TWO METHODS OF PLOTTING FROM LANDSAT IMAGERY USING ANALOGUE INSTRUMENTS

Presented paper
Enrico Clerici and Ian Harley
Department of Surveying
University of Queensland
St. Lucia, Queensland, Australia 4067

ABSTRACT
Rectification techniques for Landsat imagery have generally been restricted to digital approaches which are costly to apply and to operate and which require access to large computers. This article describes two simple methods of plotting planimetric detail using a Zeiss Stereotop and a Kern PG2. These methods are based on a general affine transformation which has been shown to give accuracies of the same order of magnitude as the nominal resolution element of Landsat. Because of their simplicity and because of the wide availability of these instruments, the proposed methods should be particularly attractive to Earth scientists.
**Introduction**

Users of Landsat imagery are often confronted with the practical problem of transferring image data to an existing map. Of course as in all mapping operations the accuracy requirements will vary with the purpose of the map. When studying methods of restitution it is that application with the most stringent accuracy requirements which is of interest. In the case of present generation Landsat imagery this is probably the updating of planimetry in topographic maps at 1:250 000. A number of independent experiments have shown that simple linear transformations can rectify bulk-processed Landsat images with sufficient accuracy to satisfy the planimetric requirements for most topographic maps at 1:250 000 scale (Bähr & Schuh, 1974; Trinder & Nasca, 1976; Berrill & Clerici, 1977). This paper describes methods of planimetric restitution of Landsat images based on a linear affine transformation. These methods make use of two widely available photogrammetric instruments, the Zeiss Stereotop and the Kern PG2, and require little or no instrument modification. Experiments indicate very good agreement of the plotted points with a topographic map at 1:250 000.

**Theoretical considerations**

The linear affine transformation of equations (1) relates to vector spaces $x,y$ (map) and $u,v$ (image) as indicated in figure 1.

![Figure 1](image)

**Figure 1** (a) parameters of affine transformation ($a_0 = b_0 = 0$)

\[
\begin{bmatrix}
  x \\
  y 
\end{bmatrix} = \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22} 
\end{bmatrix} \begin{bmatrix}
  u \\
  v 
\end{bmatrix} + \begin{bmatrix}
  a_0 \\
  b_0 
\end{bmatrix} 
\]

(b) under the transformation of equation 1(c) a unit square is transformed into a parallelogram

Since

\[
|e_x| = |e_y| = 1; |e_u|^2 = a_{11}^2 + a_{21}^2; |e_v|^2 = a_{12}^2 + a_{22}^2
\]

Then

\[
\cos \alpha_1 = \frac{(e_u \cdot e_x)}{|e_u||e_x|} = \frac{a_{11}}{(a_{11}^2 + a_{21}^2)^{1/2}}
\]

and

\[
\cos \alpha_2 = \frac{(e_v \cdot e_y)}{|e_v||e_y|} = \frac{a_{22}}{(a_{12}^2 + a_{22}^2)^{1/2}}
\]
If we let \[ \lambda_x^2 = a_{11}^2 + a_{21}^2 \]
and \[ \lambda_y^2 = a_{12}^2 + a_{22}^2 \]
then equation 1(a) may be written in the form
\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} = \begin{bmatrix}
  \lambda_x \cos \alpha_1 & \lambda_x \sin \alpha_1 \\
  \lambda_y \sin \alpha_1 & \lambda_y \cos \alpha_1
\end{bmatrix} \begin{bmatrix}
  u \\
  v
\end{bmatrix} + \begin{bmatrix}
  a_o \\
  b_o
\end{bmatrix} \quad \ldots (b)
\]
Thus it would appear that in the plotting problem equations 1 present us with six unknowns, namely two translations \((a_o, b_o)\), two rotations of the axes \((\alpha_1, \alpha_2)\) and two scale factors \((\lambda_x, \lambda_y)\).

However if, as in conventional photogrammetric absolute orientation and scaling, an overall scale factor may be mechanically applied and if the two translations \(a_o, b_o\) and one of the rotations, say \(\alpha_1\), may be applied to the plotting sheet, we see that the instrument used for restitution need provide a means of solving only for the remaining two unknowns.

This may be shown symbolically by substituting in equation 1(b)
\[ a_o = b_o = \alpha_1 = 0 \]
and by letting the overall scale factor be \(k = \lambda_y \cos \alpha_2\); then equation 1(b) may be written as
\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} = k \begin{bmatrix}
  \sec \alpha_2 & \tan \alpha_2 \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  u \\
  v
\end{bmatrix} \quad \ldots (c)
\]
\[ \text{i.e.} \quad x = C u \]
Thus the unknowns become \(\alpha_2\), the angle of shear, and the ratio, \(s = \frac{\lambda_x}{\lambda_y}\), of the scale factors in the two coordinate directions. As stated above \(\lambda_x, \lambda_y\), the determination of the remaining four unknowns may be thought of as absolute orientation and scaling.

Method based on the Zeiss Stereotop

In (Clerici & Harley, 1979) it is shown that equation 1 can be effected in the Zeiss Stereotop using its x-parallax (model deformation) correction device. A brief description of the method is given here; for more detail see (Clerici & Harley, 1979).

The x-parallax correction device can be set to shift the right-hand photocarrier differentially in \(x\) with respect to the left-hand one in such a way that the instrument will implement the transformation of equation 1(c); the only instrumental modification necessary is the connection of the pantograph directly to the right-hand photocarrier. When planimetric details are scanned monocularly in a Landsat image placed on the left-hand photocarrier the output of the pantograph will give the rectified detail at a scale determined by the pantograph setting. Provision of a take-off point for the standard Stereotop pantograph on the right-hand photocarrier is a minor engineering task; see figure 2. The main limitation of the use of the Stereotop is indeed its pantograph which has only a limited range of enlargement (up to \(2.5 \times\)). Plotting of diapositive without the appropriate illumination attachment proves to be rather inconvenient; however this device is available as an accessory.
Methods based on the Kern PG2

In the Kern PG2 the mechanical projection is performed independently in the XZ and YZ planes, it being possible to set different principal distances $c_x$, $c_y$ in each plane. Thus the ratio,

$$ s = \frac{\lambda_x}{\lambda_y} $$

of the scale factors in the two coordinate directions may be set directly as

$$ s = \frac{c_y}{c_x} $$

See equation 1(c) and figure 3. In the following discussion two ways are indicated for the setting of the shear angle of affine transformation.

First method

The registration marks on two identical Landsat images are set on the marks engraved on the photocarriers (inner orientation). This operation is followed by an initial (approximate) setting of $c_x$ and $c_y$ on both projectors and by "relative orientation". If the instrument is in good adjustment all the "parameters of relative orientation" should be very close to their zero values. One then obtains a flat "stereo-model" of approximately 60% of the area covered by the Landsat image.
Assuming that the planimetric detail is to be taken from the right-hand image, to set the shear angle $\alpha_2$ a small rotation $\Delta\kappa$ is introduced on the right-hand photocarrier. This will introduce X-parallax as well as Y-parallax in the "model"; see figure 4.

If the Z value is kept constant and the floating mark is kept on the ground by eliminating X-parallax with $b_x$, a shear $\alpha_2$ is obtained corresponding to the $x$-component of $\Delta\kappa$ in the model space. Of course $\Delta\kappa$ will also introduce Y-parallax; however this was not found to be a seriously disturbing factor during plotting; in areas of the model where Y-parallax is largest, care must be taken to follow ground detail with the right-hand floating mark. Using this approach no modifications need be made to the instrument.

Second method

The method depends on the PG2 being equipped with the earth curvature correction device. In order to obtain in the model space a differential change in X coordinates which is a linear function of Y, as required by equation 1(c), the parabolic cam of the device is replaced by a lineal as shown in figure 5. Using this method planimetric detail is taken from a Landsat image on the right-hand photocarrier; while this is viewed monocularly for measurement, it might be noted that it is possible at the same time to view an image from a different Landsat spectral band with the left eye in order to facilitate interpretation.

Figure 5 Earth curvature correction device adapted for plotting of Landsat images.
It should be pointed out that while the range of differential x displacement which can be obtained in the second method using the modification shown in figure 4 permits the measurement of 70 mm chips (at a scale of approximately 1:3 000 000), the pantograph of the PG2 does not provide sufficient magnification to enable one to plot at a scale of 1:250 000 from these film chips. It will be necessary to further modify the earth-curvature correction device to provide sufficient differential x displacement to permit mapping from the whole of a standard Landsat image at a scale of 1:250 000. Either the thickness of the plane parallel glass plate or its range of rotation or both of these must be increased. A total range of x displacement of about 0.7 mm at image scale is possible with the standard device whereas it needs to be increased to about 1.7 mm to permit the use of standard Landsat pictures.

Tests

Stereotop

The feasibility of restitution with the Zeiss Stereotop has been tested by plotting hydrological details in a band 7 Landsat image of the Brisbane region on an existing 1:250 000 scale topographic map. A visual interpretation of the results indicated remarkably good agreement. (Clerici & Harley, 1979).

Kern PG2

A more stringent evaluation of the methods employing the Kern PG2 was possible since model coordinates could be measured with the instrument used for the test.

First method

As test data for the first method hydrological detail in a band 7 Landsat image and a topographic map at 1:250 000 were again used. "Absolute orientation", or matching of the image to the map, was obtained by introducing an approximate $Z_0$ and approximate values for the x and y principal distances. The final values of $c_x$, $c_y$, and $\Delta \alpha$ were obtained by trial and error. Matching features rather than points proved to be more effective. Availability of recognizable features widely spaced along the instrument coordinates axes facilitates making the settings.

\[ m = \pm \frac{A}{l} \]

\((A = \text{area of figure})
\(l = \text{length of line})\)

Figure 6

Portions of three rivers were digitized by obtaining two sets of instrument coordinates, one by observing the image, the other by tracing corresponding detail on the map with the pantograph microscope. To obtain an estimate of how these two sets of data agreed, the area of the figure indicating mismatch between a plotted line and its equivalent on the map was divided by the length of the line as indicated in figure 6.
The results obtained are given in the table. These are remarkably good results considering that the planimetric accuracy specifications of the map itself are ±0.3 mm at 10% level of significance. The results in the table include interpolation errors; however these were kept very small by over-sampling during digitization.

Second method

At the time of the tests the modifications necessary to increase the range of x-displacement of the parallel plate device had not been carried out. It was not therefore possible to repeat the map test at 1:250 000. With a standard rectangular grid (Wild A5) in the photocarrier an affine transformation was performed by the PG2 at a model:image scale of approximately 1:1; the shear angle was set at -0°55 and s at 1.004 0 corresponding to typical Landsat values; 24 grid points were observed and an "absolute orientation" in plan was carried out numerically on the 4 corner points only. The rms values of the deviations from the ideal values on all points were 15 μm in both X and Y at image scale.

Conclusions

Both methods using the PG2 are shown to be sufficiently accurate for planimetric map revision at 1:250 000. Although the second method described for the PG2 requires an engineering modification to the standard equipment it is more elegant than the other, and easier to use; while both are geometrically correct, the former depends on the elimination of X-parallax in the presence of some disturbing Y-parallax. However results of tests indicate that this can be effectively achieved and it has the great advantage that any PG2 may be used without modification. In the first method it is necessary to have two identical images of any scene to be measured; the second method permits the simultaneous viewing of a scene in a different spectral band.

Restitution with the PG2 has significant advantages with respect to the Stereotop; for example, its superior optical system allows magnification of up to 8X, and its stable pantograph allows plotting of standard Landsat imagery at 1:250 000. Of course the Stereotop is a relative inexpensive instrument which is perhaps more readily available to potential users of Landsat imagery such as geologists, geographers, soil scientists, foresters and others.
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References


