Factorization Method for GNSS Parameter Estimation

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Abstract: Kalman filter algorithm is sensitive to computer roundoff and numeric accuracy sometimes degrades to the point where the results cease to be meaningful. The primary goal of this research is to discuss an algorithm that involve matrix factorization and try to take advantage of the simplicity and versatility of the Kalman filter without falling victim to its potential numeric inaccuracy and instability. In order to verify its efficiency and reliability, we put the factorization method into GNSS parameter estimation. Result shows that factorization method can avoid divergent phenomenon caused by computer roundoff, and the dimension of covariance matrix reduce greatly.

Keywords: Global Navigation Satellite System; Kalman filter; Household transformation; least squire; parameter estimation

Instruction

With the developing of GNSS, its application range has expanded to various industries and fields. Many states and unions have already developed their own satellite positioning system, such as GLONASS in Russia, Galileo in Europe union and Compass in China. So nowadays how to estimate GNSS parameters more accurately is a hot point. The traditional method is least square estimation, but not so suitable for kinematic positioning. Kalman's elegant solution of the discrete linear estimation problem has had a major impact in GNSS filed. Unfortunately, numeric accuracy and stability problems have often prevented calculators from successfully computer mechanizing it. Numerical experience (Bellantoni(1967) and Leondes(1970))has shown that Kalman filter algorithm is sensitive to computer roundoff. Thus estimation practitioners have been faced with a dilemma. As we know, numerical difficulties have occurred in problems with singular or nearly singular estimate covariance matrices. So an alternative and more consistently reliable solution to the numerical instability problem is to perform some or all of the computations using extra precision arithmetic. It can reduce the effects of roundoff errors versus an increase in computation and computer storage requirements. Factorization method based on Household transformation is such a solution, it has inherently better stability and numerical accuracy than does the Kalman filter. This paper we begin our investigation of factorization method with a review of the classical least square and Kalman filter algorithm.

1. Least square algorithm

Suppose that we are given the linear GNSS observation equation at epoch k .

$$v_k = A_k \delta x_k - l_k \tag{1-1}$$

Where v_k is an vector of observation errors, A_k is the coefficient matrix, δx_k is the vector of variables that are to be estimated, l_k is the observation. Then

$$A_k^T P_k A_k \cdot \delta x_k = A_k^T P_k l_k \tag{1-2}$$

Where P_k is the weight matrix of observation. Thus, δx_k can be estimated with its covariance matrix.

In stationary model, we take X_k as virtual observation in $k+1$ epoch. Then

$$\begin{cases} v = X_{k+1} - X_k = \delta x'_{k+1} - l' \\ v_{k+1} = A_{K+1}\delta x'_{k+1} - l_{k+1} \end{cases} \tag{1-3}$$

Where $l' = X_k - X_{k+1}^0$. And that

$$\begin{bmatrix} I \\ A_{k+1} \end{bmatrix}^T \begin{bmatrix} D_{X_k}^{-1} & \\ & P_{k+1} \end{bmatrix} \begin{bmatrix} I \\ A_{k+1} \end{bmatrix} \delta x'_{k+1} = \begin{bmatrix} I \\ A_{k+1} \end{bmatrix}^T \begin{bmatrix} D_{X_k}^{-1} & \\ & P_{k+1} \end{bmatrix} \begin{bmatrix} l' \\ l_{k+1} \end{bmatrix} \tag{1-4}$$

Which simplifies to

$$\delta x'_{k+1} = (D_{X_k}^{-1} + A_{k+1}^T p_{k+1} A_{k+1})^{-1} \cdot (D_{X_k}^{-1} l' + A_{k+1}^T P_{k+1} l_{k+1}) \tag{1-5}$$

During processing, receiver clock offset is a white noise parameter, each epoch will have a new parameter. For $k+1$, we do not need the previous receiver clock offset, we can remove it from coefficient matrix.

Suppose $a_{j,k}$ is the element of coefficient matrix, c_k is the constant matrix. If we want to remove the parameter of number i, coefficient matrix and constant matrix have to be changed as:

$$\begin{cases} a_{j,k} = a_{j,k} - a_{j,i} a_{i,k} / a_{i,i} \\ c_k = c_k - c_i a_{k,l} / a_{i,i} \end{cases} \tag{1-6}$$

From $k+1$ epoch, there will be new parameter corresponding to number i .

2. Kalman filter algorithm

In kinematic positioning, parameters estimated by k epoch can not be directly used as virtual observations in $k+1$ epoch. Kalman filter theory establishes the state equation between two epoch according to the kinestate of receiver.

Suppose $\overline{X}_{k+1}$ is the predicted value of $k+1$ epoch based on X_k , $\overline{D}_{X_{k+1}}$ is its covariance matrix.

$$\overline{X}_{k+1} = \Phi_k X_k \tag{2-1}$$

$$\overline{D}_{X_{k+1}} = \Phi_k D_{X_k} \Phi_k^T + Q_k \tag{2-2}$$

Where Φ_k is state-transition matrix, Q_k is process noisy matrix.

$$\Phi_k = diag(1,1,1,0)$$

$$Q_k = diag(Q'_{dx}\Delta t, Q'_{dy}\Delta t, Q'_{dz}\Delta t, \sigma_{dt}^2)$$

σ_{dt}^2 is the noise of receiver clock offset, $Q'_{dx}, Q'_{dy}, Q'_{dz}$ express the movement of receiver.

Take $\overline{X}_{k+1}$ as virtual observation for $k+1$ epoch. According to formula(1-3)and(1-4), we can update $\overline{X}_{k+1}$ and $\overline{D}_{X_{k+1}}$.

$$(\overline{D}_{X_k}^{-1} + A_{k+1}^T P_{k+1} A_{k+1})\delta x_{k+1} = \overline{D}_{X_k}^{-1}(\overline{X}_{k+1} - X_{k+1}^0) + A_{k+1}^T P_{k+1} l_{k+1} \quad (2\text{-}3)$$

Using matrix decomposition theory that deduced by bibliography [2], formula(2-3) can write as

$$D_{X_{k+1}} = (I - J_{k+1}A_{k+1})\overline{D}_{X_{k+1}} \quad (2\text{-}4)$$

$$X_{k+1} = \overline{X}_{k+1} + J_{k+1} l_{k+1} \quad (2\text{-}5)$$

Formula (2-4) and (2-5) is the classical Kalman filter recursion formula.

Generally Kalman filter algorithm can obtain desired result, but for some specific conditions things are quite different. First I want to introduce you a concept roundoff. Roundoff errors are a side effect of computer arithmetic using fixed or floating point data words with a fixed number of bits. For example, let I denote the identity matrix. Consider the filtering problem with measurement sensitivity matrix

$$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1+\delta \end{bmatrix}$$

and covariance matrices $P = I$ and $R = \delta^2 I$,Where $\delta^2 < roundoff$ but $\delta > roundoff$. In this case, the product HPH^T will be singular and is not invertible.

In order to solve this problem, factorization method is introduced.

3. Factorization Method based on Household transformation

Recall the least square performance functional from section 1

$$J(x) = \|A_i \delta x - l_i\|_2 = \min \quad (3\text{-}1)$$

and let H , H be an orthogonal matrix. Because of property of orthogonal matrix[4], we can write

$$J(x) = \|HA_i \delta x - Hl_i\|_2 = \min \quad (3\text{-}2)$$

In fact, $J(x)$ is independent of H and this can be exploited. We shall show how H can be chosen using Household transformation in bibliography [4].

For an arbitrary matrix $A \in R^{m\times n} (m \ge n)$, there exists an orthogonal transformation $H \in R^{m\times m}$ such that

$$HA = \begin{bmatrix} s & \vdots & \\ & \vdots & \tilde{A} \\ \mathbf{0} & \vdots & \end{bmatrix} \quad (3\text{-}3)$$

Where s and $\tilde{A}$ are computed directly from A , and the matrix H is only implicit, computer mechanization requires no additional computer storage other than that used for A . We use the properties [4] of the elementary Household transformation H that

$$Hl_i = l_i - \gamma\overline{u} \qquad \gamma = (l_i^T \overline{u})\overline{u} + v \quad (3\text{-}4)$$

Where $\overline{u}$ is a unit vector of u , and u is the normal to the reference plane . v is that part of l_i that is orthogonal to u . Formulation of H as a precursor to computing Hl_i requires an order of magnitude more computation than does the direct evaluation of Hl_i using formula (3-4). Further, this formula shows that storage of the matrix H is not necessary. So formula (3-2) can write

$$J(x) \left\| \begin{bmatrix} R \\ 0 \end{bmatrix} \delta x - \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\|_2 = \min \quad (3\text{-}5)$$

Where $R \in R^{n\times n}$, is an upper triangular matrix, $Hl_i = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$,

$z_1 \in R^n$, $z_2 \in R^{m-n}$.

By reducing the least square performance functional to the form (3-3), one can see by inspection that the minimizing δx must satisfy

$$R\delta x - z_1 = 0 \quad (3\text{-}6)$$

These results are more elegant than is the brute force construction via the normal equation. More importantly, the solution using orthogonal transformation is less susceptible to errors due to computer roundoff.

To prove this assertion we take this theory into GNSS pseudo-range positioning. Suppose there are m satellites at epoch k , with a priori matrix P_{k-1}^0 . P_{k-1}^0 is a orthogonal matrix and can be decomposed to $P_{k-1}^0 = R_0^T R_0$,where R_0 is an upper triangular matrix. Take R_0 as virtual observation for k epoch.

$$\begin{bmatrix} v_0 \\ v_k \end{bmatrix} = \begin{bmatrix} R_0 \\ A_k \end{bmatrix}_{(m+4)\times 4} \delta x_k - \begin{bmatrix} X_{k-1}^0 - X_k^0 \\ l_k \end{bmatrix}_{m+4} \quad (3\text{-}7)$$

According to form (3-5) and (3-6), (3-7) can be written as

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ & r_{22} & r_{23} & r_{24} \\ & & r_{33} & r_{34} \\ & & & r_{44} \end{bmatrix} \cdot \delta x_k - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0 \quad (3\text{-}9)$$

Where $r_{i,j}$, b_i is the element of R and z_1 . δx_k and its covariance matrix can be estimated as $P_k = (R^T R)^{-1}$. And P_k can be taken as virtual observation for $k+1$ epoch. We can find that form (3-9) cause a reduction of the numerical ranges of the variables. Loosely speaking, one can say that computations which involve numbers ranging between 10^{-N} and 10^N are reduced to ranges between $10^{-N/2}$ and $10^{N/2}$.

4. Result analysis

IGS station “shao” is used for our test. Time range is 24 hours, ionospheric delay is eliminated by LC combination; tropospheric delay via Saastmoinen model to correct the zenith delay and adopts GMF as mapping function; relativistic effect is modeled by the position and velocity of satellites; coordinates and clock offsets of satellite are calculated by broadcast ephemeris.

Figure 4.1 gives the plot of position error using least squire method, Kalman filter method and factorization method respectively in stationary model. It is found that all three methods are elegant.

Table 4-1 stationary positioning results of three methods(RMS)

	X(m)	Y(m)	Z(m)
Least squire	0.207	0.327	0.181
Kalman filter	0.203	0.283	0.311
Factorization	0.207	0.327	0.181

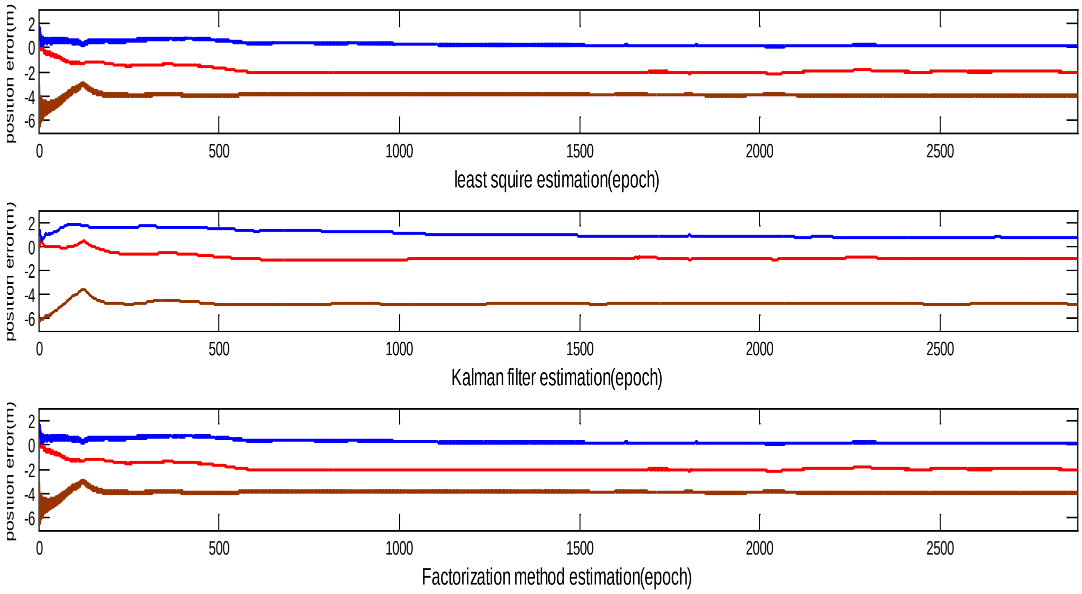

Figure4.1 stationary positioning results for three methods

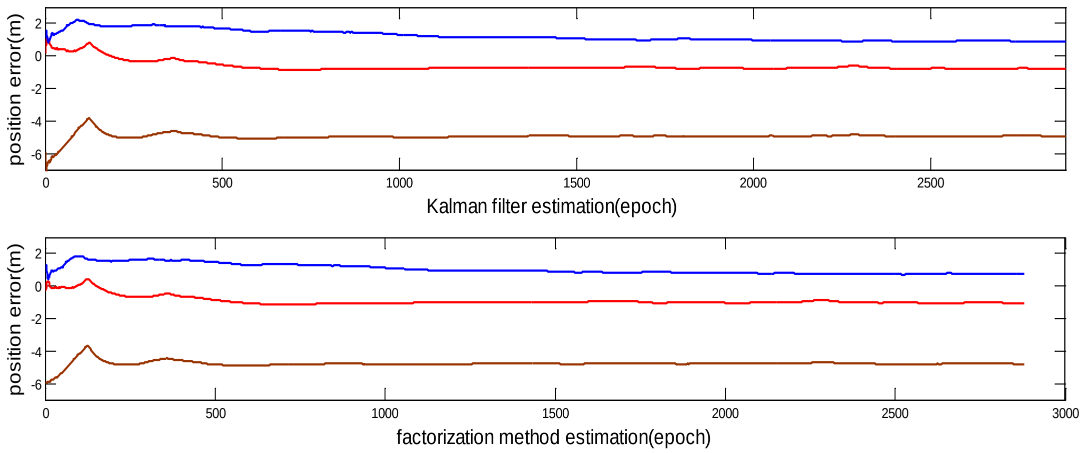

Figure4.2 kinematic positioning results for Klaman filter and factorization

Table 4-2 dynamic results of two methods(RMS)

	X(m)	Y(m)	Z(m)
Kalman filter	0.188	0.277	0.306
factorization	0.188	0.277	0.306

Table 4-1 summaries the RMS of three methods and find that estimations from least squire method are extremely the same as that from factorization method. It is not occasional because there are the same in origin. Estimations from Kalman filter method are somewhat have little difference with others. This is mainly because its process of receiver clock offset is different.We also suppose that the IGS station data is kinematic, and solving it in kinematic model.

Figure 4.2 gives the position error plot of Kalman filter method and factorization method in kinematic model.

Table 4-2 and figure 4.2 show us that Kalman filter method and factorization method can obtain the same result in kinematic positioning. This paper we do not encounter computer roundoff in Klanman filter processing, but this method discussed here can be used in GNSS parameter estimation, no mater stationary or kinematic.

5. Conclusion

This paper reviews classical least squire algorithm and Kalman filter algorithm, analyses the transmission of covariance matrix and explain their strategies in receiver clock offset estimation. Due to that Kalman filter algorithm is sensitive to computer roundoff and numeric accuracy sometimes degrades to the wrong point, this paper gives Factorization method based on Household transformation to solve the problem, and obtain some results.

(1)estimation is more convenient, R^{-1} is not needed for calculation and nearly singular covariance matrices can be well solved.
(2) Numeric storage is half than that use Kalman filter method.
(3)Receiver clock offset can be eliminate directly from R matrix.
(4)dimension of digital is greatly reduced,which can avoid computer roundoff.

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