SORA-PR: A program for rectification of tilted photographs of plane objects

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SUMMARY

The universal concept of the Avioplan OR 1 differential rectifier
permits it to be used for tasks for which, until now, conventional
rectifiers have been used. The data required for the photographic
transformation can be derived via a program in a general purpose
computer. Such a program is described and a practical example is
shown.It is particularly important to note that with this new recti-
fication procedure a combined perspective and affine rectification
is possible. This application is particularly important in the
production of orthophotos from aerial photographs of plane, but
sloping ground.

1.   INTRODUCTION

At the 13th ISP Congress in Helsinki Wild Heerbrugg Ltd introduced
the digitally controlled differential rectifier Avioplan OR 1/4/
and the Institute for Photogrammetry of the Technical University,
Vienna, introduced the program SORA-OPS (Software for Offline
Rectification with the Avioplan – Orthophoto and Stereo orthophoto
/1,3/ which had been developed under contract from Wild Heerbrugg
Ltd. In the four years since that Congress this hardware and soft-
ware have been widely distributed throughout the world. Several
other applications of digitally controlled differential recti-
fication, with practical examples, have been published in the last
few years. Reference /5/ will serve as representative of several
such publications. It deals with the production of orthophotos of
dome surfaces and extracts from this original publication are
being presented at this Congress in a separate paper /6/.

References /2,7/ attracted the greatest attention in practice and
deal with the use of the OR 1 for rectification of tilted photo-
graphs of plane objects. The technique opens up to architects,
monument conservationists and archaeologists a method for the
production of pictures with all the properties of exact perpen-
dicular photographs from photographs taken without special prepa-
ration with either metric or amateur (non-metric) cameras.

The technique has something to offer in classical aerial survey
also, however. There are many cases where the area covered by an
aerial photograph can be approximated with sufficient accuracy by
horizontal or sloping plane. In such cases the aerial photograph
can be rectified without the relatively large quantity of digital
data required to define an accidented ground surface.

These considerations led Wild Heerbrugg Ltd to commission the Institute for Photogrammetry of the Technical University, Vienna, to develop a computer program which would be fully suitable for practical production. Further conditions were that it must be possible to supply the new program as an addition to already de- livered SORA-OP or SORA-OPS programs and that it must similarly be suitable for 60-bit, 32-bit and 16-bit computers. The program has the name SORA-PR (Plane Rectification).

2. DIFFERENTIAL RECTIFICATION VIA THE AVIOPLAN-SORA SYSTEM

The data, on magnetic tape, required to drive the Avioplan OR 1 consist of the $\xi\eta$-coordinates of a deformed grid. The distorted picture is then transformed into a new photographic image which corresponds to a square grid /1/. Fig. 1 shows the two grids for the case of rectification of a tilted photograph of a plane object, referred to in the following description as perspective rectifi- cation.
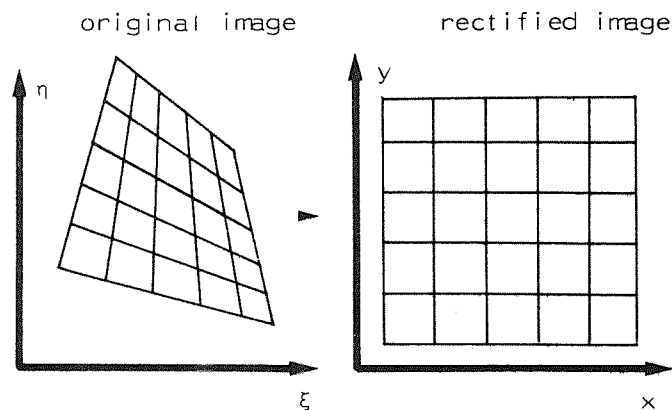
original image   rectified image

Fig. 1: Basic principle of the differential rectification
of a perspective image

Within a grid quadrilateral the OR 1 operates linearly. The opera- tion can therefore be described as a bilinear transformation:

$$\xi_i = a_o + a_1 x_i + a_2 y_i + a_3 x_i y_i$$

$$\eta_i = b_o + b_1 x_i + b_2 y_i + b_3 x_i y_i$$

(1)

A perspective rectification via the OR 1 and SORA can therefore only be theoretically correct in the intersection points of the grid. There will be errors in other points whose magnitude will depend upon the scan width and the tilt of the photograph. This so-called interpolation error can be studied with the help of the cross-ratio of perspective projection. The centers of the grid sides are computed firstly with the bilinear transformation(1) and secondly by means of the cross-ratio.

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Reference /7/ gives a detailed review of this error source. It shows that the interpolation error can be neglected for all practical work. It comprises, for example, 0.1mm for an orthophoto in the scale of 1:100 and with 6mm scan width made from a P31 photograph with a tilt of 18^g and a scale of approx. 1:300.

The operations, shown in more detail in /3/, by which the coordinates of the deformed grid are obtained in SORA from the defined square grid in the XY-plane permit the following interesting special cases in the rectification of tilted photographs of plane objects:

- both the inner and outer orientation can be determined from control points so that photographs from non-metric cameras as well as from metric cameras can be rectified.

- In the perspective rectification of non-metric photographs the 8 parameters of perspective transformation describe the mathematical relation between the object plane and the image plane. Even in this case however, the more general relations of central projection with the 6 elements of outer orientation and 3 elements of inner orientation are selected. One parameter, usually in this case an estimated principal distance, is then assumed as known.

- The control points for determination of the outer orientation and possibly also for the inner orientation need not lie in the object plane of the object which is to be rectified, but can lie in front of or behind this plane. In this case, if there are more than 4 control points the principal distance of a non-metric camera can also be computed.

3.   THE PROGRAM SORA-PR

The following Sections show some details of the method of definition of the object plane, the types of transformation and the definition of the area to be rectified.

3.1. Definition of the object plane

The object plane (e) is defined in a 3-dimensional coordinate system XYZ by the equation

$$(e): \ A \cdot X + B \cdot Y + C \cdot Z + D \ = \ 0 \eqno(2)$$

The following cases can be differentiated for the determination of the parameters A,B,C and D:

3.1.1 Parallel plane

The plane (e) lies parallel to and at distance D from the XY plane. This distance is defined in the program by the input of a specially coded point with a Z coordinate. If more then one such point is input the distance D is computed as the arithmetic mean of all the Z coordinates.
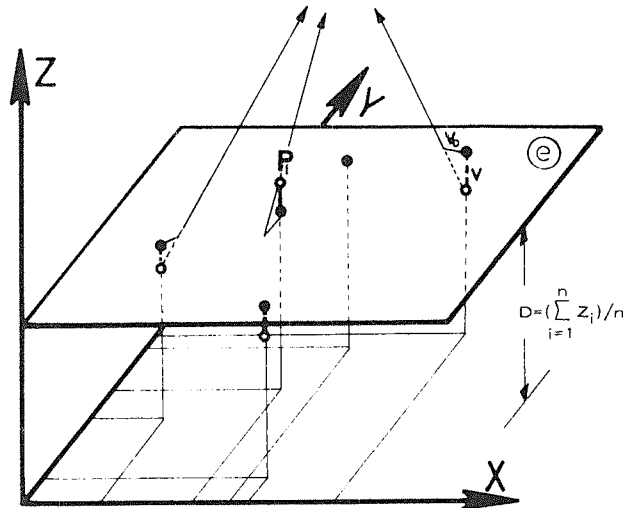
Fig. 2: Determination of a parallel object plane

The parameters of the plane are then:

$$A=B=0, \quad C=-1, \quad D=(\sum_{i=1}^{n} Z_i)/n \qquad (3)$$

The distances v give a good indication whether the approximation of the object by a parallel plane is justified for the particular task. These values are therefore computed by the program and, what is more important for the user, their projections v_o in the recti-fied end-product are output (Fig. 2).

If no such points are input in SORA-PR the program assumes the relatively common special case:

$$A=B=D=0 \quad C\neq0 \qquad (4)$$

i.e.the XY plane is the plane of projection.

3.1.2 Inclined plane

The object plane is inclined relative to the XY plane (Fig. 3). The parameters of this plane are derived from the XYZ coordinates of points which define the inclined plane. If there are more then 3 points the parameters are determined by an adjustment in which the sum of the squares of the distances v is a minimum (Fig. 3).
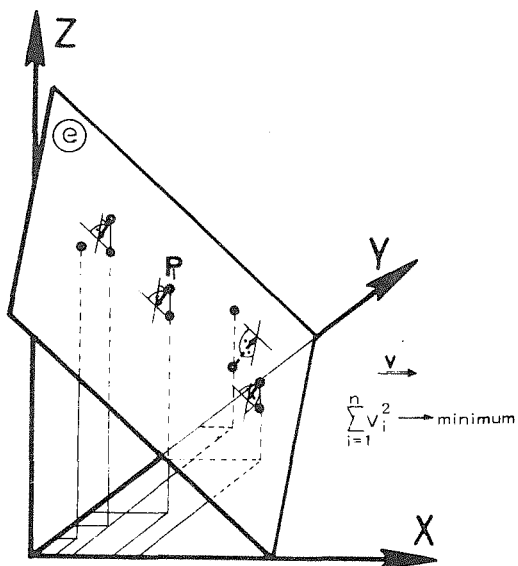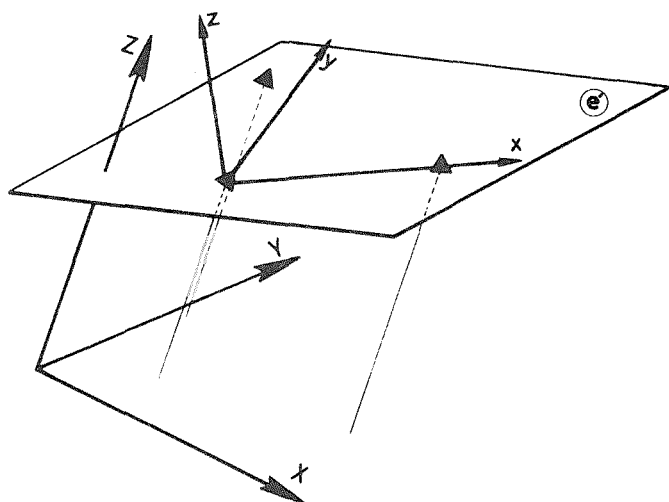
Fig. 3: Adjustment of an
inclined object-plane

A first approximation to the plane
(e') is derived from the 3 points
which define the biggest area. A
coordinate system is then defined
in this plane as shown in Fig. 4.
The coordinates xyz (z = 0) of this
plane are related to the coordinates
XYZ of the major system via the
equations of a spatial rotation
(R = rotation matrix) and trans-
lation:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = R \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} + \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} \qquad (5)$$

After determination of the six
parameters (3 rotations, 3 trans-
lations), of equations (5) from
the three points shown in Fig. 4
the XYZ coordinates of all points
which define the inclined plane
are transformed into the approximate
plane (e').

The error equations

$$v = ax + by + d - z \qquad (6)$$

then represent, with sufficient
accuracy, the adjustment of the
plane for the determination of the
parameters a, b and d.
Finally the parameters a,b,c (= -1)
and d are transformed into the
parameters A,B, C and D of
equations (2). For this we set
equations (5) into equations (6):

Fig. 4: Definition of the
xyz coordinate system of the
inclined plane (e')

$$a(r_{11}X+r_{12}Y+r_{13}Z+x_o)+b(r_{21}X+r_{22}Y+r_{23}Z+y_o)+$$

$$c(r_{31}X+r_{32}Y+r_{33}Z+z_o)+d=0$$

i.e.

$$(a \cdot r_{11}+b \cdot r_{21}+c \cdot r_{31})X+(a \cdot r_{12}+b \cdot r_{22}+c \cdot r_{32}) \cdot Y+ \qquad (7)$$

$$(a \cdot r_{13}+b \cdot r_{23}+c \cdot r_{33}) \cdot Z+(a \cdot x_o+b \cdot y_o+c \cdot z_o+d)=0$$

The equations (7) and (2) are equivalent, however, so that we have finally:

$$A = a \cdot r_{11} + b \cdot r_{21} + c \cdot r_{31}$$

$$B = a \cdot r_{12} + b \cdot r_{22} + c \cdot r_{32}$$

$$C = a \cdot r_{13} + b \cdot r_{23} + c \cdot r_{33} \qquad (8)$$

$$D = a \cdot x_o + b \cdot y_o + c \cdot z_o + d$$

Just as with a parallel plane, after the adjustment of an inclined plane the distances v and their projections v_o in the rectified endproduct are output by SORA-PR.

3.2. Types of transformation

3.2.1 Perspective rectification

The left diagram of Fig 5 shows the basic principle of perspective rectification by means of differential rectification. The object plane is assumed to be inclined relative to the XY plane to which the control points for determination of the outer orientation of the photograph are referred. Obviously the object plane (e) can lie parallel to the XY plane or can be identical with it (see Section 3.1.1).

PERSPECTIVE RECTIFICATION   combined PERSPECTIVE and AFFINE RECTIFICATION
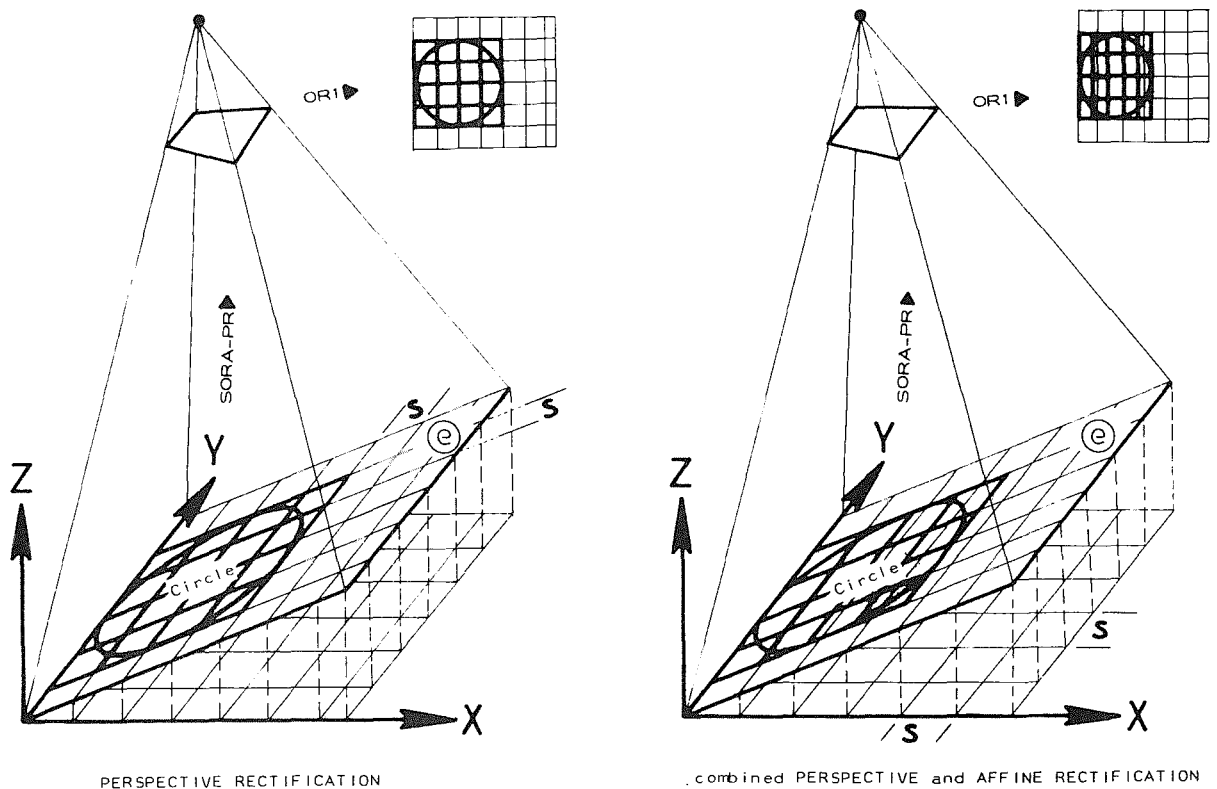
Fig. 5: Basic principle of perspective rectification and combined perspective and affine rectification in the Avioplan OR 1

The computation of the data for the Avioplan begins with the definition of a square grid of interval S in the object plane (e). The parameters of the outer orientation and possibly also of the inner orientation of the photograph are then determined from the control points. Finally the XYZ coordinates of the grid points are transformed into the photograph which is to be rectified. The result is the set of coordinates of the deformed grid of Fig.1, with which the distorted image will be transformed in the OR 1 into a new photograph corresponding to the square grid. A circle in the object plane is obviously reproduced in the rectified picture as a circle.

The variant with a tilted plane occurs most often in practice in architectural photogrammetry. The coordinate system in which the passpoints and the points for definition of the object plane are determined usually does not lie in the plane of the facade. After the transformation, however, a true-to-scale image of the (rotated) facade plane is available.

3.2.2 Combined perspective and affine rectification

In aerial photogrammetry one usually requires not a rectification in an inclined plane, but an orthogonal projection of the object plane(e) in the XY plane. This transformation comprises a perspective rectification and an affine rectification. The square grid with interval S is defined in this case in the XY plane. The rest of the procedure follow the same lines as shown in the Section above. A circle in the object plane will be reproduced in the rectified image as an ellipse.

As is well-known a combined perspective and affine rectification is not a straightforward process in a conventional rectifier and this method of differential rectification is technically superior. It also offers the possibility of producing rectified photos, without mass digitzing of the surface, of areas in which conventional rectification could not be applied because the radial displacements would be too large.

3.2.3 Enlargement

SORA PR permits the OR 1 to be used as a simple enlarger. The square grid in this case is defined directly in the plane of the original photograph. The permissible enlargement ratio lies in the range 0.27 (= 0.66 x 0.4) to 15 (= 6 x 2.5), the extreme limits of the range of the OR 1 /4/.

3.2.4 Affine Enlargement

Different enlargement ratios are applied in the two coordinate directions of the photograph. This difference is achieved by defining a rectangular grid. This technique can be used for example to remove affine shrinkage in photographic and cartographic products.

3.3. Definition of the area to be rectified

For perspective rectification (Section 3.2.1) and combined perspective and affine rectification (Section 3.2.2) it is possible to define the area to be rectified either in the XY plane of the object coordinate system or in the picture. For enlargements (Section 3.2.3) and affine enlargements (3.2.4) the area to be projected can of course only be defined in the picture.

3.3.1 Limits defined in the XY plane

The coordinates of two specially coded points and the width of the area required are input (Fig. 6).

PERSPECTIVE RECTIFICATION   combined PERSPECTIVE and AFFINE RECTIFICATION
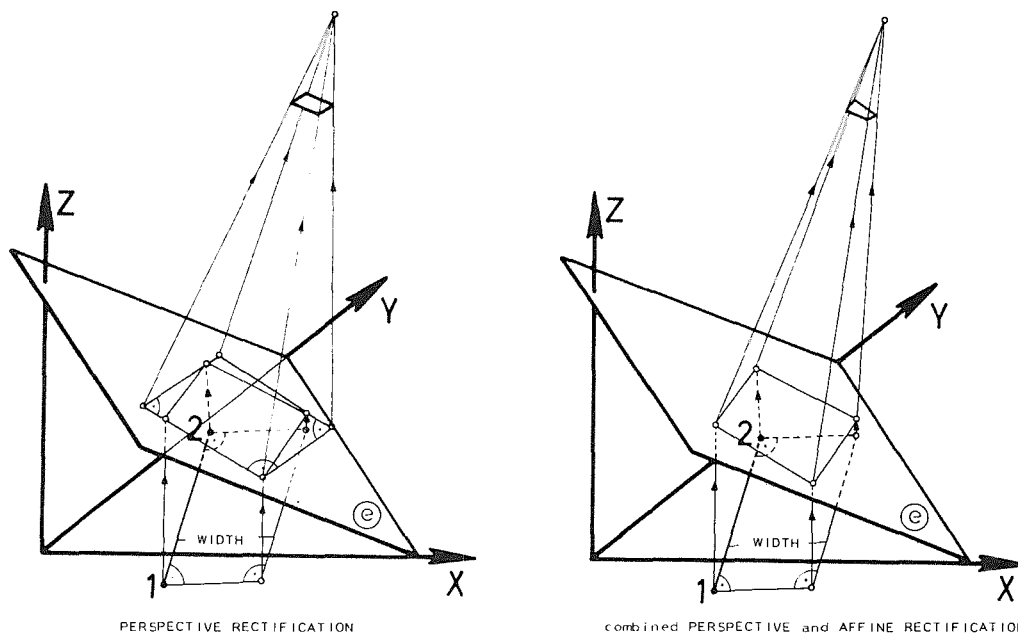
Fig. 6   Definition in the XY plane of the area to be rectified

In the case of combined perspective and affine rectification these data define directly the area to be rectified. For the case of perspective rectification in an inclined plane the rectangle defined in the XY plane is transformed into the object plane. The resulting quadrilateral is then replaced by a circumscribing rectangle which defines the area to be rectified.

3.3.2 Defining the limits in the picture

The limits can be defined in the picture with the coordinates of 3 or 4 points. If only 3 points are input, the perpendicular distance of the third point from the straight line between the first two points is computed and, as shown in Fig. 7, a rectangle is defined.

PERSPECTIVE RECTIFICATION   combined PERSPECTIVE and AFFINE RECTIFICATION
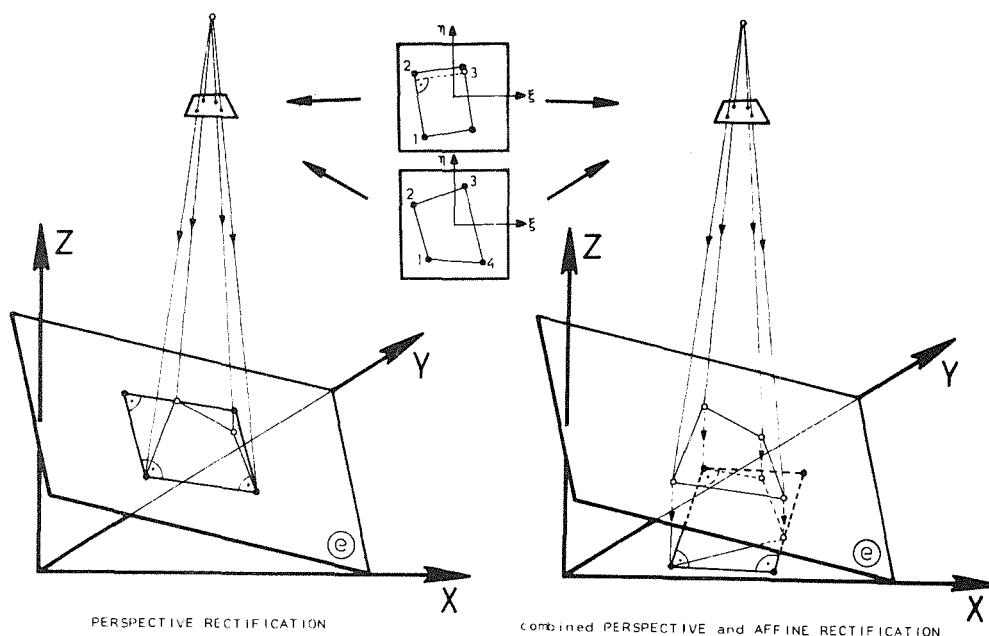
Fig. 7:   Definition in the picture of the area to be rectified

There are then 4 points available which are transformed into
the object plane (e) by means of the perspective equations. For
a perspective rectification (Section 3.2.1) this quadrilateral
in the object plane is replaced by a circumscribing rectangle
which finally defines the area to be rectified.

In the case of combined perspective and affine rectification
(Section 3.2.2, right-hand diagram of Fig. 7) the quadrilateral
in the object plane is projected orthogonally in the XY plane
and there replaced by a circumscribing rectangle. For enlarge-
ments(Section 3.2.3) and affine enlargements (Section 3.2.4)
there is obviously no projection into an object plane. If 4
image points are input the quadrilateral in the picture is re-
placed by a circumscribing rectangle. If no such points are
input, the program assumes that an area of 23cm x 23cm is to be
enlarged.

4.   PRACTICAL EXAMPLES

The technical data for 3 practical examples are shown in
Table I.

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	Facade	"Carnuntum" aerial photo	German base map
Camera	Wild P32 f = 64mm	Hasselblad 500 EL/M	RMK 30/23
Tilt (g)	24	23.5	2.5
Approx. distance (m)	30	600	3800
Rect. type	3.2.1	3.2.1	3.2.2
Scan width (mm)	8	8	16
Scale of rectification	1:100	1:2000	1:5000
Format (cm)	28 x 17	27 x 19	40 x 40
Time in CDC Cyber 74 (sec) CPU	2.3	3.2	5.3
I/O	7.3	9.0	12.7

Table I.

Example 1 shows a facade of a building in Vienna. The photograph was taken with a Wild P32 camera (Fig. 8a).

In order to define the plane several points in the plane of the facade were measured in a local coordinate system which did not lie exactly in the plane of the facade. In addition to these points several control points in front of the plane of the facade were also available for the determination of the elements of the outer orientation. Fig. 8b shows the perspective rectification in the plane of the facade.

Example 2 involved a strongly tilted aerial photograph taken with a Hasselblad camera for archaeological research in the region of the Roman settlement of Carnuntum, east of Vienna (Fig. 9a). The control points were taken from an orthophoto map at a scale of 1:2'000, which had been independently produced some time earlier.

Example 3 was a combined perspective and affine rectification of an aerial photo for the 2km x 2km area of a German base map, 1:5000 scale.

Fig. 8a: Facade photographed with a Wild P 32

Fig. 8b: Rectified photo (reproduced at half scale)

5. CONCLUSIONS

The technique of rectification with the system Avioplan
OR 1/SORA-PR can be characterized as follows:

- It is possible to rectify photographs with very large tilts,
  a process which is not possible with conventional rectifiers.

- Combined perspective and affine rectification, a far from
  straightforward procedure with conventional rectifiers, permits
  the production of orthophotos even in regions of strong slope
  without having to digitize the terrain surface.

- Control points lying outside the object plane can be used for
  the orientation of the pictures.

- The complete orientation is made numerically with a record of
  the results, in particular of the residual errors in the control
  points which do not lie in the object plane.

- Work in the darkroom is limited to loading and unloading of
  the film into or out of the drum of the OR 1.

- In view of the very short computing times and the use of
  relatively large scan widths in the OR 1, if the slopes are
  small, the procedure is very economic.

ACKNOWLEDGEMENTS

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