THE EXPERIENCE OF USING OF WAVELET ANALYSIS FOR INVESTIGATION OF THE SEA LEVEL VARIABILITY ON THE BASE SATELLITE ALTIMETRY INFORMATION

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Abstract: The capabilities of continuous wavelet-transformation for research of temporary variability of the sea level in the north-west part of the Pacific ocean are estimate. The advantage of the wavelet analysis to research the nonstationary time series in comparison with Fourier analysis are demonstrated. The application of the wavelet-analysis (Matlab version) on example of the time series altimetry information from satellites T/P, ERS-1/2 are examined. With using the wavelet spectrograms of time series of the sea level the main scales of the fluctuations are determined. The hypotheses about the mechanisms of the external delivery of energy are proposed. Based on the estimation of slopes the areas of maxima energy in the wavelet spectrograms the direction of nonlinear energy flow of oscillations on temporary scales is determined. That the energy flow from large-scale motions to small-scale is connected to nonlinear generating of turbulence are demonstrated.

Keywords: Wavelet analysis, satellite altimetry, Pacific ocean, sea level

1. INTRODUCTION

Wavelet analysis is becoming a common tool for analyzing localized variations of power within time series. In the article is shown, that the wavelet-analysis has been mostly adapted to study a structure inhomogeneous and non-stationary oceanographic processes and the practical advisories of a research through wavelet-analysis in modification MATLAB of time series of a level of ocean are given. At exposition of a mathematical leg of the problem we further follow to articles [Astafeva N.M., 1998; Vitjazev V.V.,2001; Smolencev N.K.,2003].

The Wavelet's theory is powerful alternative to the Fourier analysis and gives more flexible engineering for signal processing particularly for analysis of time series.

The classical Fourier analysis is founded on a capability of the researches of functions in temporal and frequency areas with a help of the direct and inverse Fourier transforms which demonstrates remarkable ability of the Fourier transform to focalize in a point the information "spread" on time about periodicity of a function at transition from a time domain in frequency. It is reached because the kernel of the Fourier transform is not localized in time but has a marginal localization in frequency area. This circumstance also makes the Fourier transform as the perfect instrument for study of processes which properties do not vary in the time.

However this circumstance makes the Fourier transform as a poor method for a research of functions with the changeability's characteristics in the time. For example the Fourier transform does not distinguish a signal representing the sum of two sine waves from a signal consisting of the same sine waves but joining sequentially (fig. 1).

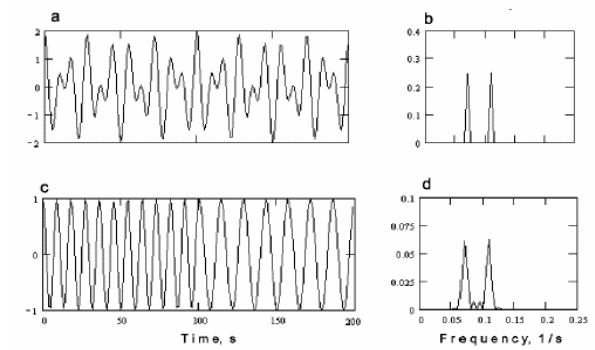

Fig. 1: Ambiguity of the Fourier transformation: *a)* - a modelling time series - the sum of two sinusoids with frequencies 0.062 and 0.105 Hz; *b)* – the periodogramm of the sums of these sinusoids; *c)* - the same sinusoids included consistently; *d)* -periodogramm of the sinusoids included consistently.

For elimination of this deficiency it is necessary to localize the Fourier transform on intervals of a final length. Many investigators are used such a reception, evaluating of a power spectrum not only on all length of the time series but also by its different parts using, for example, the help of the Fourier transformation window, which allows to receive time changes of the spectrum. Here it is important to underline that the transformation window has a stationary value for a breadth, characterizing a length of an interval ΔT which in turn determines by a measure of the time resolution while the line width determines a measure of the frequency resolution. It is known that both these performances are inversely proportional. It is necessary to add that there is a problem of a choice of the time area window width in using of the window transformation. Too wide window can provide a reasonable representation of the low-frequency components but his width will be superfluous for harmonics with high frequency as all interesting irregularities in high-frequency area of the spectrum will smooth out. On the contrary, the narrow window will enable to study the variations of high-frequency components but it will not be adequate for low-frequency harmonics.

Wavelet-transformation of a signal is a generalization of the spectral analysis in which basis classical Fourier transformation lays. Bases used for this purpose are named wavelet-functions of two arguments - scale and shift. The concept of the frequency of the classical spectral analysis is replaced here with scale and to block with «short waves» all time base is entered a time shift of functions. As against the traditional Fourier transformation the wavelet-transformation provides a bidimensional representation of a researched one-dimensional signal in a frequency plane the frequency – the position. Thus an analogue of the frequency is the scale of argument of the basic function so its position is characterized by its shift. It allows to locate the large and small details of the signals simultaneously locating them in a time scale. In the whole, the wavelet-analysis can be characterized as the located spectral analysis or the spectral analysis of local indignations. If you make window function dependent on a frequency in a manner so the windows became wide for the low frequencies and wider for high frequencies the window Fourier transformation passes in a new class of transformations which has received the name of wavelet-transformation.

The term "wavelet" occurs from English word wavelet which a literal translation means the small wave. The basic is here that the wavelet-transformation not simply "cuts" researched object on pieces but allocates from it components of different scales and that each component is analyzed with that degree of detail which corresponds to his scale.

2. WAVLET ANALYSIS

Let's give now the formal definitions underlying the wavelet analysis.

Definition 1. The continuous wavelet-transformation of function $f(t) \in L^2(R)$ is:

$$W(a,b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} f(t) \Psi^* \left(\frac{t-b}{a} \right) dt, \qquad \textbf{(1)}$$

were $a, b \in R, \quad a \neq 0$.

Thus, basis of wavelet's are functions of type $\Psi\left(\frac{t-b}{a}\right)$, where *b* - shift, *a* - scale.

Definition 2. Function $\Psi(t)$ n included in equation (1) is called wavelet (analyzing, basic or parent wavelet). In the formula (1) symbol * designates procedure of complex interface.

The parameter *a* determines the wavelet size and is called as a scale. Its analogue in the Fourier-analysis is the period (frequency) of harmonious fluctuation. It is necessary to say that concept of the scale is much wider (though also less evident) than concept of the period. It is connected by that the functional kernel in the Fourier transformation is fixed once and for all while the wavelet transformation of the same function can be received with the help of various basic wavelets (i.e. in different systems of scales). The parameter *b* sets a time wavelet localization and is called as a shift. This parameter has no analogue in the Fourier transformation.

The basic properties of wavelets are a time-frequency localization and equality to zero of the zero moments. The time-frequency localization is prominent feature analyzing wavelet. It means, that wavelets and their Fourier transformation are essentially different from zero only on small time and frequency intervals and are different from zero (or equal to zero) outside of these intervals very little. To provide convertibility of the wavelet-transformation the zero moment of wavelet-transformation should be equal to zero. For applications it happens important that not only the zero moment, but also *m* the highest moments were equal to zero. Wavelets, possessing such property, appear useful at the analysis of time series has polynomial trends. Ignoring a trend, they allow to investigate high-frequency components of lines.

Let's consider some wavelet examples which are the most suitable as it seems to us for the solving of the tasks put by us.

MHAT wavelet

This wavelet turns out double differentiation of Gauss function. The name of it wavelet's - MHAT (Mexican HAT) has taken place from a characteristic kind of his picture - like sombrero. MHAT-wavelets are well located both in time and in frequency area. We shall note also that MHAT-wavelet has zero values of the zero and first moments.

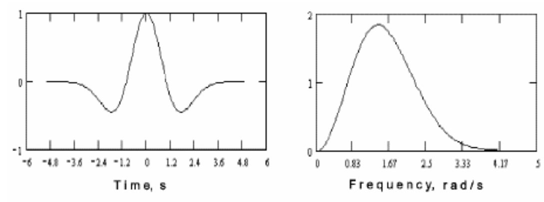

Fig. 2: Wavelet MHAT (on the left) and its Fourier transformation (on the right).

Wavelet Morlet

Analytical representation wavelet's Morlet is set by the following expression:

$$\Psi(t) = e^{-t^2/\alpha^2} \left[e^{ik_0 t -} e^{k_0^2 \alpha^2 /4} \right].$$

Wavelet Morlet is the lonely flat wave modulated Gauss. Parameter α sets width Gauss function, parameter k_0 - a frequency of a flat wave. Usually choose α = 2 and k_0 = 2π. At these values with sufficient accuracy it is possible to accept: $\Psi(t) = e^{-t^2/\alpha^2} e^{i2\pi t}$

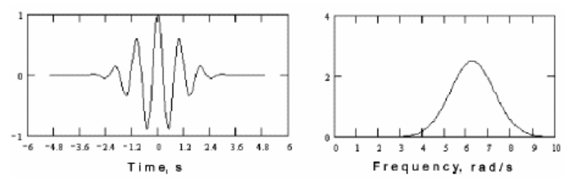

Fig. 3: Real part wavelet Morlet (at the left) and its transformation Fourier (on the right).

Let us note that wavelet Morlet has two first the moment are equal to zero. Generally, than above the order analyzing wavelet, the more the zero moments he has, the better transformation allocates features of an initial signal.

Representations of results of wavelet-transformation can be different. We shall note some from them, the most important for the practical application. We shall stop more in detail on continuous one-dimensional wavelet transformation. We shall notice that result of wavelet-transformation of one-dimensional lines *f (t)* is the two-dimensional file of amplitudes of wavelet-transformation - values of coefficients *W (a, b)*. Distribution of these values in space *(a, b)* gives the information on evolution of the relative contribution a component of different scale in time. Each line of received two-dimensional matrix W (a, b) describes change in time of a component of initial lines with period T which calculation depends on characteristics used wavelet.

For complex wavelets the result of continuous wavelet transformation, i.e. the received file of coefficients, can be submitted as values of amplitudes and phases. More useful to analyses is shown not amplitudes and its positions on time scale but the localization of local extrema (maxima or minima). Such graphic representations have received the name – 'sceleton'.

The picture continuous wavelet-transformation depends from chosen wavelet. For example, the module of coefficients of transformation calculate with using wavelet Morlet for testing harmonious signal is displayed as a continuous line of a parallel axis of time which center is located on frequency of a signal and corresponds to number of a line in a matrix of coefficient: $a=N(\log_2(pT_0/k_0T)-1.$.

Thus thickness of a line is determined by parameter k_0 and decreases at his increase. The module of coefficients of transformation with the help wavelet MHAT in this case on continuous wavelet-transformation will be displayed as periodically repeating pieces parallel to each other and an axis and which center is located on frequency of a signal and corresponds to number of a line in a matrix of coefficients: $a=N(\log_2(pT_0/T)-1,5$. The center of an arrangement of pieces on time corresponds to positions of maxima (at a conclusion of amplitude of factors) or maxima and minima (at a conclusion of the module) a harmonious signal.

The information on features of initial time series is made in asymptotic behaviors of coefficients *W (a, b)* at small values *a*. If coefficients on small scales miss, in the given point there is a feature.

At use of continuous wavelet-transformation it is necessary to take into account, that each value *W (a, b)* is calculated on a piece of initial lines which length increases with increase in scale *a* and determined by the size of wavelet on each line of a matrix. For a line with number *a* length analyzing wavelet $L=T_0 2^{(1-A/N)}$. Initial and final on time sites of a file have extending areas, in which reliability of coefficients *W (a, b)* gradually decreases at approach edges as coefficients are calculated on the pieces which are overstepping the bounds of initial time series.

The wavelet analysis has been applied for research satellite altimetric data. The file altimetric information is received from satellites TOPEX/POSEIDON, ERS-1/2 and represents maps of anomalies of a sea level from October, 1992 till February, 2002 (time step 7 days). As bases function have been chosen wavelet Morlet which, from our point of view, are well adapted to research of the processes containing climatic and synoptic components [Astafeva N.M,1998; Vitjazev V.V,2001; Vorobjov V.I., Gribunin V.G. ,1999].

For an example we shall carry out the continuously wavelet analysis of a sea level in point with coordinates 62.89 N., 161.66E, located in Kuril area in currents Oyasio near of the coast of Kamchatka peninsula. The initial time series and continuously wavelet transformation showed on figure 4.

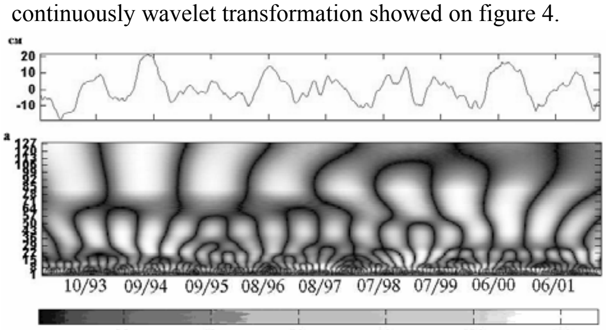

Fig. 4. The deviations of sea level of ocean in Kuril area (above) both its wavelet-spectrum and a scale (below).

A range of variability of a level are from -20 up to 22 sm. On the figure of initial realization annual fluctuations sea level and fluctuation with various scales are observed.

Using wavelet transformation we can determine, which scales (or frequencies) define variability of a sea level on different intervals of time. On the wavelet-spectra the expressed maxima of energy corresponding to the periods of 2 years (*a* = 80) are observed. But if in first half of realization: since 1992 till 1997 maxima of energy are distributed as well in area of lower frequencies till 3-4 years, thus not being distributed on the frequencies corresponding to the intermediate periods from 1,5 till 2 years (on fig. 4 we precisely see the dark areas corresponding to reduction of energy) after 1997 a picture return: maxima of energy, not being distributed on more low-frequency area pass in area of higher frequencies, that is the zone of extreme values of energy is distributed in an interval of scales from 2 years about one year while in area of lower frequencies in this time interval of any increases in energy it is not marked.

Besides this, on fig. 4 with a various degree of intensity the located maxima of energy for annual (*a* = 43) and semi-annual (*a* = 21) scales are marked. 1994 and 2000 years marked as the characteristic years responsible for the most essential contribution of annual fluctuations in variability of a level while with 1995 on 1998 the contribution of annual fluctuations to variability of a level is least significant. Semi-annual fluctuations which also are traditionally considered as essential at variability of a level of ocean, bring the contribution much smaller, that is distinctly visible on intensity of energy on fig. 4. Semi-annual fluctuations for 1994, 1999 (and partly 2001year) are most significant. Fluctuations of higher frequencies (2-3 months) are marked also in several parts of realization, in particular in the winter 1994 and, especially, in summer months 1997-99 and 2001 years.

The comparative contribution of annual and semi-annual fluctuations to variability of a sea level can be estimated using of linear coefficients. Linear coefficient of wavelet transformation was calculated using MATLAB (fig. 5).

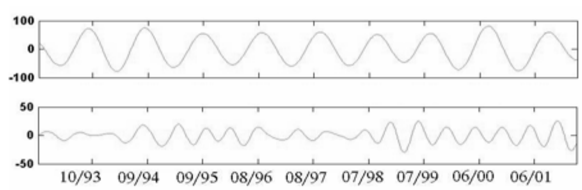

Fig. 5. The linear coefficients of wavelet-transformation for annual (above) and semi-annual fluctuations of the sea level.

Range of variability of linear coefficient of annual fluctuations from -90 up to 90 while semi-annual he in some times is less. That gives the basis to judge the essential contribution of these fluctuations to examined variability. Thus it is important to note, that the contribution of these fluctuations to variability of a level of ocean various in various time intervals. For annual fluctuations this feature is marked less precisely, than for semi-annual. For example, the amplitudes of variability of annual fluctuations for 1995-99 year hardly less than in other time intervals. For semi-annual fluctuations the marked non-uniformity of the contribution is allocated especially brightly: in the certain time intervals, for example, with 1992 up to the middle of 1994 and in 1997-98 the contribution of semi-annual fluctuations in general is insignificant for variability of a level.

Seasonal variability of fluctuations of a level in this area is investigated well on the basis of sea level information from coastal moreographs. However representations about seasonal variations of a level at open ocean were rather limited before occurrence satellite altimetric measurements.

Amplitudes of an annual harmonic on all examined area change in limits from 5 up to 18 sm. Amplitude of a wave with the annual period often represent as an annual solar component the tidal forces and estimate methods of the harmonious analysis. Legitimacy of such identification causes the certain doubts. It is obvious, that variations of displacement of the sea level by annual tidal forces, much less than annual variations of a meteorological origin. Therefore the harmonious analysis actually allocates amplitude and a phase of an annual harmonic of a seasonal fluctuation of vertical displacement of a sea level.

In an examined region the contribution seasonal steric components in a variation of a level exceeds 50 % and places is especial in east region of Pacific ocean, he achieves 70 %. After the exception steric's components significant amplitudes displacement of a sea level (up to 5 sm) with the expressed winter maxima are characteristic only for current Kuroshio (between 30 and 40N and before 180E). Usually, researchers believe that contribution the static effects of atmospheric pressure ("invert barometer") is 25-30 % from size of seasonal variations of a sea level. Static vertical displacement of a level in Pacific ocean much less than observable values and also differ under the phase characteristics. In northern region of Pacific ocean, it is especial in the areas adjoining to continents, the continental type of an annual cycle of atmospheric pressure with a maximum in January - February and a minimum in July, i.e. return to an annual fluctuation of temperature of air prevails. Accordingly the seasonal rhythmic of static displacement of a sea level of ocean coincides with an annual fluctuation of temperature of air. In the middle latitude of the region of Pacific ocean (from 40 up to 62N and to the east 160E) owing to influence of cyclones the oceanic type of an annual course of atmospheric pressure with a minimum in December - January and a maximum during the summer is expressed, i.e. the seasonal fluctuation of atmospheric pressure coincides with a seasonal fluctuation of temperature of air. It is possible to assume, that variations of a level of these scales not only are connected to static effects of atmospheric pressure and 0radiating balance. They are connect with dynamic processes too, including low-frequency gradient-vorticity waves which mechanism of occurrence is connected to variability of a tangential pressure of a wind and gradients of atmospheric pressure [Belonenko T.V.,2004].

These representations about the comparative contribution of annual and semi-annual fluctuations of a sea level can be considerably added on the basis of the wavelet-analysis, both concerning mechanisms of their generation, and about interannual variability of intensity of these fluctuations.

Coming back to a question on comparison of the wavelet-analysis and its advantages concerning the spectral analysis, we shall note, that the marked features at the spectral analysis in general in any way are not allocated.

Let's pay attention also that areas of the increased energy often have the certain inclination concerning system of coordinates. For example, the extensive area of a low-frequency maximum of energy of 1996-97 is inclined to the right. It means, that the maximum of energy corresponding to scale of 2 years, is gradually passed on other scales aside lower frequencies. There is a redistribution of energy on scales from higher to lower. The located maxima of energy on the wavelet-spectra can mean areas of power supply from external sources. The extended areas of the maximal energy are connected, probably, with transfer of energy from one time scales to another. When this inclination is directed from area of the big periods to area low - there is a stream of energy from large-scale movements to small-scale, connected with nonlinear generation of the turbulence caused by dynamic instability of large-scale movements. When in the return side there is a power supply of large-scale movements from small-scale. Similar processes have received the name in physics «movements with negative viscosity». Corresponding coefficients of a turbulent exchange have a negative sign. The similar phenomena are well-known enough in meteorology when power supply of large-scale jet currents in high layers of an atmosphere occurs from cyclones.

Inclinations the line of the maximal energy, which we can see on the sceleton of the wavelet-transformations (fig. 6), schematically display the marked transfer of energy between different scales. We distinctly understand, for what time interval such transfer of energy occurs and in what direction and for what is absent.

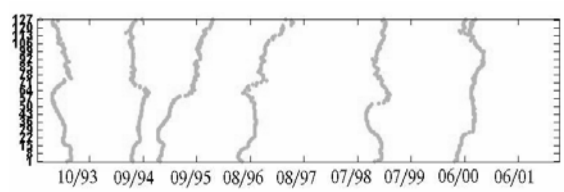

Fig. 6. Sceleton of the wavelet-transformation for the sea level fluctuation in Kurils area.

3. CONCLUSION

The wavelet-analysis promptly develops now and draws attention scientific in various applied areas, including draws in the field of natural sciences. In these investigations we have shown opportunities of the wavelet-analysis to study oceanographic fields by the example of research of variability of a sea level of the ocean. This method opens wide prospects in this direction.

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