Error analysis of weak lidar signals inverting

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Abstract - It is considered the problem of the software development for the control of the reliability with which the characteristics of atmospheric aerosols are determined from the results of lidar measurements. The errors of these characteristics strongly depend on the algorithms developed for weak signals processing. The rigorous solution was used for the lidar techniques development needed to determine the extinction coefficient of the atmosphere. It was carried out here inverting the backscattering signals returned from a homogeneous atmosphere and measured by the system transmitting pulses from one point in space. The homogeneity criterion was found and used for computerized testing of the lidar systems. The error analysis was carried out using the experimental results. The analysis of the results shows it was found a number of effective algorithms for weak signals processing based on the new rigorous solution of the equation and useful for lidar software development.

Keywords: lidar, aerosols backscattering, weak signals

1. INTRODUCTION

It is not a simple problem to develop lidar methods for the aerosols characteristics determination from the results of lidar measurements of weak backscattering signals. The software development is needed for the control of the reliability with which the aerosol parameters are determined from the lidar data. The errors of these parameters strongly depend on the data-processing technique. The purpose of this paper is discussing of the lidar equation solutions developed for weak signals processing and the results of the analysis of the effectiveness of these solutions.

2. ELASTIC LIDAR DATA-PROCESSING TECHNIQUE

The appropriate choice of the algorithms to be used for lidar data analysis is based on the rigorous solution [1] of the lidar equation

$$P(\vec{R}i,\vec{r}_j)=P_*(\vec{R}_i)+Af\beta(\vec{r}_j)\cdot T_{i,j}^2,$$

$$T_{i,j}=exp\left(-\int\limits_{C_{i,j}}\sigma(\vec{r})\,dr\right), \qquad (1)$$

where P_* = the power of the background light, $\boldsymbol{T_{i,j}}$ = the transmittance from the lidar position to the scattering volume position, $P_{i,j}$ = the backscattering signals measured by the system transmitting pulses from points in space $i=1,2,...$, $\vec{r}_j$ = the scattering volume positions, $\vec{R}_i$ = the transceiver positions, f = the beam convergence factor, A = the constant, β = the backscattering coefficient, σ = the extinction coefficient, $C_{i,j}$ = the segment $[\vec{R}_i,\vec{r}_j]$.

This solution was developed in terms of the solution of the equations

$$\frac{1}{W_i}\left((ln\beta)_i''+2\left(\frac{1}{R_i^2}-\sigma_i'\right)\right)+W_i=\left(ln P_i'\right)',$$

$$i = 1, 2,\ldots 8 \qquad (2)$$

where

$$(ln\beta)_i''=\frac{\partial^2 ln\beta}{\partial x^2}cos^2\alpha_i+\frac{\partial^2 ln\beta}{\partial y^2}sin^2\alpha_i+2\frac{\partial^2 ln\beta}{\partial x\partial y}sin\alpha_i cos\alpha_i,$$

$$W_i=(ln\beta)_i'-2\left(\frac{1}{R_i}+\sigma\right),$$

$$(ln\beta)_i'=\frac{\partial ln\beta}{\partial x}cos\,\alpha_i+\frac{\partial ln\beta}{\partial y}sin\,\alpha_i,$$

$$\sigma_i'=\frac{\partial\sigma}{\partial x}cos\alpha_i+\frac{\partial\sigma}{\partial y}sin\alpha_i,$$

α_i = the elevation angle.

3. SOLUTIONS FOR THE SYSTEM OF EQUATIONS

The rigorous solution was used to determine the extinction coefficient. It was carried out here inverting the backscattering signals returned from a homogeneous atmosphere and measured by the system transmitting pulses from one point in space. The homogeneity criterion

$$\left(ln P_i'\right)'=-\frac{1}{r_i}\left\{2(1+\sigma_* r_i)+\frac{1}{1+\sigma_* r_i}\right\}, \qquad (3)$$

($P_i = P_i(r_i)$, σ_* = const), was found and used for computerized testing of the lidar systems (see also [2]).

The system of the equations (2) can be rewritten if the homogeneity criterion is satisfied:

$$u_1+u_2\,cos\alpha_i+u_3\,sin\alpha_i+u_4\,cos2\alpha_i+$$
$$+u_5\,sin2\alpha_i++u_6 z_i+u_7 z_i\,cos\alpha_i+ \qquad (4)$$
$$+u_8 z_i\,sin\alpha_i=0,$$

where

$$u_1=4(\sigma-\sigma_*)\sigma++\frac{1}{2}\left\{\left(\frac{\partial\,ln\beta}{\partial\,x}\right)^2+\left(\frac{\partial\,ln\beta}{\partial\,y}\right)^2+\frac{\partial^2\,ln\beta}{\partial\,x^2}+\frac{\partial^2\,ln\beta}{\partial\,y^2}\right\},$$

$$u_2=2\left\{(\sigma_*-2\sigma)\frac{\partial\,ln\beta}{\partial\,x}+\frac{\partial\sigma}{\partial\,x}\right\},$$

$$u_3 = 2\left\{(\sigma_* - 2\sigma)\,\frac{\partial\, ln\beta}{\partial y} + \frac{\partial\sigma}{\partial y}\right\},$$

$$u_4 = \frac{1}{2}\left\{\begin{array}{l}\left(\frac{\partial\, ln\beta}{\partial x}\right)^2 - \\ -\left(\frac{\partial\, ln\beta}{\partial y}\right)^2 + \frac{\partial^2\, ln\beta}{\partial x^2} - \frac{\partial^2\, ln\beta}{\partial y^2}\end{array}\right\},$$

$$u_5 = \frac{\partial\, ln\beta}{\partial x}\,\frac{\partial\, ln\beta}{\partial y} + \frac{\partial^2\, ln\beta}{\partial x\,\partial y},$$

$$u_6 = 2(\sigma - \sigma_*),\quad u_7 = -\frac{\partial\, ln\beta}{\partial x},\quad u_8 = -\frac{\partial\, ln\beta}{\partial y},$$

$$z_i = \frac{1 + 2\sigma_* r_i}{(1 + \sigma_* r_i)\, r_i}.$$

The solution of the system (4) can be written as

$$\frac{\partial\sigma}{\partial x} = \frac{\partial\sigma}{\partial y} = \frac{\partial ln\beta}{\partial x} = \frac{\partial ln\beta}{\partial y} = \frac{\partial^2\, ln\beta}{\partial x^2} = \frac{\partial^2\, ln\beta}{\partial y^2} = \frac{\partial^2\, ln\beta}{\partial x\,\partial y} = 0,$$
$$\sigma = \sigma_*. \qquad (5)$$

4. ANALYSIS OF THE STATISTICAL ERRORS

The first step in the investigation was to invert the backscattering signal measured by the system transmitting pulses from one point in space.

The statistical error δ of the measured parameter (of the extinction coefficient σ) depends on the statistical error δP of the backscattering signal and on the derivatives $\partial\sigma/\partial P_i$ in linear approximation (see, for example [3]). The statistical error δP was approximated here as the value proportional to the squared root from the power P. The proportionality coefficient C was found using experimental data [1]. The theoretical analysis shows the existence of the minimal value $\delta = \delta_{min}$, depending on the atmospheric and lidar parameters and equal approximately $C\,(A\beta)^{-1/2}$. The results of the error analysis are shown in Fig.1-3.

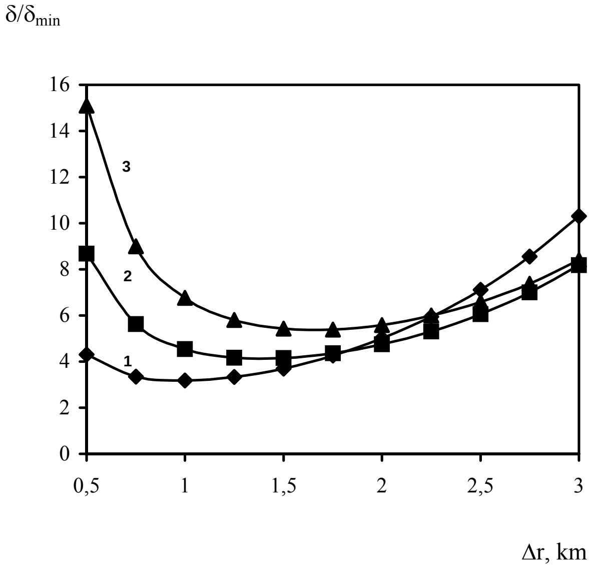

Fig.1. Relative error δ/δ_{min} dependence on the step Δr for different distances r, $P_*/P = 0.05$, $\sigma = 0{,}3$ km^{-1}
1 – r = 0,5 km, 2 – r = 1,0 km; 3 – r = 1,5 km

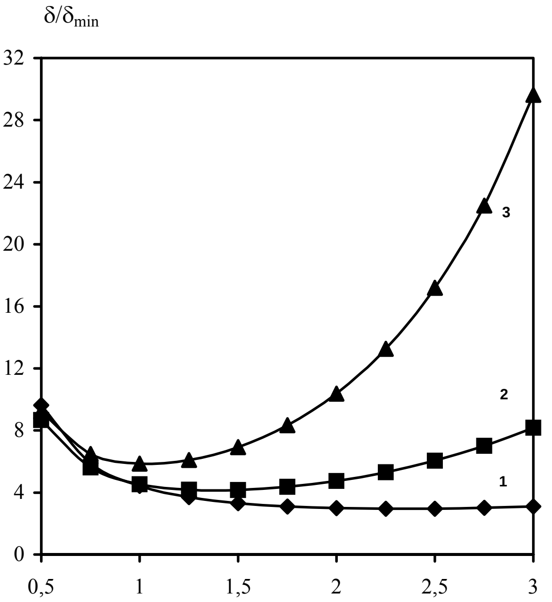

Fig.2. Relative error δ/δ_{min} dependence on the step Δr for different extinction coefficients, $P_*/P = 0.05$, r = 1,0 km
1 - $\sigma = 0{,}1$ km^{-1}, 2 - $\sigma = 0{,}3$ km^{-1}, 3 - $\sigma = 0{,}5$ км$^{-}$

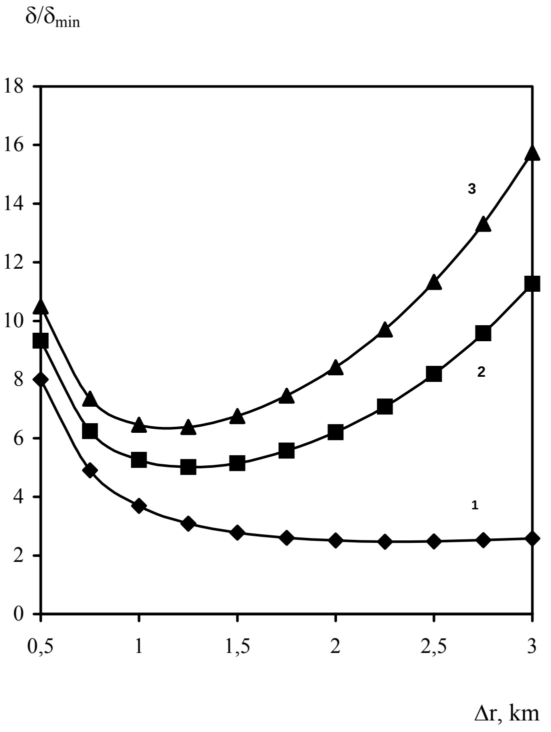

Fig.3. Relative error δ/δ_{min} dependence on the step Δr for different P_*/P, $\sigma = 0{,}3$ km^{-1}, r = 1,0 km
1 - $P_*/P = 0$; 2 - $P_*/P = 0{,}1$; 3 - $P_*/P = 0{,}2$

The relative statistical error δ/δ_{min} is presented as a function of several variables Δr, r, σ, P_*/P, where Δr is the beam path segment (the step), r is the distance between the scattering volume and lidar, P is the backscattering signal power measured at the distance r.

The analysis of the results shows the dependence of the error δ/δ_{min} on the step Δr has a minimum. This fact is associated with the light extinction from one hand and with the dependence of the statistical error δP on the power P from the other hand. The statistical error δ can be decreased drastically by using the appropriate step Δr.

5. CONCLUSIONS

Here were found the effective solutions of the lidar equation including unknown power of the background light. The effectiveness of these solutions essentially depends on the atmospheric conditions. The accuracy of lidar measurements can be improved using the dependence of the results on the measurement range. The algorithms for weak signals processing based on the new rigorous solution of the equation are useful for lidar software development.

6. REFERENCES

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