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ANALYTICAL PHOTOGRAMMETRY APPLIED TO THE  
MEASUREMENT OF LARGE STRUCTURES

M.A.R.Cooper & M.R.Shortis  
Department of Civil Engineering  
The City University, London

ABSTRACT

Examples of the practical application of analytical photogrammetry to the measurement of large structures (including measurement of deformation) are described. Results and accuracies are given and suggestions for improving the methods are made.

## INTRODUCTION

What is a large structure? For the purpose of this paper, it can be defined as one that is difficult or dangerous to measure directly by surface contact because of its size. Photogrammetry therefore immediately becomes a possible method of measurement because it does not demand contact with any part of the surface of the structure as a necessary condition for its successful application, although a small amount of contact measurement may improve its accuracy.

There are two aspects of any type of measurement that must be well-defined before any work starts if the measurement is to be satisfactory. These are firstly, the accuracy required and secondly, the way in which the results are to be presented. If these aspects are not properly defined, the measurement is not likely to be completely successful. It is necessary for the photogrammetrist to discuss accuracies (which determine costs) and the form of the results with the engineer and, if necessary, question the engineer's assessment of the accuracies required. This is often the most difficult part of the photogrammetrist's work, but it can be avoided completely if the photogrammetrist is also the engineer (or if the engineer is also the photogrammetrist).

## THE SOUTH ELEVATION OF THE SOUTH TRANSEPT OF ST. PAUL'S CATHEDRAL, LONDON

The photogrammetric measurement of the elevation is being carried out as part of a systematic programme of monitoring the movement of the Cathedral. This programme was begun about 1935 by Freeman Fox and Partners and the Surveyor of the Fabric.

Measurements include levelling and the monitoring of thermal contraction and expansion of cracks, some of which began to appear very soon after (and possibly during) the construction by Sir Christopher Wren in the late 17th century. Photogrammetry presented an opportunity to measure a large part of the exterior without the need to erect scaffolding which is both expensive and an eyesore. A precision of  $\pm 10\text{mm}$  in the absolute positions of points on the elevation was aimed at. Photogrammetric surveys have so far been carried out in 1978 and 1980. Just after the 1978 survey, plumb-lines were established on the facade and vertical profiles were produced by taking offsets from the wires to the masonry. These profiles necessitated the erection of scaffolding which photogrammetry does not require. The plumb-line profiles were used as the standard with which the photogrammetric work was compared. The form of results was to be graphical, showing vertical profiles for comparison with previous and later work.

The control survey (1978). Three datum points were established, each in the form of a buried concrete pillar about 0.5m deep with a brass bolt in the top. They were placed at 25m intervals in an approximate straight line roughly parallel to the facade of the building and about 45m from it. The distances between any one datum point and the other two were measured twice (once in each direction) using a Tellurometer MA100. Differences in level of the bolts were found using a Watts Autoset level, reading all three hairs. The levels were tied to a height-datum point in the form of

a bolt in the kerb about 55m from the facade of the building. A Wild T2 theodolite was set up in turn over each of the three approximately co-linear datum points and horizontal angles between the other two were measured. At the same time, horizontal and vertical circle readings were taken at the three datum points to nine points on the facade of the building. These points are shown circled in Figure 1 and they provided the control for the photogrammetry. They were not deliberately marked in any way, but were well-defined features, such as a serif on a carved letter.

The computation of co-ordinates was based on one fixed datum point and a fixed direction from it to a distant point, related by a measured horizontal angle to one of the other two datum points. The method of computation was by 'variation of co-ordinates' in three dimensions, using slope distances, horizontal angles, vertical angles and differences in level as weighted, but uncorrelated, measurements. The a priori weights were derived from the standard errors of the means of repeated measurements, using the well-known equation  $w = n(n-1)/\Sigma v^2$ , with  $n > 3$ . The co-ordinate system adopted had the XY-plane horizontal with the Y-axis directed towards the face of the building and roughly perpendicular to it; the Z-axis was vertically upwards. Table 1 shows details of the maximum and root-mean-square (rms) standard errors of the nine control points on the facade, determined as a result of the 'variation of co-ordinates' computation and incorporating the a posteriori variance factor  $\sigma_o = 1.26$ .

The photography (1978). One stereopair of normal, horizontal axis photographs was sufficient to give full coverage of that portion of the facade where monitoring was to be carried out. The camera used was a Zeiss (Jena) UMK 10/1318 with glass plates and Ilford FP4 emulsion. The photographs were taken from the platform of an articulated hydraulic hoist, normally used for cleaning street lights, at an elevation of about 10m above ground level. The separation of the camera stations was about 6.3m and the camera base was about 40m from the facade and roughly parallel to it. Thus the base/distance ratio was about 1/6 and the scale of photography was 1/400. A 35mm camera with a long focal-length lens was used to obtain large-scale records of the nine control points on the facade. These records were used to aid identification of the particular features that were surveyed from the datum points.

The photogrammetry (1978). The stereopair was measured in a Zeiss (Jena) Stocometer with IGR encoders on each of the four axes. Data were registered by a RETAB registration and formatting unit and fed directly into a PDP11/V03 mini-computer. The computation of the co-ordinates of the perspective centres and the directions of the perspective axes was carried out by solving linearised observation equations for the 12 unknowns, using the principle of least squares. All stereocomparator measurements were given equal weight. Co-ordinate and parallax residuals at the 9 control points are shown in Table 2. Stereocomparator readings were made to points on the profiles and these were transformed to (XYZ) co-ordinates according to the orientation parameters already computed. One of these profiles is illustrated in Figure 1 and plotted in Figure 2. Because of the way the photographs had been taken (with axes approximately perpendicular to and base approximately parallel to the facade) a vertical line on the photograph could be made almost coincident with the y-axis of the stereocomparator. Thus, although a profile is not marked (except at one

terminal) it was possible to locate and measure points on it by running along the y-axis of the stereocomparator. The profiles were on the surfaces of smooth faced blocks of stone so that any small departure from the true profile in the XZ-plane would not introduce a significant error in the Y-direction. In any case, transformation of the stereocomparator measurements showed that the maximum departure from a true profile in the XZ-plane was 0.05m, over which distance there was no significant change in the facade in the Y-direction. This was confirmed by comparing the profiles from the stereocomparator measurements with those from a Zeiss (Jena) Stereometrograph F, the latter profiles being true profiles derived from the scaled stereomodel. Horizontal profiles were similarly recorded. In addition, an elevation of the facade at a scale of 1/50 was produced from the stereoplotter. This drawing was used to show the locations of the profiles and of the control. It also served (together with the photographs) as a useful reference for discussions between the photogrammetrists, the engineers and the Cathedral staff.

The control survey (1980). The same procedures as in 1978 were followed for the measurement and computation of the control on the facade. No significant change in the positions of the datum points was detected. There was a change in the disposition of the control on the facade. The three lower points in Figure 1 were replaced by four targetted points in 1980. The original three points were not particularly good for photogrammetric measurement and it was hoped to improve the accuracy by replacing them with targetted points. This did not entirely run counter to one of the main reasons for using photogrammetry because the targets were small, made of adhesive paper, were in position only for the few hours necessary for the survey and photography and could be placed and removed safely and easily. The results of the 1980 survey are summarised in Table 1 and are based on a variance factor  $\sigma_0 = 0.96$ .

The photography (1980). It was not possible to use the hoist for the 1980 photography, so the camera was mounted on a tripod on the ground. Therefore, the format contained more 'dead ground' than in 1978. The stereopair was taken with a base/distance ratio of about 1/3.5 and a scale of about 1/500. Agfa Pan 30 on glass plates was used.

The photogrammetry (1980). Stereocomparator measurements were again the main source of photogrammetric information. Residuals at the 10 control points are shown in Table 2 and derivation of the (XYZ) co-ordinates of the profile points was by the same methods as in 1978 with stereocomparator readings as the basic data. These 1980 measurements for one of the profiles are illustrated in Figure 2.

Conclusions. Examination of Tables 1 and 2 shows that both sets of photogrammetric measurements have achieved the precision of 'better than 10mm' in the Y-axis, although the 1980 measurements are rather worse than the 1978 measurements. The smaller scale of photography and the 1980 observer's relative lack of experience of stereoscopic measurement will account for some of this loss of precision. The base/distance ratio of 1/3.5 may also have added to the difficulties of the observer. The increased amount of 'dead ground' in the format has weakened the photogrammetric solution compared with the earlier measurement. The deterioration has apparently not come from the control survey; both sets of measurements have given similar precisions as can be seen from Table 1.

As far as any deformation is concerned, Figure 2 illustrates a significant outward lean between Z-values of about 5m and 18m. The evidence is that this is not recent, and may even have taken place during construction; chains then emplaced at the 22m level may have been put in to tie back the masonry and restore verticality in the portion of the elevation above 22m. The agreement between the photogrammetric measurements and the plumb-line measurements is good. However at two points (Z-values 18m and 26m approximately) there are discrepancies of about 20mm between the plumb-wire measurements and the photogrammetry. Some of these discrepancies were caused by the points measured being either in shadow (for the lower) or on a bright part of the masonry (for the higher) and not ideal for photogrammetry. The evidence is that photogrammetry can be used successfully in the context described. Moreover, the stereopairs and control co-ordinates together constitute a unique source of quantitative and qualitative information that is impossible to obtain by any other means.

#### THE DEFORMATION OF A TOWER CRANE UNDER LOAD

The purpose of this investigation was to use photogrammetric methods to measure the deformations of a Record Potain G/3/25 tower crane under different loads. These measurements were needed for comparison with theoretical deformations from structural analysis. The crane stands over 50m high and has a jib radius of 55m. The main difficulty in photogrammetric measurement is the fact that the crane is a very slender object in relation to its length and will occupy only a very small part of any photographic image, thereby causing problems in the photogrammetric solution for the camera positions and orientations. Moreover, the control cannot be positioned on the crane (unless it is possible to survey the control independently under each loading condition) because of the deformation that is being measured. It is not generally possible to set aside a crane for a long period solely for research purposes. The capital cost of the machine is high and the time necessary to survey the control under each loading condition is rarely available. Therefore a method has to be found that will cause the crane to be unproductive (in terms of work on the site) for as short a time as possible - say one hour as a maximum. The accuracy of the deformation measurement should be about 20mm and the results should be in a form compatible with the input requirements of a computer programme for structural analysis, i.e. as digital information in a co-ordinate system related to the crane in the 'null' or unloaded position.

The geometrical and economic problems outlined above were overcome to some extent by using a crane that happened to be close to a tall building. Control points could be placed on the building and surveyed whilst the crane was working normally. In this way, the crane would be needed only for the period of the photography, and the area of the photograph format which could be used for measurement increased. However, the usual conglomeration of construction site materials and services placed a severe restriction on the geometrical configuration of the control survey and photogrammetry. This is illustrated perhaps too clearly in Figure 3.

The control survey. A base-line about 18m long was set out on a flat concrete slab and measured by ground taping with a steel band. The base-line was inclined by about  $10^{\circ}$  to the line of the building and about 70m from it. Horizontal and vertical angles were observed (using a Hilger & Watts 1-second theodolite) from the terminals of the base-line to 10 targetted points on the building behind the crane. These points are indicated by circles in Figure 3. Distances were measured to them using the Tellurometer MA100. A 'variation of co-ordinates' computation was carried out to give the co-ordinates of the control points. The precision was indicated by a maximum a posteriori semi-major axis of the error ellipses of 6mm.

The photography. Stereophotographs were taken, using Zeiss (Jena) UMK 10/1318 cameras with glass plates carrying Ilford FP4 emulsion, from tripods at the terminals of the base-line. The crane, with driver and banksman, was made available for one hour only between 12.30 and 13.30, so time was short. If one camera had been used, then several changes of position would have been necessary to give a stereopair for each loading condition. Moreover, in the time taken to change the position of the camera, the crane might have suffered some accidental displacement (caused by wind, for example) which would have introduced false parallax. For these reasons, synchronised pairs of photographs were taken, the first pair with the jib roughly parallel to the base-line. The load had been placed on the ground vertically below the jib. The hook was fully retracted. This pair of photographs defined the datum position from which all deformations were measured. The load was then lifted and trolleyed successively to radii of 50m, 30m, 15m and 5m. Stereopairs were taken for each radius, after allowing a few minutes to elapse each time so that oscillations died down. The sequence of operations necessary to obtain the five stereopairs took only 40 minutes.

The photogrammetry. The photography is an example of the kind that is extremely difficult to set up in a stereoplotter; the usual empirical methods of relative and absolute orientation will not work when only a small part of the format is available for measurement. In any case, the need for the data to be presented in digital form was met by the Zeiss (Jena) Stecometer and ancillary equipment described in the foregoing account of the work on St.Paul's Cathedral. Similar computing procedures were followed here.

The results of the measurements with the load at 50m radius are shown schematically in Figure 4. The maximum and root-mean-square (rms) residuals at the control points are shown in Table 3. Nineteen points were selected on the crane at points of interest in the structural analysis. These points were not targetted, but were easily identifiable points in the stereomodels, such as the centre of a bolt.

Conclusions. The main source of error comes from the spatial separation of the control from the measured object. The figures quoted in Table 3 refer to the control and not necessarily to the crane, where there could be an undetected scale error. However, since deformations are required rather than absolute positions, this is of less importance. The deformations are predominantly in the XZ-plane and for each stereopair the rms residuals at the control were less than 20mm in this plane. Whether or not the same precision applies to measurements on the crane is uncertain; deformations

measured were apparently of the required precision. The accuracy could be improved by firstly having the crane closer to a back-ground where control can be better distributed and secondly, by positioning the cameras so that the geometrical configuration is stronger. Each of these conditions is often impossible to achieve in practice on a construction site, without taking the crane out of the normal work programme.

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Year		X	Y	Z
1978	max	2.0mm	3.5mm	2.2mm
	rms	1.4mm	2.8mm	1.4mm
1980	max	1.8mm	3.6mm	2.1mm
	rms	1.0mm	2.0mm	1.0mm

Table 1. Standard errors of the co-ordinates of the control points  
(St Paul's Cathedral)

Year		X	Y	Z	pZ
1978	max	7.6mm	7.6mm	7.5mm	4.6mm
	rms	3.3mm	4.7mm	4.0mm	2.5mm
1980	max	5.6mm	7.0mm	5.8mm	4.1mm
	rms	3.2mm	6.1mm	2.6mm	2.2mm

Table 2. Co-ordinate and parallax residuals at the control points.  
(St Paul's Cathedral)

	X	Y	Z	pZ
max	0.010m	0.025m	0.028m	0.009m
rms	0.006m	0.017m	0.016m	0.005m

Table 3. Co-ordinate and parallax residuals at the control points.  
(Tower crane)



Figure 1

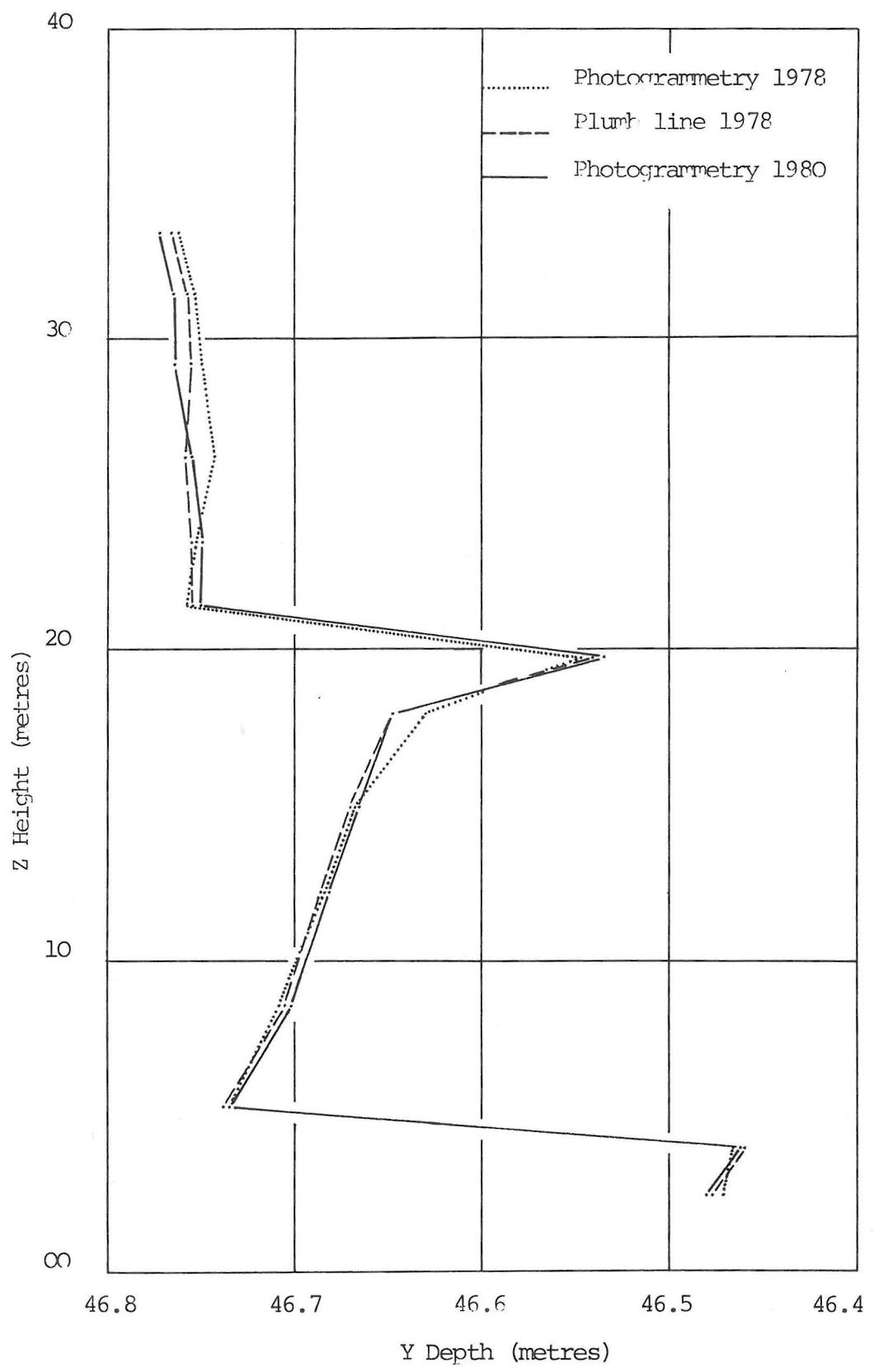


Figure 2

Figure 3



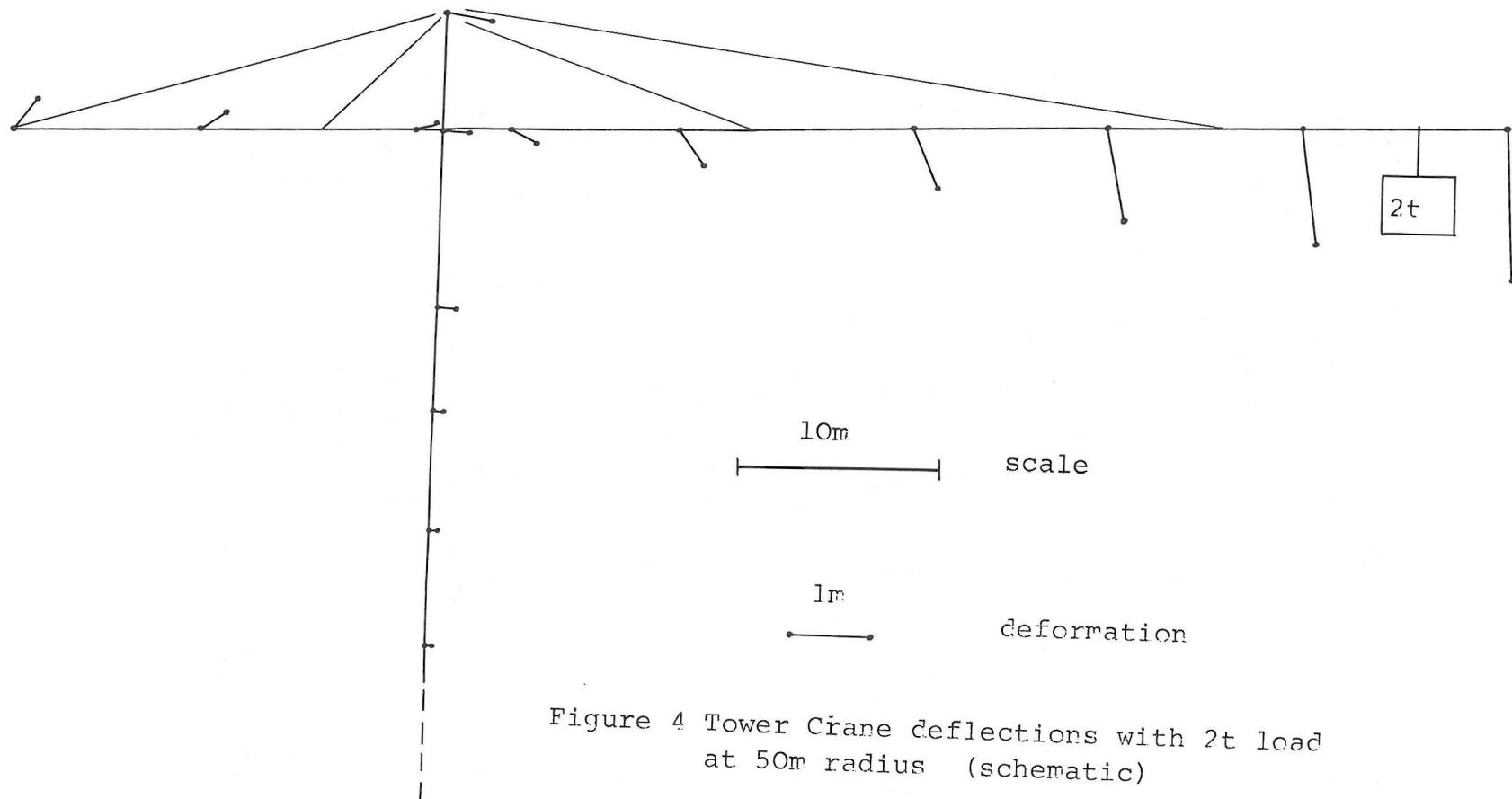


Figure 4 Tower Crane deflections with 2t load  
at 50m radius (schematic)