiative transfer equation is offered for the case of the strong anisotropic light scattering. The method is based on the subtraction from the exact solution the small angle approximation in the form (Goudsmit, 1940). As the small angle approximation contains all the singularities of the exact solution, the rest is a smooth function, the determination of which does not present difficulties by any numerical method.

The advantage of the small angle approximation (Goudsmit, 1940) is its analytic form as a series on the surface harmonic. On the basis of the addition theorem for the Legendre polynomials this analytic form essentially simplifies the determination of the source function in the radiative transfer equation. In many respects such approach is similar to (Evans, 1993), however essentially exceeds it on the convergence rapid. In the article (Boudak, 1990) the small angle modification of the spherical harmonics method (MSH) being the generalization of the approach (Goudsmit, 1940) on the arbitrary medium geometry is offered. In (Astakhov, 1994) the MSH generalization on a vector case is carried out. It allows taking advantage of the method (Boudak, 2003) for the solution of the boundary-value problem (3).

MSH is based on the definition of the continuous dependence of the decomposition coefficients of the angular radiance distribution on the surface harmonic from their numbers. It allows reducing the infinite set of the ordinary differential equations of the spherical harmonics method to one partial equation permitting the analytical solution.

In order to pass to the equation set of spherical harmonics method in the vectorial case the generalized spherical harmonics are introduced into consideration (Kuščer, 1959):

$$\ddot{\mathrm{Y}}_l^n(\mu) = \mathrm{Diag}\left\{\mathrm{P}_{n,+2}^l(\mu), \mathrm{P}_{n,+0}^l(\mu), \mathrm{P}_{n,-0}^l(\mu), \mathrm{P}_{n,-2}^l(\mu)\right\}, \qquad (4)$$

where $\mathrm{P}_{n,s}^l(\mu)$ are generalized Legendre polynomials, s=±0, ±2; and all the functions contained in (3) are represented as the series in terms of the generalized spherical harmonics:

$$\left[\ddot{x}_{CP}(\cos\gamma)\right]_{rs} = \sum_{l=0}^{\infty}(2k+1)x_{rs}^k \mathrm{P}_{r,s}^k(\cos\gamma),$$

$$\vec{p}(\tau,\nu,\varphi) = \sum_{n=-\infty}^{+\infty}\sum_{k=0}^{\infty}\frac{2k+1}{4\pi}\mathrm{e}^{in\varphi}\ddot{\mathrm{Y}}_k^n(\nu)\vec{p}_k^n(\tau), \qquad (5)$$

where $\nu = (\hat{\mathbf{l}},\hat{\mathbf{l}}_0)$ is cosine of the zenith angle and φ is the azimuth angle $\hat{\mathbf{l}}$ with the reference to the direction of the radiation incidence;

$\mu = \nu\mu_0 + \sqrt{1-\mu_0^2}\cdot\sqrt{1-\nu^2}\cos\varphi$;

μ_0 is cosine of the incident angle, $\left(\ddot{x}_k\right)_{rs} = x_{rs}^k$.

Let's determine the continuous dependence $\vec{p}^n(k,\tau)$ of the coefficients $\vec{p}_k^n(\tau)$ on the numbers. In the case of the strong anisotropy this dependence is a slow monotonic decreasing function, that allows expanding it in a Taylor series with the maintenance of the first two terms

$$\vec{p}_{k\pm1}^n(\tau) \equiv \vec{p}^n(k\pm1,\tau) \approx \vec{p}^n(k,\tau) \pm \frac{\partial}{\partial k}\vec{p}^n(k,\tau). \qquad (6)$$

The assumption (6) transforms the infinite equation set of the spherical harmonics method to one partial equation. Therefore after carrying out the corresponding transformations (Astakhov, 1994) the solution for the boundary-value problem can be represented in the following form:

$$\vec{p}_{MSH}(\tau,\nu,\varphi) = \sum_{n=-1}^{+1}\sum_{k=1}^{\infty}\frac{2k+1}{4\pi}\mathrm{e}^{i2\pi\varphi}\ddot{\mathrm{Y}}_k^{2n}(\nu)\vec{p}_k(\tau), \qquad (7)$$

where $\vec{p}_k(\tau) = \sum_{i=1}^{4}\frac{\exp(\Lambda\tau\zeta_i/\mu_0)\left(\zeta_i\ddot{\mathbf{1}} - \ddot{x}_k\right)^{\mathrm{V}}}{\frac{d}{dz}\det\left(z\ddot{\mathbf{1}} - \ddot{x}_k\right)\Big|_{z=\zeta_i}}$ is the expansion coefficient of the matrix Green function for a plane layer of the turbid medium;

ζ_i are the eigenvalues of the characteristic equation

$$\det(\zeta_i\ddot{1} - \ddot{x}_k) = 0\,. \qquad (8)$$

The obtained solution neglects the length variance of the scattered ray trajectories and the backscattering. For the full solution of the boundary-value problem (3) let's present the angular distribution of the Stokes vector-parameter in the following form

$$\vec{p}(\tau,\mu,\varphi) = \vec{p}_{MSH}(\tau,\nu,\varphi) + \vec{p}\,'(\tau,\mu,\varphi)\,, \qquad (9)$$

where $\vec{p}\,'(\tau,\mu,\varphi)$ is a smooth rest, for which we have the boundary-value problem

$$\mu\frac{\partial}{\partial\tau}\vec{p}'(\tau,\hat{\mathbf{l}}) + \vec{p}'(\tau,\hat{\mathbf{l}}) = \frac{\Lambda}{4\pi}\oint\ddot{\mathrm{S}}(\hat{\mathbf{l}},\hat{\mathbf{l}}')\vec{p}'(\tau,\hat{\mathbf{l}}')d\hat{\mathbf{l}}' + \vec{\mathrm{F}}(\tau,\hat{\mathbf{l}}), \qquad (10)$$

$$\begin{cases}\vec{p}'(\tau,\hat{\mathbf{l}})\Big|_{\Gamma_1} = 0,\\ \vec{p}'(\tau,\hat{\mathbf{l}})\Big|_{\Gamma_2} = -\vec{p}_{MSH}(\tau,\hat{\mathbf{l}});\end{cases}$$

where the source function is determined by the expression

$$\vec{\mathrm{F}}(\tau,\hat{\mathbf{l}}) = \sum_{n=-\infty}^{+\infty}\sum_{k=0}^{\infty}\frac{2k+1}{4\pi}\mathrm{e}^{in\varphi}\ddot{\mathrm{Y}}_k^n(\nu)\ddot{\mathrm{T}}_k^n\vec{p}_k^n(\tau)\,, \qquad (11)$$

matrix $\ddot{\mathrm{T}}_k^n$ is a linear combination of the matrices of the recurrence relations for the generalized Legendre polynomials (Gel'fand I.M., 1963) and the matrix $\ddot{x}_k$. At the definition of the expression (11) it is necessary to take into account a transformation matrix (Kuščer I., 1959) from CP-representation, in which the solution (7) is obtained, to SP-representation in (10). The return to SP-representation makes all the matrices in (10) real.

We'll solve the boundary-value problem by the regular discreet ordinate method, which allows taking into account the complex boundary conditions.

The most of real observable phenomena can be described using the proposed model. The charts in Fig.2 demonstrate calculations of PPC of the polarization degree as a function of the particle size in the suspension, where r_m is a modal radius of the particle size distribution. The figure demonstrates that the obtained solution corresponds to PPC of the real existing underlying surface.

4. THE MULTI-ANGULAR VIDEOPOLARIMETRY

Basing on the above mentioned mathematical model the analysis of the influence of the layer parameters on the PPC shape has been carried out. The analysis has made it possible to determine the following PPC parameters, which are sensitive to the variations of layer characteristics and independent on the observation scheme:

1. The angle of the polarization inversion (where PPC passes through zero);
2. The value of the polarization degree in the points of maximum and minimum;
3. The slope of the PPC in the inversion point, which is also the characteristic of the inner structure of the substance layer (to be more exact, the size of particles).

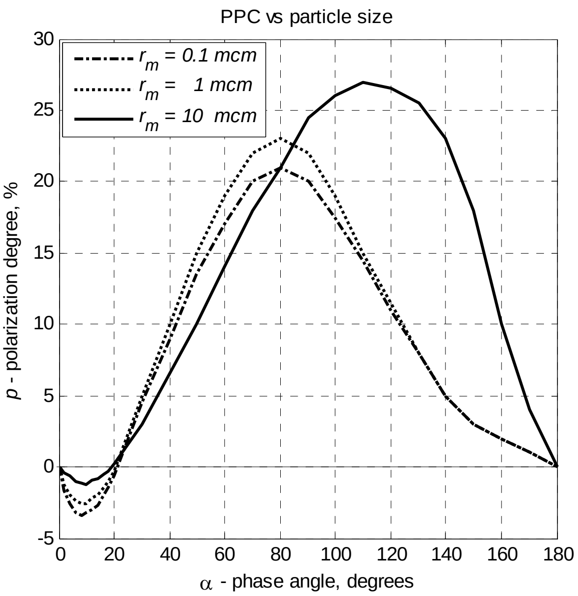

Figure 2. The influence of the particles size on the PPC form.

The stability of these invariants is a basis for the classification and identification of natural objects. The unambiguous interpretation of outcomes of the remote sensing is possible only by the definition of PPC characteristics over all Stokes parameters.

The proposed multi-angular method includes measuring of PPC constants. An original structure of a device for their measurement has been developed.

It has been shown that PPC can be approximated over 4 points which determine angular channels of the polarimetric system. On the basis of the analysis a scheme of the 4-angular passive type polarimeter for the remote sensing has been developed. The principle of its work consists of the mechanical system ensuring a periodic change of polarization filters in front of each of four radiation receivers (see fig. 3). Each of the receivers is oriented a different sight angle. Using calculations based on the mathematical model, a range of phase angles as wide as $\Delta\alpha=80^o$ may be recommended as a maximum coverage angle over four angular channels. Thus such structure turns sequentially each of four angular channels into a set of four polarization channels. A distinctive feature of the developed device is its ability to carry out measurements of the polarization parameters at several phase angles at the same time.

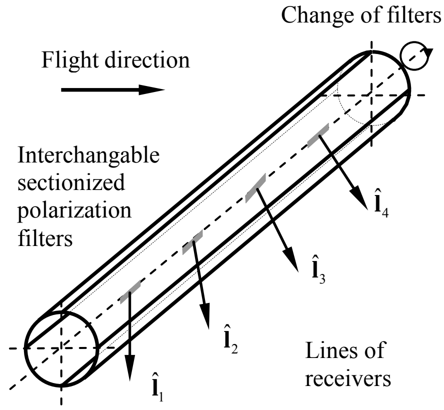

Figure 3. The influence of the particles size on the PPC form.

5. CONCLUSIONS

The solution of the vectorial radiative transfer equation has an analytic form with a few parameters, which allows solving the inverse problem. This provides the solution of the problems for reconstructing the optical parameters of the underlying surface using the measured data of the polarization of the reflected radiation.

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