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PHOTOGRAMMETRIC TRACKING OF A MOVING PARTICLE  
IN RUNNING WATER

Abstract

In a laboratory experiment, the three-dimensional position and the velocity of a floating particle in running water are measured by analytical photogrammetry. Photographs were taken with two Zeiss-Jena UMK cameras through the glass wall of a flume. The particle was dropped into the running water in the flume and illuminated with a stroboscope. The position of the particle is measured with a standard mean error of 0.1-0.3 millimeters.

# PHOTOGRAMMETRIC TRACKING OF A MOVING PARTICLE IN RUNNING WATER

Ole Jacobi

## Introduction

In some hydraulic experiments there is a need for a measuring technique which can track the movement of a small particle without disturbing the flow of the water in which the particle is moving.

In collaboration with the Institute of Hydrodynamics and Hydraulic Engineering, the Institute of Surveying and Photogrammetry has developed a photogrammetric measuring technique for this kind of experiment.

The photogrammetric tracking was used in connection with an experimental investigation of the turbulent open channel flow. This experiment was carried out by B. Mutlo Sumer and Rolf Deigaard and is described in References (1) and (2). The photogrammetric part of the experiment was carried out by Ole Mærsk-Møller and the author and was first described in 1979 (3).

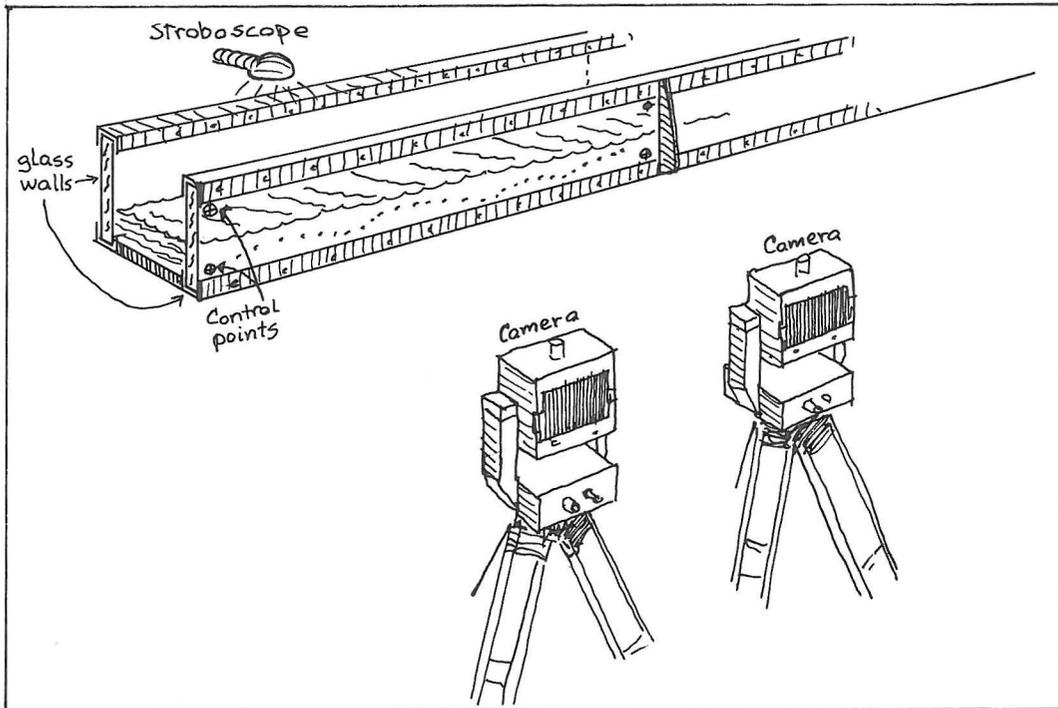
## Law of Refraction

Photogrammetric measurements can be made through transparent materials if the refractive index of the material is known, and if the surfaces that separate the materials can be described mathematically. In experiments with underwater photogrammetry, the surface is usually flat, either a glass wall or the surface of still water.

A light ray going through one material in a straight line will be bent according to Snells law when passing a boundary between two materials with different refractive index

$$\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1}$$

where  $i_1$  is the angle of incidence,  $i_2$  the angle of reflection,  $n_1$  the refractive index of the material where the light ray comes from, and  $n_2$  is the refractive index of the material where the light enters.



### The Experimental Set-Up

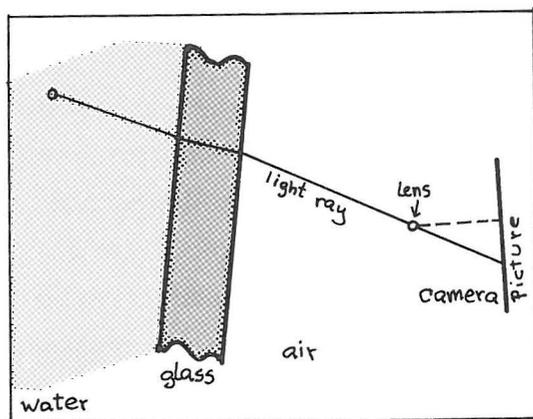
The aim of the experiment was to track the movement in three dimensions and in time of a particle flowing in running water. The water was running in a square flume with vertical walls of thick glass. This made it possible to observe the flowing particle from outside the flume. The particle in motion was a three-millimeter plastic sphere with a specific gravity slightly greater than 1.000. Two Zeiss-Jena UMK cameras were placed three meters from the glass wall, 1.3 meters apart. The entire experimental set-up was covered with a black plastic tent in order to keep the flume and the cameras in darkness. The experiment was started by opening the shutters of both cameras and dropping the particle in the water upstream. The particle was followed with a stroboscope that flashes in short bursts, 11.6 times per second. Every time a flash lit up the particle, a dot was pictured in each of the photographic films. When the particle reached the downstream end of the flume, it had moved 2.5 meters, and some 150 dots had been pictured on each photograph.

Control points were placed on the front glass wall and inside the back wall under the water in order to control the orientation of the cameras. These control points made it possible to control the refraction of the light rays in the water and in the glass wall. The control points were measured by intersection with theodolites and placed in a three-dimensional coordinate system defined by the glass wall and the bottom of the flume. The relative accuracy of the control points was 0.1 millimeter. In order to secure that all control points were pictured on the photographic film, special light arrangements were made for each point.

As the particle moves in the direction of the photogrammetric basis, there is a great risk of combining points in the two pictures that do not belong to each other. To ensure a true stereoscopic interpretation of the two pictures, an ordinary camera flash with a strong light was used at intervals of about one second while the particle was being photographed. These flashes brought out special points on the two pictures, and with the aid of these points it was no problem to make a correct stereoscopic interpretation.

### Measurement

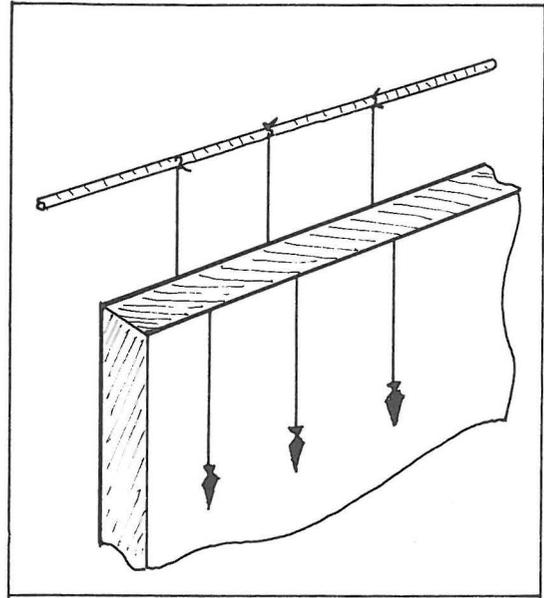
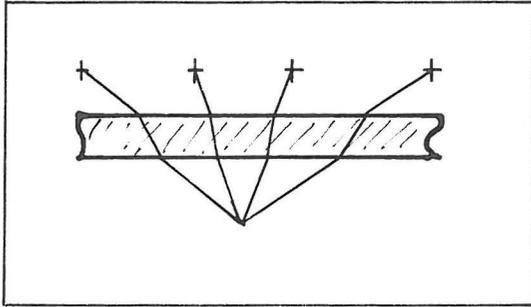
The pictures were measured in a Zeiss-Jena Stecometer. The mean square error of the picture coordinates arising from the adjustment of the control points was between 0.003 mm and 0.006 mm in all of the 111 pairs of pictures measured.



In calculating the position of the floating particle, the refraction of the light rays from the border between the water and the glass wall and from the border between the glass wall and the surrounding air had to be taken into consideration. The glass wall must have an even surface, and the thickness and the refractive index of the glass must be known.

The refractive index of the glass was measured with two theodolites. Four plumb lines were placed behind the glass wall, and their positions were intersected with the theodolites. The direction to the plumb lines was measured above the glass wall and through the glass wall, and the difference

between the measured angles was used to calculate a refractive index for the glass.

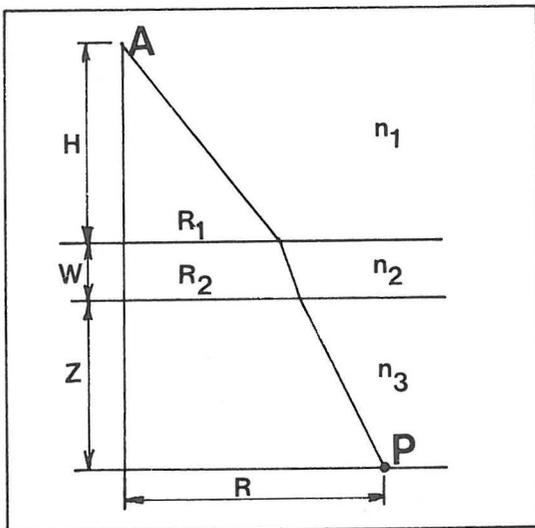


### Computing the Path of the Light Ray

For an analytical computation by the method of least squares it is necessary to have two procedures for the calculation of the light rays:

- The projection center, A, and the object point, B, are known. Calculate the path of a light ray going from B to A.
- The emerging rays from two cameras are known. Compute the point where the two rays intersect.

There is no direct solution to case a), but by setting up two equations, a solution can be found by iteration. A more detailed explanation is given by J. Höhle (4).

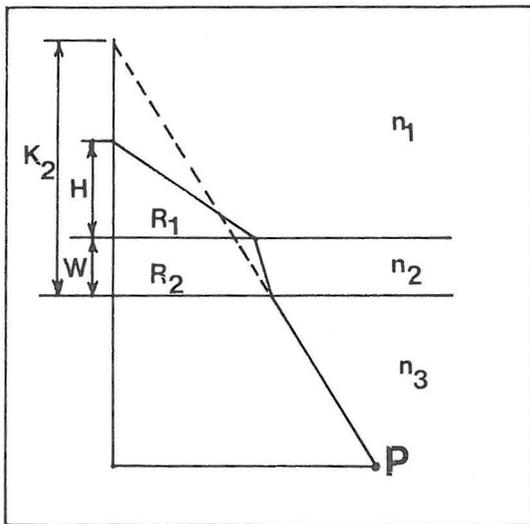


If A is the projection center, H the distance to the glass wall, W the thickness of the glass wall, and P is the object point with the distance Z from the glass wall,  $n_1$ ,  $n_2$ , and  $n_3$  the refractive indices of air, glass, and water, respectively, the two distances,  $R_1$  and  $R_2$ , can be calculated by

$$R_1 = R_2 - \sqrt{\frac{W^2}{\frac{n_2^2}{n_1^2} \cdot (1 + H^2/R_1^2) - 1}}$$

$$R_2 = R - \sqrt{\frac{Z^2}{\frac{n_3^2}{n_2^2} \cdot (1 + W^2/(R_2 - R_1)^2) - 1}}$$

When  $R_1$  is computed, the object point is moved to the front surface of the glass wall, and the photogrammetric calculations can proceed in a normal manner.



In case b),  $R_1$  can be found from the known direction of the light ray and from the distance  $H$  between camera and glass wall:

$$R_2 = R_1 + \sqrt{\frac{W^2}{n_2^2 \cdot \frac{(R_1^2 + H^2)}{R_1^2} - 1}}$$

and

$$K_2 = \sqrt{\frac{n_3^2}{n_2^2} \cdot \frac{(R_2 - R_1)^2 + W^2}{(R_2 - R_1)^2} \cdot R_2^2 - R_2^2}$$

The direction of the light ray in the water after the reflection can be described by  $K_2$  and  $R_2$ . The same calculations are carried out for the light ray coming from the other camera, and by intersecting the two lines the object point can be found.

The computer program and the equations were tested by calculating the control points on the hindmost wall of the flume. These control points were placed under the water and therefore seen through water, glass, and air.

### Results

111 pairs of pictures were measured with about 150 points in each picture. Three-dimensional coordinates were calculated to each point with an accuracy of about 0.2 millimeters. This accuracy was more than sufficient for

the hydraulic experiment. This experiment has shown that close-range photogrammetry can be carried out with analytical methods in hydraulic laboratory experiments with the same accuracy as in ordinary close-range photogrammetry. The photogrammetric method can speed up the hydraulic experiments because the time-consuming measurements are made on pictures after the hydraulic experiments have been finished. The photogrammetric method has a large capacity and a good accuracy, but the cost of the instruments is high. It is also important to notice that special computer programs are needed to solve the refraction of light rays. With a different set-up of the hydraulic experiment, new computer programs must be made in order to make allowance for the physical placing of cameras, water, and boundaries.

### References

- (1) Sumer, B.M., and Deigaard, R., 1979, Experimental investigation and motion of suspended heavy particles and bursting process, Series Paper No. 23, Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark.
- (2) Sumer, B.M., and Deigaard, R., 1979, Experiments on particle motion near the bottom in turbulent open channel flow, EUROMECH 120, Fluid Mechanics Department, Technical University of Denmark, Paper No. 3.2.
- (3) Jacobi, O., 1979, Photogrammetric tracking of a moving particle in running water, EUROMECH 120, Fluid Mechanics Department, Technical University of Denmark, Paper No. 3.3.
- (4) Höhle, J., 1971, Zur Theorie und Praxis der Unterwasser Photogrammetrie, D.G.K. Reihe C, Heft 163.