APLICATION OF SURVEYING AND PHOTOGRAMMETRIC METHODS IN STUDIES OF GEOMETRICAL FORMS OF VAULTS IN HISTORIC MONUMENTS

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APPLICATION OF SURVEYING AND PHOTOGRAMMETRIC METHODS IN STUDIES OF GEOMETRICAL FORMS OF VAULTS IN HISTORIC MONUMENTS.

I. Principles of investigations.

Results of geodetic or photogrammetric meassurements of historic monuments are usually represented in the form of drawings /diagrams/ in suitable scale, with vaults represented by the means of two vertical, perpendicular cross-sections. These documents are not sufficient for cognitive purposes in the field of history of art, especially in connection with studies of the geometrical form of vaults, where - as it was found during our investigations of several structures - there are hidden many hitherto unknown secrets of the ancient art of erecting vaults. Inaccurate identification of the vault form may lead not only to improper scientific, historical recognition, but also to improper results of reconstruction, repair or preservation works.

These needs demand a comprehensive investigation of historic vaults and domes. In general, this means investigation of the whole surface of the vault, not only of its selected cross-sections, and can be done by analytical-numerical methods or by the methods of topographical analysis /graphical methods/.

The analytical-graphical methods consist in approximation of the surface with an assumed model, on the base of a given set of points, and then testing of the adequacy of approximation on the base of magnitude of residuals. It was found from the hitherto performed investigations, that in general, the vaults have shapes of second degree surface, with accuracy of up to few milimetres. So in most cases the equation of a second degree surface is a sufficient model for generalization of vault surfaces; after reduction to the canonical form it usually gives a triaxial ellipsoid or an ellipsoid of revolutions with differentiated parameters or, in the case of the so-called

monastery vaults - an elliptical cylinder.

The aquation, describing the whole surface or a part of it, can give us a very valuable information allowing a full recognition of the geometrical form, and in the case of need, its accurate reconstruction.

Topographical methods, based on the representation of shape of the vault by the means of a contour plan, are characterized by better visuality, but do not furnish full specification of geometrical parameters of the surface. Thanks to their visuality, these methods may be very halpful for planning of the approximation procedure, and besides they allow a rough determination of the regular zones on the vault, zones of possible deformations, and of the type of surface /determination of the generalizing model/. For some types of vaults, /like, for example, the "Treasury" chamber in the Lublin cathedral/, the topographical methods are the only obtainable base for further investigations.

II. Results of present investigations.

1. Chamber in the Wiśnicz renaissance castle, /XVI c./.

The measurements were performed by the Cracow Geodetic Enterprise. Coordinates of points on the vault were determined by the photogrammetric method. The observed points were selected in vertices of a regular grid, which horizontal projection would create a net of aquares with 10 cm sides. The analysis of geometrical form of all segments of the vault was done on the base of the results of approximations with several second degree surfaces, in consecutive iterations.

The horizontal cross-section of the chamber is an irregular quadrilateral with sides 4.20 m, 4.40 m, 4.60 m and 4.71 m long /fig.1/, the uppermost point of the vault is 4.40 m above the floor.

The vault is divided by decorative plate moulds into 14 fields, arranged into a starlike pattern. The then architect has oonsciously deformed the regular form of the vault along the edges of the moulds, in order to adjust the curvature between the vault and the plate thus obtaining a positive artistic effect. The pattern of the plate moulds creates an

impression of a symmetric setting, though in reality there exist departures created by the irregular shape of the outer limits of the vault /the irregular quadrilateral/.

The characteristic feature of the vault are its acoustical effects, connected with two opposite corners, /fig.1 - corners 1 and 3/.

Another characteristic feature of the vault is its irregular setting in the vertical cross-section, which is composed of three concentric zones; the first zone comprises fields 1 to 12, the second zone - field 13 placed about 5 cm above the first zone, and the third zone, situated again about 5 cm above the former, field 14. The setting reveals in the widening of the mould plates close to the edges of the raised zones. But thanks to the small value of the rise, the vault creates an impression of uniformity.

The irregular arrangement of the vault surface demanded the approximations to be carried out indepenently for particular fields, for example 1 or 3, or for groups of fields, like for example 1+2+3+4. As the result, it was found that the particular fields or groups are second degree surface of different parameters and types; in most cases they are ellipsoids. Accuracy of the approximations was characterized by the average departure of a point from the approximated surface, as measured along the normal to the surface. These departures ranged from 3 to 10 mm.

It was found from the accuracy analysis, that smallest residuals pertain to the group 1+3, which plays the main role in the accoustical effects; the surface, as determined with the average normal departure of a point equal $\pm$ 5 mm, is approximately an ellipsoid of revolutions.

Oppositely situated groups of fields: 1+5+6 and 3+9+10, are not placed symmetrically, namely: i/ joint approximation of these fields shows in the vertical cross-section an ellipsoid with axis inclined 1.2° from the level /fig.2/, ii/ corresponding, but opposite architectural details are not on the same level.

Independent fields 13 and 14 have purely decorative character, irregular field 14 is a well justified exeption in the whole setting, field 13 resembles an oblate ellipsoid.

As an example of the mathematical model of the zone 1÷3 we present the following equation of an ellipsoid, obtained after several iterations

$$\frac{X^2}{2.98^2} + \frac{Y^2}{3.05^2} + \frac{Z^2}{2.21^2} = 1$$

The surface, playing the main role in the acoustical effects and determined with the accuracy of ±5 mm, is most probably an ellipsoid of revolutions.

The deformations of regular geometrical forms, introduced by the past architect into the vault as a whole and into its separate parts, have the character of the so called "perspective and composition corrections", used in the past to enhance artistic values of a masterpiece, /see "Kościół św. Anny w Krakowie" by J.Gomoliszewski, Kraków 1957/.

2. Chamber in the Cameldolite Monastery in Bielany /Kraków/, XVII c.

The measurement of the chamber and its vault was performed with the surveying methods. The chamber represents an irregular quadrilateral with sides 5.28 m, 5.55 m, 5.35 m, 5.45 m long, /fig.3/. Top of the vault is situated 5.48 m above the floor level. The vault is composed of two elliptical cyllinders intersecting at right angle. Intersection edges disappear close to the central part of the vault, which is introduced as a smoothing surface in order to level the upper parts of cylinders of slightly different parameters. The intersection of cylinders creates the so-called "monastery vault". The lines of intersection are two ellipses. The vault and walls of the chamber have no decorative elements.

Investigations have shown, that the cylindrical surfaces had had to be built in agreement with a-priori defined geometrical assumptions; this is proved by small normal departures of vault points from the approximating surfaces - the average residual is ± 10 mm - and they are results of not only the construction inaccuracies, but alsp, of the introduction of intermediate smoothing surface, and of the buildup of deformations caused by repairs, plastering, whitewashing and other

preservation activities.

Approximation of the intersection lines with general second degree equation yielded ellipses for both of them; maximum residuals were about 10 mm, majority of residuals was of a few milimetres. The vault has acoustical effects along both diagonals.

3. Chamber in the Pieskowa Skała renaissance castle /XIV-XVI c./.

The measurements of the chamber and its vault was done with surveying methods. The chamber has again the shape of an irregular quadrilateral, with sides 6.5 m, 5.46 m, 6.01 m, 6.58 m long /fig.4/. The uppermost point of the vault is 4.46 m above the floor level. The surface of the vault does not contain any decorative elements nor deflections and is apparently uniform.

The approximation computations were performed in two stages. During the first stage it was found that the vault is not a uniform second degree surface; using a special computer program for approximation /with multiple rejections of outlying points in each iteration/, six distinct fields were distinguished. In the second stage several approximations were performed for oppositely situated corner fields in pairs / 1+2 and 3+4 / and fields 5 and 6. It was found that:

i. zones 1+2 and 3+4 are triaxial ellipsoids with different parameters,

ii. zone 5 is a triaxial ellipsoid more flattened than the preceeding two,

iii. the inner zone 6 is an irregular surface created by intersection of two ellipses of different parameters.

The chamber is characterized by acoustic effects along both diagonals.

4. Vault of the "Treasury" chamber in the Lublin cathedral /XVIII c./.

The photogrammetric survey of the vault was performed by our students - members of the Photogrammetric Section, Geodesy Scientific Circle of the University of Mining and Metallurgy

in Cracow - under supervision of Mr. W.Borowiec, PhD and Mr. A.Wróbel, MEng. Computations of spatial coordinates of points were performed by the Photogrammetry Workshop of the Cracow Geodetic Enterprise.

The investigated vault is a unique masterpiece, in which the problem of construction of suitable geometrical form was excellently harmonized with demands of the painter's perspective. The whole surface of the vault, 20 m^2 large, resembling an ellipsoidal shell, is covered by a painting exercised in XVIII c. by J.Majcher. Oval mould plate surrounds the base of the vault and forms a frame for the painting /fig.5/; the unpainted stripes on the left side of the painting are traces of cracks induced during war.

The main part of the painting shows two rows of columns in antique style, finished with decorative mould plates. The columns represent a supporting structure carrying with the help of arches a dome of a tall structure. Together with the rest of the picture taken as a background, they create an impression of magnificent perspective depth, though the curvature of the surface is relatively small. Height difference between its highest and lowest points does not exceed 50 cm. To an observer, standing at the entrance the columns seem vertical and very tall, though in reality they are painted in nearly horizontal position, and their lenght does not conform with the impression. Moving inside, we notice the columns to incline, but without any change of their cylindrical form.

Photogrammetrically determined coordinates of selected points of the vault allowed us to draw a contour map of the surface, with contours spaded every 5 cm. During the topographical analysis of the map the shape and geometrical properties of the vault were determined. The method, used for the first time in analysis of vaults, allowed us to discover many hitherto unknown secrets of the old architecture.

The geometry of contours /fig.6/ implies, that the surface is not symmetric, nor homogeneous. The main axis of the vault, as determined by graphical methods, is in horizontal plane broken in the highest point of the vault. The point is shifted for 35 cm with respect to the geometric centre of the chamber.

The deflection angle of the vault axis equals to 1/24 of the full angle /15°/. A detailed analysis of the contours has shown, that every horizontal cross-section of the vault is composed of two elliptical arcs having common centres, equal semi-minor axies and different semi-major axies /fig.7/. Their connection has a uniform character, resulting in a regular quasi-elliptical line on every level. So the vault surface is close to an ellipsoid /quasi-ellipsoid/.

The deflection of the ridge line /in horizontal projection has created steeper slope of the surface under the columns, than opposite to them. Simultaneously, because of the steepening and soothing of curvature, the slope along columns is uniform. Thanks to this the columns seem to be straight and cylindrical independently of the position of the observer. So the fragments of the surface along the colonnade /marked with broken lines on fig.8/ can benearly considered planes. Independently of the colonnade, rectilinearity is also preserved by edges of the plate moulds above the columns, /they are perpendicular to the axies of columns/, thanks to small flat segments left in these regions.

Along certain lines which are straight on the contour plan, passing through the centre of the vault and creating multiplies of the 15° angle with the main axies, there occur noticeable changes in the curvature of contours. This notice suggests with great probability, that the erection of the vault was based on a radial and symmetric system of 24 axies, itersecting in its gwometric centre /fig.8/.

We can see on the fig.6, that axies and edges of columns converge in the very point in which the geometric axis of the vault intersects the axis of the chamber, in other words the perspective centre is placed in the point of intersection of the two axies /on fig.6 and 8 the broken geometric axis of the vault is the vertical projection of the ridge line of this surface/. As the ridge line joins the highest points of transversal cross-sections, perspective centre placed on this line helps to enhance the slenderness of columns and the apparent depth of their image. The deflection of the ridge line, consciously planned by the architect, besides the already mentioned effects

underlines the depth of perspective of the whole painting.

The authors, well aware of the fact, that in Poland, as well as in other countries, there is a lot of priceless monuments of this kind, find it advisable to spread wide and employ the described methods of investigations. The investigations play a substantial role in recognition of the architectural art of past master, which is still not known in full today. The authors expect a growing interest in the problem.

In the workshops of the Cracow Geodetic Enterprise /KPG/ there are now under elaboration several problems related to inventory surveys of historical monuments, together with elaboration of photogrammetric surveys performed by students of the University of Mining and Metallurgy during scientific expeditions called BARI. The latest problem under elaboration is the inventory survey of an ancient rotunda in Salonika, Greece /IV c.B.C./.

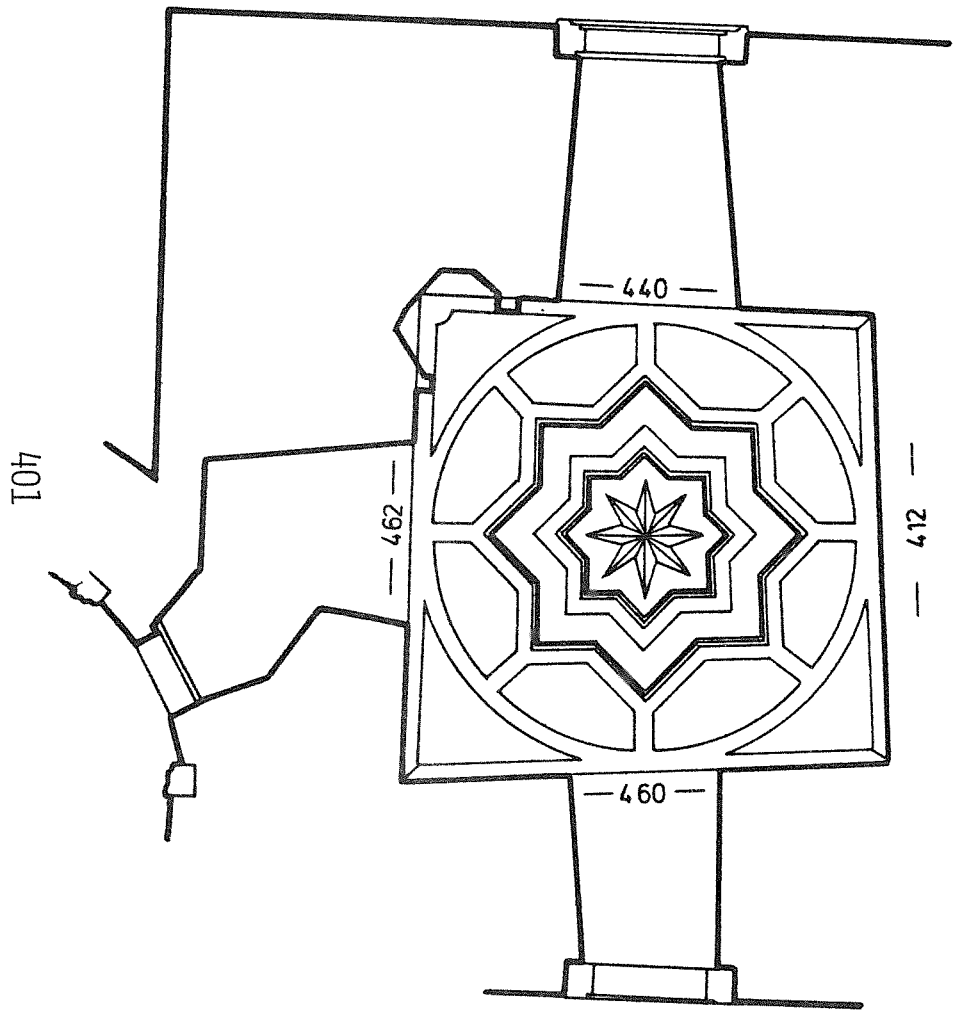


fig. nr 1

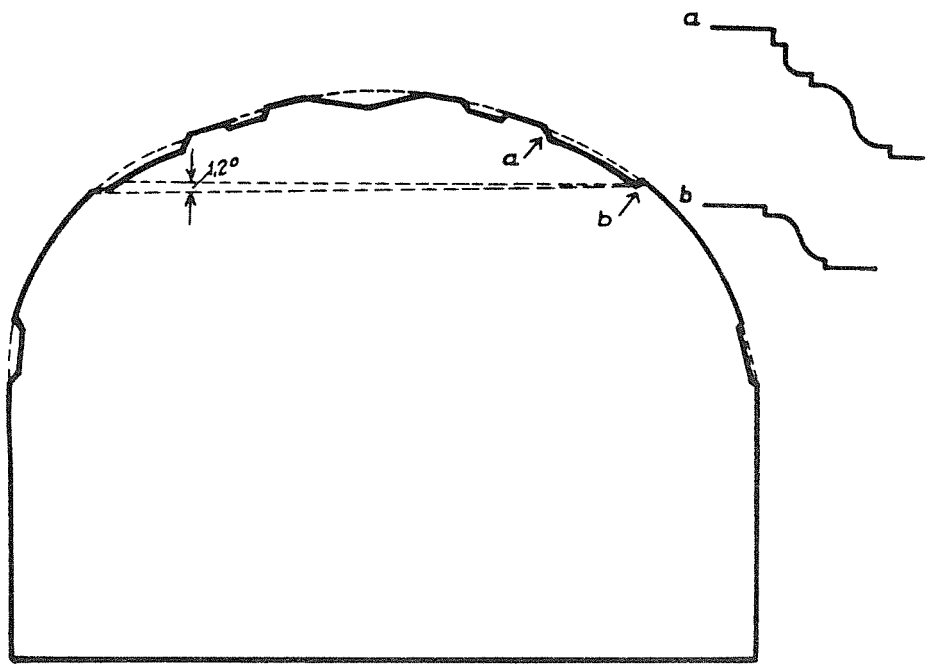


fig. nr 2

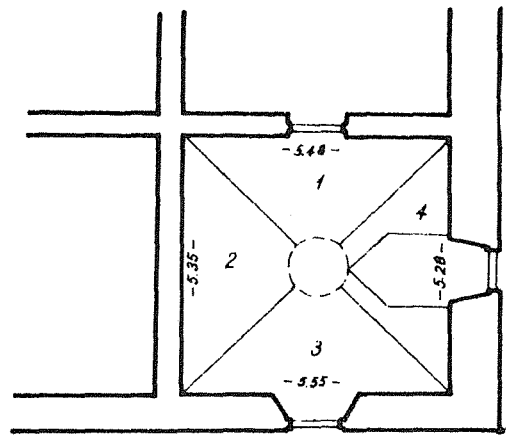


fig. nr 3

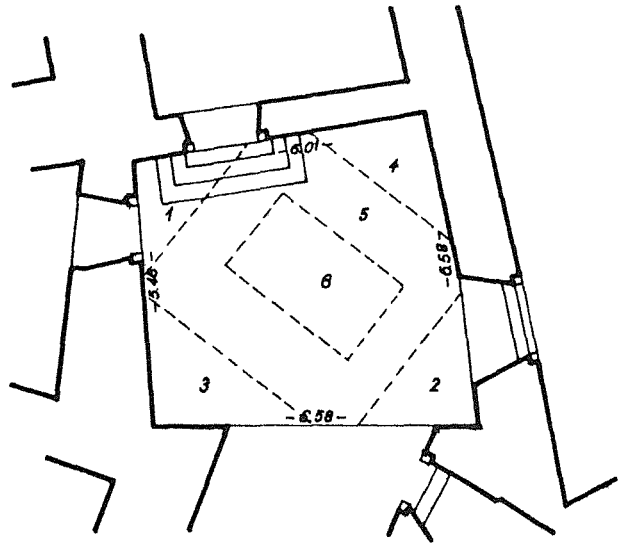
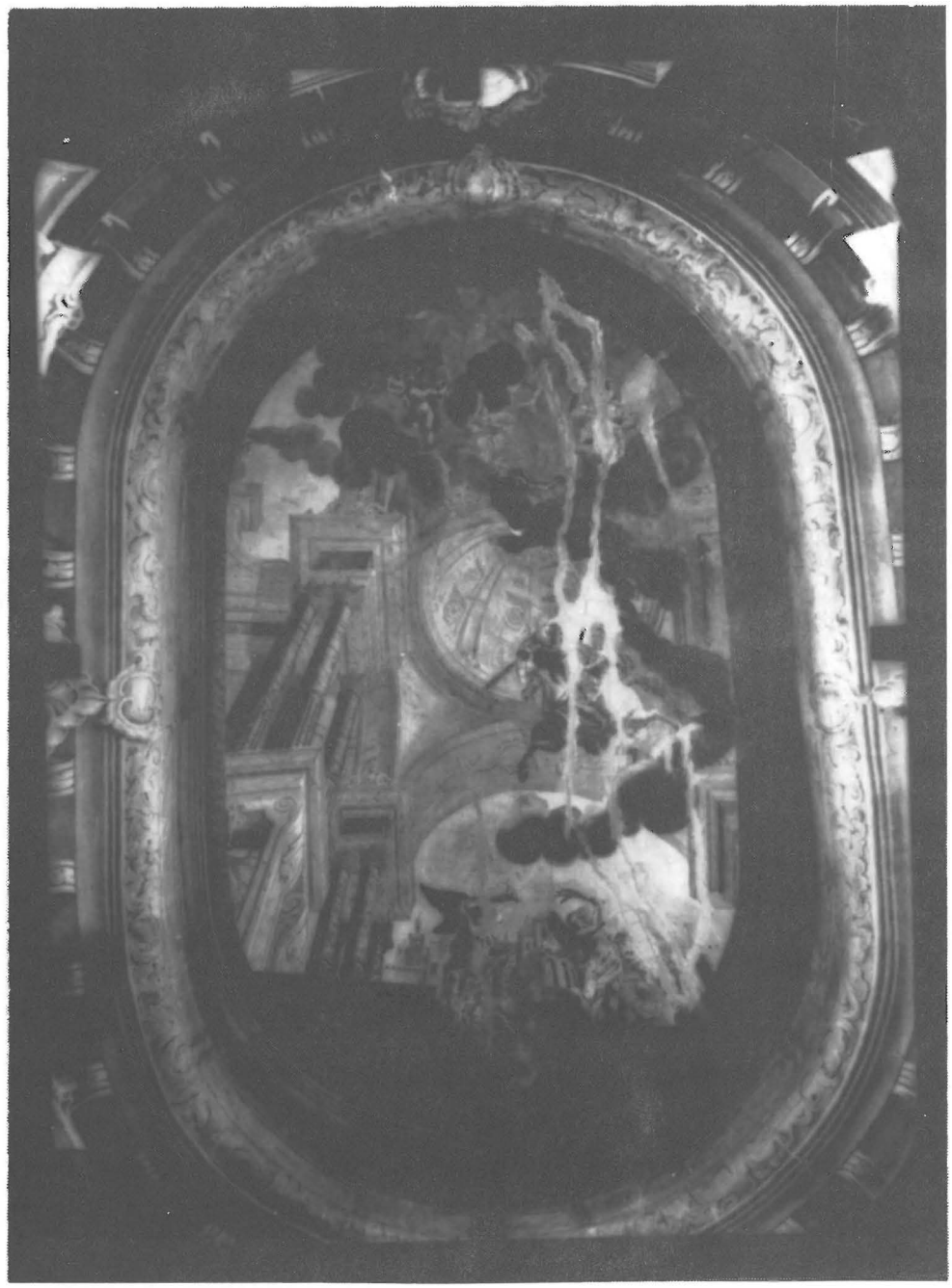
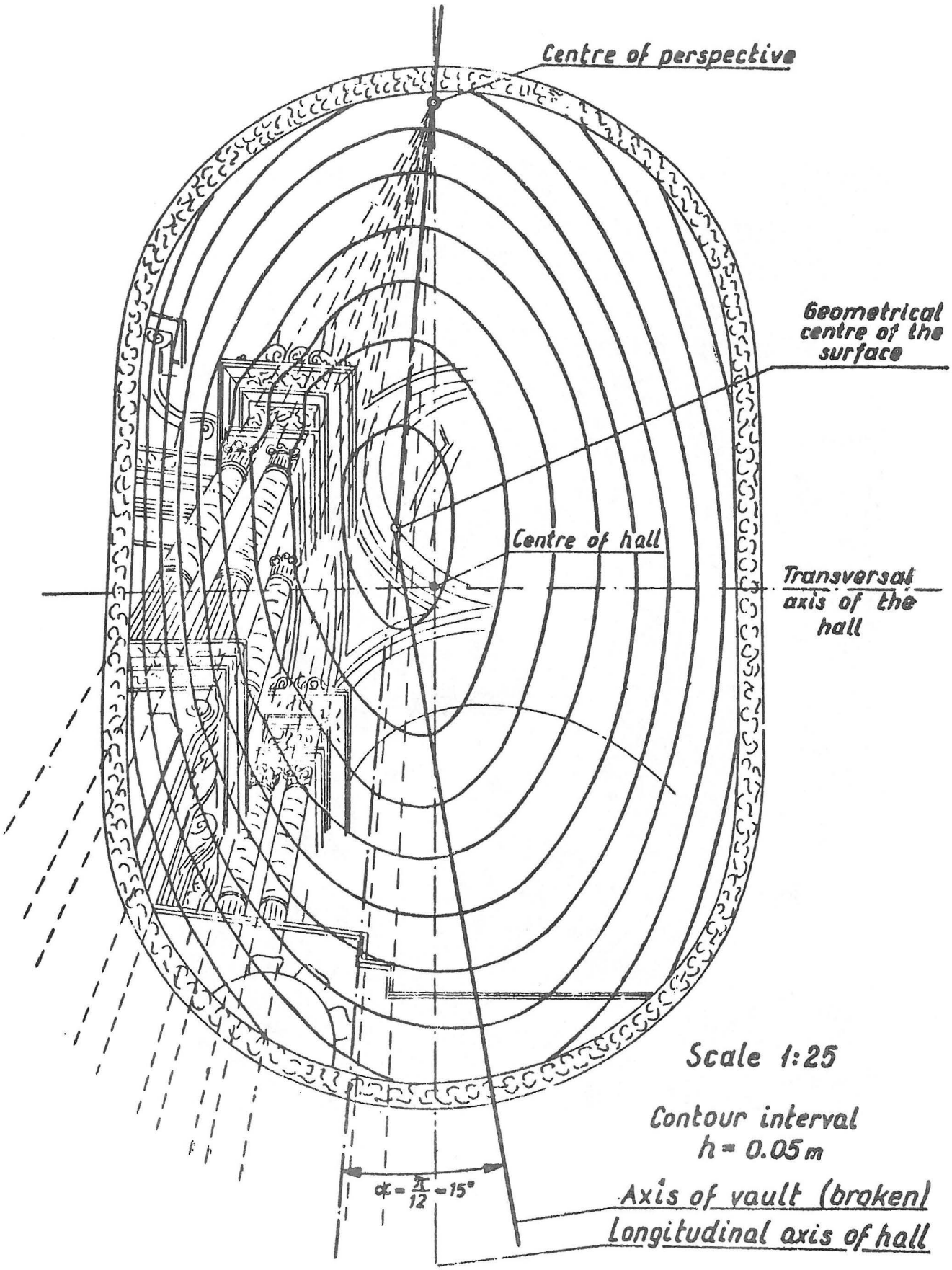
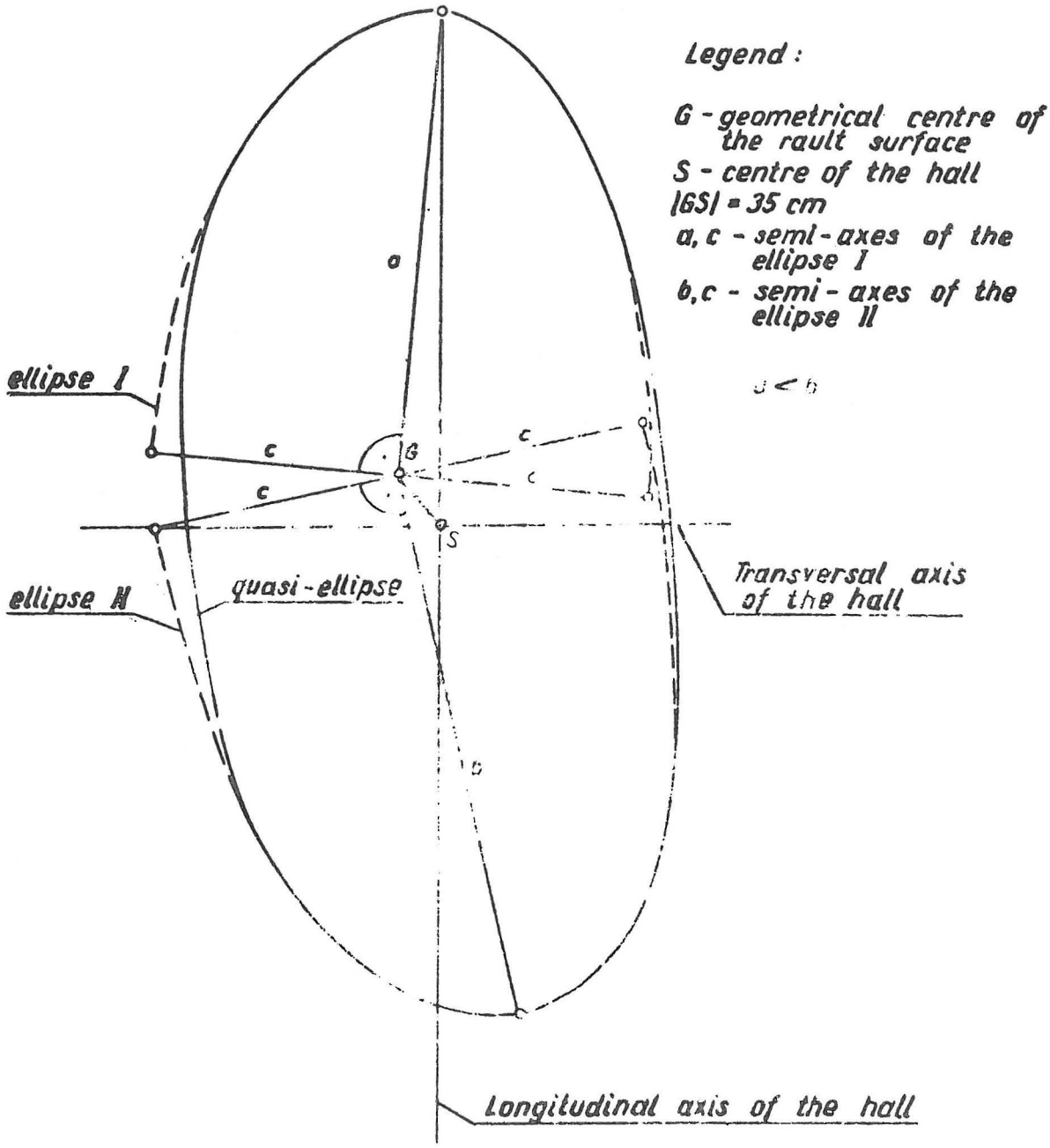


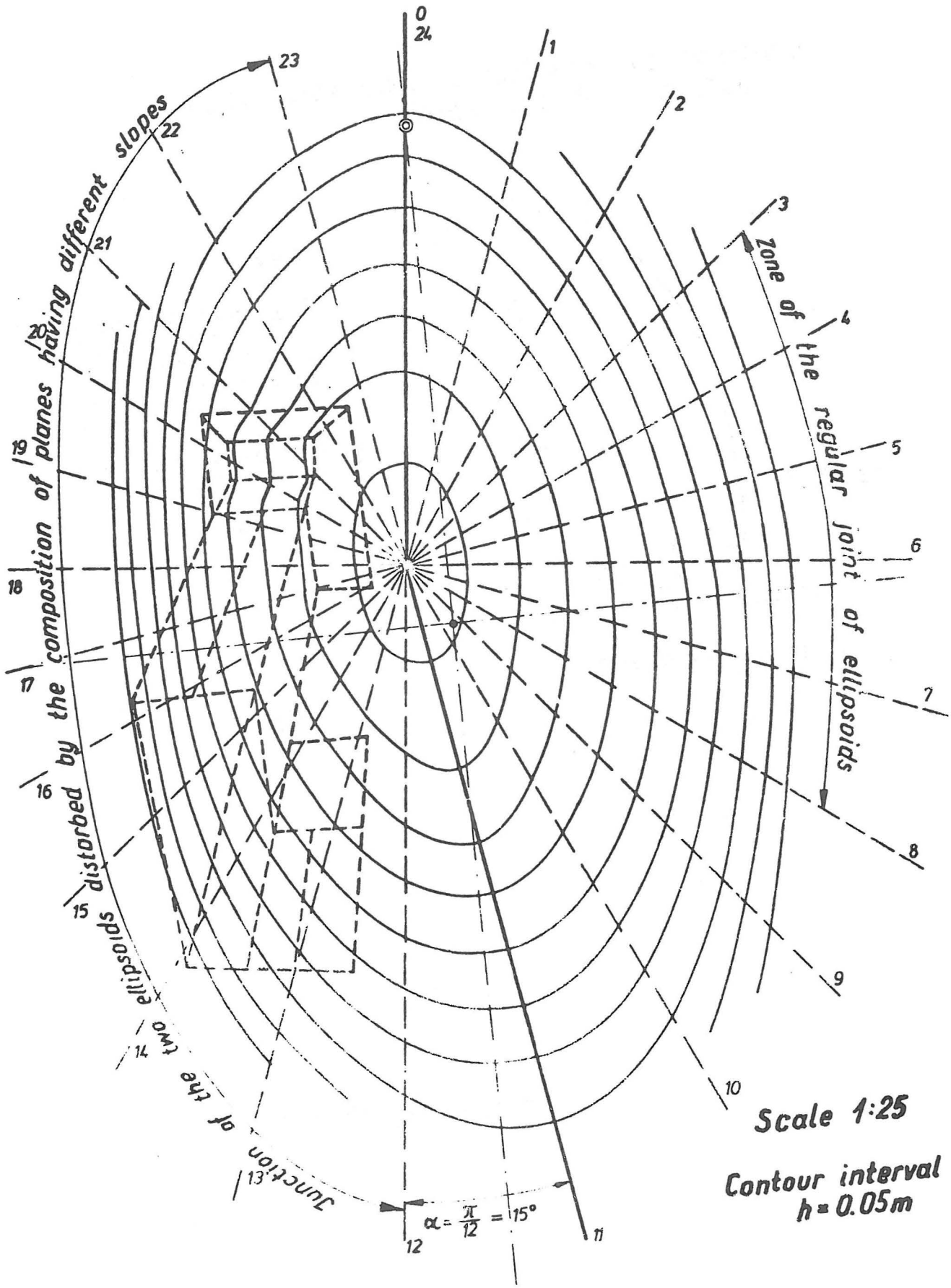
fig. nr 4




Centre of perspective
Geometrical centre of the surface
Centre of hall
Transversal axis of the hall
Scale 1:25
Contour interval h = 0.05 m
$\alpha = \frac{\pi}{12} = 15°$
Axis of vault (broken)
Longitudinal axis of hall

Geometric construction of horizontal cross -sections of the vault




Zone of the regular joint of ellipsoids
Junction of the two ellipsoids disturbed by the composition of planes having different slopes
Scale 1:25
Contour interval
h = 0.05m
α = π/12 = 15°