XIV CONGRESS OF THE
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Presented Paper

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ANALYTICAL PHOTOGRAMMETRIC MEASUREMENTS RELATED TO HYPERBOLIC COOLING TOWERS

Stereotriangulation analytical method for measuring hyperbolic towers is presented and the deviations from the designed shape are established employing the least squares method. Error analysis of the stereotriangulation model used is presented as well.

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Abstract. Stereotriangulation analytical method for measuring hyperbolic towers is presented and deviations from the designed shape are established employing the least squares method. Analysis of errors of the stereotriangulation model used is presented as well.

1. INTRODUCTION

An accurate knowledge of the detailed deviation of the surface of a cooling tower from a perfect figure can be of great value into the civil engineering. The best solution to this problem is provided by the analytical photogrammetry,wherein the coordinates of the photographic images of special target points attached to the surface of the structure are measured. The relative spatial coordinates (X,Y,Z) of the target points are obtained by means of photogrammetric triangulation of the corresponding rays from two or more suitably distributed camera stations. The solution in no way depends upon an accurate knowledge of either the location of each exposure station or the orientation of each camera (7).
The photogrammetric theory employed in our study for the least squares adjustment and error propagation of analytical photogrammetric triangulation, was adopted directly from the general photogrammetric adjustment derived by Brown (6). The mathematical model employed is based on projective methods, Direct Linear Transformation (DLT). The Projective Transformation Parameters (PTP) are computed iteratively using relative control points, by considering as unknowns not only the PTP coefficients of the cameras, but also the X,Y,Z coordinates corresponding to some of the measured images (4).

2. THE MATHEMATICAL MODEL OF ANALYTICAL STEREOTRIANGULATION

a) The condition and constraint equations.

The fundamental projective relations between the cartesian space coordinates X_j, Y_j, Z_j of a point and the plate coordinates x_{ij}, y_{ij} of its photographic image, may be put into the form:

$$x_{ij} = xp_i + c_i \frac{A_i(X_j-X_i^c)+B_i(Y_j-Y_i^c)+C_i(Z_j-Z_i^c)}{D_i(X_j-X_i^c)+E_i(Y_j-Y_i^c)+F_i(Z_j-Z_i^c)}$$

(2.1)

$$y_{ij} = yp_i + c_i \frac{A'_i(X_j-X_i^c)+B'_i(Y_j-Y_i^c)+C'_i(Z_j-Z_i^c)}{D_i(X_j-X_i^c)+E_i(Y_j-Y_i^c)+F_i(Z_j-Z_i^c)}$$

where $j=(1,2,\ldots,n)$; $i=(1,2,\ldots,m)$;

x_{ij},y_{ij} are the measured plate coordinates, properly corrected for comparator errors and lens distortion;

xp_i,yp_i are the plate coordinates of the principal point of the i-th photograph;

c_i is the calibrated principal distance of the i-th exposure;

X_i^c,Y_i^c,Z_i^c are the object space coordinates of the perspective center of the i-th exposure station;

$$T_i = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix}_i$$ is the rotation matrix relating the coordinate axes of image space to those of object space.

The nine direction cosines of the rotation matrix may be expressed in terrestrial photogrammetry uniquely in terms of three angles, the azimuth α, the elevation ω and the swing angle κ. The following expressions can be derived from equations (2.1) by dividing the numerator and the denominator by $-V_i = -(DX^c+EY^c+FZ^c)_i$ (1):

(2.2) $$x_{ij}=\frac{a_1^iX_j+a_2^iY_j+a_3^iZ_j+a_4^i}{a_9^iX_j+a_{10}^iY_j+a_{11}^iZ_j+1}, \quad y_{ij}=\frac{a_5^iX_j+a_6^iY_j+a_7^iZ_j+a_8^i}{a_9^iX_j+a_{10}^iY_j+a_{11}^iZ_j+1}$$

where:

$$a_1^i=-(xpD+cA)_i/V_i;\quad a_2^i=-(xpE+cB)_i/V_i;\quad a_3^i=-(xpF+cC)_i/V_i$$

$$a_4^i=(xpD+cA)_iX_i^c/V_i+(xpE+cB)_iY_i^c/V_i+(xpF+cC)_iZ_i^c/V_i$$

(2.3) $$a_5^i=-(ypD+cA')_i/V_i;\quad a_6^i=-(ypE+cB')_i/V_i;\quad a_7^i=-(ypF+cC')_i/V_i$$

$$a_8^i=(ypD+cA')_iX_i^c/V_i+(ypE+cB')_iY_i^c/V_i+(ypF+cC')_iZ_i^c/V_i$$

$$a_9^i=-D_i/V_i;\quad a_{10}^i=-E_i/V_i;\quad a_{11}^i=-F_i/V_i$$

The relations (2.2) are the basic equations for the DLT, where the eleven parameters are considered as being independent(2). In order to obtain an exact solution for the triangulation, it is necessary to establish five constraints between them, which must be enforced. The five necessary constraints for each station i, are (5):

(2.4) $$F_1^i=(Q_1-Q_2+(Q_5^2-Q_4^2)/Q_3)_i=0;\quad F_2^i=(Q_6-Q_4Q_5/Q_3)_i=0$$

$F_3^i=(Q_4/Q_3-xp)_i=0$; $F_4^i=(Q_5/Q_3-yp)_i=0$; $F_5^i=(Q_1/Q_3-(Q_4/Q_3)^2-c^2)_i=0$

in which:

$$Q_1^i=(a_1^2+a_2^2+a_3^2)_i \qquad Q_4^i=(a_1a_9+a_2a_{10}+a_3a_{11})_i$$

(2.5)

$$Q_2^i=(a_5^2+a_6^2+a_7^2)_i \qquad Q_5^i=(a_5a_9+a_6a_{10}+a_7a_{11})_i$$

$$Q_3^i=(a_9^2+a_{10}^2+a_{11}^2)_i \qquad Q_6^i=(a_1a_5+a_2a_6+a_3a_7)_i$$

xp_i, yp_i, c_i, are the precalibrated parameters of the interior orientation.

When two or more cameras photograph common points, it is possible to go further by considering as unknowns not only the PTP, but also the $X_jY_jZ_j$ coordinates corresponding to some of the measured images. In the present treatment, the unknown coordinates of the relative control points are not eliminated from the original condition equations, but instead are carried through the solution as unknowns along with the unknown coefficients of PTP. As a consequence, the final solution would involve only the inversion of a 11mx11m matrix,despite the fact that many more unknowns (2mn condition equations+5m constraint equations) have been involved in the original equations.

b) The least squares adjustment.

The photogrammetric model under consideration is assumed to involve m exposure stations (i=1,2,...,m) and a total of n relative control points (j=1,2,...,n). It is assumed that approximations are known for each of the unknown coefficients of PTP and for each of the unknown coordinates of the control points (3). According to (2.2), the linearizated condition equations arising from all n control points and all m stations may be formulated. The appropriate matrix equation for this ensemble is:

$$\text{(2.6)} \qquad \underset{(2mn.1)}{v} + \underset{(2mn.11m)}{B}\ \underset{(11m.1)}{\Delta} + \underset{(2mn.3)}{\bar{B}}\ \underset{(3n.1)}{\bar{\Delta}} = \underset{(2mn.1)}{E}$$

For stereotriangulation the all constraint equations (2.4)are interpreted as additional observations with a suitable high weight which enforces the zero variances.

The linearizated constraint equations arising from all m stations are expressed by the system:

$$\text{(2.7)} \qquad \underset{(5m.1)}{\overset{o}{v}} + \underset{(5m.11m)}{\overset{o}{B}}\ \underset{(11m.1)}{\Delta} = \underset{(5m.1)}{G}$$

Now that all of the condition and constraint equations have been defined by (2.6) and (2.7),the general system of error equations may be expressed in matrix form as:

$$\text{(2.8)} \qquad \begin{bmatrix} \underset{(2mn.1)}{v} \\ \underset{(5m.1)}{\overset{o}{v}} \end{bmatrix} + \begin{bmatrix} \underset{(2mn.11m)}{B} & \underset{(2mn.3)}{\bar{B}} \\ \underset{(5m.11m)}{\overset{o}{B}} & 0 \end{bmatrix} \begin{bmatrix} \underset{(11m.1)}{\Delta} \\ \underset{(3n.1)}{\bar{\Delta}} \end{bmatrix} = \begin{bmatrix} \underset{(2mn.1)}{E} \\ \underset{(5m.1)}{G} \end{bmatrix}$$

The covariance matrix associated with the merged observation equations is:

$$(2.9)\quad \underset{(2n+5m)\cdot(2n+5m)}{\lambda} = \begin{bmatrix} \underset{(2n.2n)}{\lambda} & 0 \\ 0 & \underset{(5m.5m)}{\overset{o}{\lambda}} \end{bmatrix} \quad \text{in which:}$$

$$(2.10)\quad \underset{(2.2)}{\lambda_{ij}} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}_{ij} \quad \text{and} \quad \underset{(5.5)}{\overset{o}{\lambda}}{}_i = k\text{Diag}(1,1..1);\ k=10^6-10^8$$

The normal equations for a least squares solution are then given by the following expression:

$$(2.11)\quad \begin{bmatrix} \underset{(11m.11m)}{N+\overset{o}{N}} & \underset{(11m.3)}{\hat{N}} \\ \underset{(3n.11m)}{\hat{N}^T} & \underset{(3n.3n)}{\bar{N}} \end{bmatrix} \begin{bmatrix} \underset{(11m.1)}{\Delta} \\ \underset{(3n.1)}{\bar{\Delta}} \end{bmatrix} = \begin{bmatrix} \underset{(11m.1)}{C+\overset{o}{C}} \\ \underset{(3n.1)}{\bar{C}} \end{bmatrix}$$

In actual computation the Δ matrix is first solved for from a set of reduced normal equations and then the $\bar{\Delta}_j$ matrices inside the $\bar{\Delta}$ matriy are solved for one at a time. The solutions of the normal equations are thus provided by:

$$(2.12)\quad \Delta = \left[N+\overset{o}{N}-\hat{N}\bar{N}^{-1}\hat{N}^T\right]^{-1}\left[C+\overset{o}{C}-\hat{N}\bar{N}^{-1}\bar{C}\right]; \qquad \bar{\Delta}_j=\bar{N}_j\bar{C}_j-\bar{N}_j^{-1}\hat{N}_j^T$$

An iterative procedure is used in the solution. The iteration is stopped when the corrections in the Δ and $\bar{\Delta}$ matrices become negligibly small. A computer program called PROIECT5 was developped by the author. Its formulation is based on the principle of observation equations as described in the above paragraph.

3. SIMULTANEOUS CALIBRATION AND BLOCK TRIANGULATION

As a result of the determinacy afforded by convergent photographs, a complete calibration of the inner cone may be recovered without absolute control in the object space. Because the method requires no knowledge of object space coordinates or knowledge of other object space relationships, we refer to it as constituting a process of self-calibration. The mathematical formulation specifically allowed, introduces a supplemental constraints on ground points by treating a priori knowledge as direct observations of these parameters.
Since the xp_i, yp_i, c_i are incorporated in PTP, the new model makes provision for coefficients K_1,K_2,K_3 for Gaussian symmetric radial distorsion and for coefficients P_1,P_2, for decentering distortion, both assumed constant over all photographs. Inasmuch as the parameters K_1,K_2,K_3,P_1,P_2 of the inner cone are to be recovered simultaneously in a block triangulation, the projective equations (2.2) are augmented with these parameters as follows, resulting equations (3.1):

$$x_{ij}+\bar{x}_{ij}(\bar{r}_{ij}^2K_1+\bar{r}_{ij}^4K_2+\bar{r}_{ij}^6K_3)+P_1(\bar{r}_{ij}^2+2\bar{x}_{ij}^2)+2P_2\bar{x}_{ij}\bar{y}_{ij}=(m/n)_{ij}$$

$$y_{ij}+\bar{y}_{ij}(\bar{r}^2_{ij}K_1+\bar{r}^4_{ij}K_2+\bar{r}^6_{ij}K_3)+2P_1\bar{x}_{ij}\bar{y}_{ij}+P_2(\bar{r}^2_{ij}+2\bar{y}^2_{ij})=(n/q)_{ij}$$

in which: $\bar{x}_{ij}=x_{ij}-xp_i$; $\bar{y}_{ij}=y_{ij}-yp_i$; $\bar{r}^2=\bar{x}^2_{ij}+\bar{y}^2_{ij}$

For an exact solution for the calibration it is necessary to establish for each station only two constraints between PTP ($F^i_1=0$ and $F^i_2=0$). In this case the relative control points must be suitably distributed along the area of the image plates. If all linearizated equations arising from all n points and all m stations are gathered, the collection of equations may be written in matrix form, as:

$$(3.2)\quad \begin{aligned} &\underset{(2mn.1)}{v} + \underset{[2mn.(5+11m)]}{B}\ \underset{[(5+11m).1]}{\Delta} + \underset{(2mn.3n)}{\bar{B}}\ \underset{(3n.1)}{\bar{\Delta}} = \underset{(2mn.1)}{E} \\ &\underset{(2m.1)}{\overset{o}{v}} + \underset{[2m.(5+11m)]}{\overset{o}{B}}\ \underset{[(5+11m).1]}{\Delta} = \underset{(2m.1)}{G} \end{aligned}$$

Following the procedure introduced by Brown (8),we admit supplemental observation equations arising from a priori knowledge regarding the X_j,Y_j,Z_j parameters carried in equation (3.1). The supplemental equations are grouped according to the set of parameters involved:

$$(3.3)\quad \underset{(3n.1)}{\bar{v}} - \underset{(3n.1)}{\bar{\Delta}} = \underset{(3n.1)}{\bar{E}}$$

where: $\bar{v}_j=\begin{bmatrix}\bar{v}_X & \bar{v}_Y & \bar{v}_Z\end{bmatrix}^T_j$; $\bar{E}_j=\begin{bmatrix}-e_X & -e_Y & -e_Z\end{bmatrix}^T_j$

$(e_X)_j=X^o_j-X^{oo}_j$; $(e_Y)_j=Y^o_j-Y^{oo}_j$; $(e_Z)_j=Z^o_j-Z^{oo}_j$

$X^o_j,Y^o_jZ^o_j$ are the observed values of the object space coordinates of the j-th ground point.

$X^{oo}_j,Y^{oo}_j,Z^{oo}_j$ are the current values of the X_j,Y_j,Z_j parameters.

The entire set of observation equations (3.2) and (3.3) can be merged into the composite equation:

$$(3.4)\quad \underset{(k.1)}{\tilde{v}} + \underset{(k.\ell)}{\tilde{B}}\ \underset{(\ell.1)}{\tilde{\Delta}} = \underset{(k.1)}{\tilde{E}}$$

in which:

$$\tilde{v}=\begin{bmatrix}v & \overset{o}{v} & \bar{v}\end{bmatrix}^T;\quad \tilde{B}=\begin{bmatrix}B & \bar{B} \\ \overset{o}{B} & 0 \\ 0 & -1\end{bmatrix};\quad \tilde{\Delta}=\begin{bmatrix}\Delta & \bar{\Delta}\end{bmatrix}^T;\quad \tilde{E}=\begin{bmatrix}E & G & \bar{E}\end{bmatrix}^T$$

$k=2mn+2m+3n$; $\ell=5+11m+3n$

The covariance matrix λ associated with the merged observation equations is:

$$(3.5)\quad \tilde{\lambda} = \text{Diag.}\begin{bmatrix}\lambda & \overset{o}{\lambda} & \bar{\lambda}\end{bmatrix}$$

where λ is the covariance matrix for the coordinates of the ground points, 3x3 block diagonal, where independence of ground points is assumed.

The normal equations for a least squares solution are then given by the following matrices:

$$(3.6)\quad \begin{bmatrix}N+\overset{o}{N} & \hat{N} \\ \hat{N}^T & \bar{N}+\bar{W}\end{bmatrix}\begin{bmatrix}\Delta \\ \bar{\Delta}\end{bmatrix}=\begin{bmatrix}C+\overset{o}{C} \\ \bar{C}-\bar{W}\bar{E}\end{bmatrix}$$

in which: $W = \lambda^{-1}$; $\overset{o}{W} = \overset{o}{\lambda}^{-1}$; $\overline{W} = \overline{\lambda}^{-1}$

The reduced system of normal equations, a (5+11m)x(5+11m) system, involving the interior projective parameters K_1, K_2, K_3, P_1 P_2 and PTP for all stations, is then:

$$(3.7) \qquad \left[N + \overset{o}{N} - \hat{N}(\overline{N}+\overline{W})^{-1}\hat{N}^T\right] \Delta = C + \overset{o}{C} - \hat{N}(\overline{N}+\overline{W})^{-1}(\overline{C} - \overline{W}\overline{E})$$

Once the vector Δ thus obtained, each vector of the X_j, Y_j, Z_j parameters can be computed in turn, from:

$$(3.8) \qquad \overline{\Delta}_j = (\overline{N}_j + \overline{W}_j)^{-1}(\overline{C}_j - \overline{W}_j\overline{E}_j) - (\overline{N}_j + \overline{W}_j)^{-1}\hat{N}_j^T \Delta$$

After the solution has converged, the vectors of measuring residuals may be obtained from:

$v = E$; $\overset{o}{v} = G$; $\overline{v} = \overline{E}$, in which $E, G, \overline{E}$, denote the final discrepancy vectors of the iterative process.

The estimate of the reference variance would be:

$$(3.9) \qquad \sigma_o^2 = \frac{v^T W v + \overset{o}{v}^T \overset{o}{W} \overset{o}{v} + \overline{v}^T \overline{W} \overline{v}}{(2mn+2m+3n)-(5+11m+3n)}$$

A computer program called ETALON was developped by the author and its formulation is based on the principle of observation equations as described in the above paragraph. Based on the PTP coefficients derived from the above paragraph, the elements of the interior orientation xp,yp,c, are averaged from:

$$xp_i = \frac{(a_1+a_2+a_3)_i + (u_4-u_5+u_6)_i}{(a_9+a_{10}+a_{11})_i}; \quad yp_i = \frac{(a_5+a_6+a_7)_i + (-u_1+u_2-u_3)_i}{(a_9+a_{10}+a_{11})_i}$$

(3.10)

$$c_i = \sqrt{\left(\frac{u_1^2+u_2^2+u_3^2}{a_9^2+a_{10}^2+a_{11}^2}\right)_i} = \sqrt{\left(\frac{u_4^2+u_5^2+u_6^2}{a_9^2+a_{10}^2+a_{11}^2}\right)_i}$$

where: $u_1^i = (a_2D - a_1E)_i$; $u_2^i = (a_3D - a_1F)_i$; $u_3^i = (a_3E - a_2F)_i$;

$u_4^i = (a_6D - a_5E)_i$; $u_5^i = (a_7D - a_5F)_i$; $u_6^i = (a_7E - a_6F)_i$

(3.11)

$$u_i' = \sqrt{(u_1^2+u_2^2+u_3^2)_i}; \quad u_i = \sqrt{(u_4^2+u_5^2+u_6^2)_i}; \quad Q_3^i = (a_9^2+a_{10}^2+a_{11}^2)_i$$

$$D_i = -a_9^i/\sqrt{Q_3^i}; \quad E_i = -a_{10}^i/\sqrt{Q_3^i}; \quad F_i = -a_{11}^i/\sqrt{Q_3^i}$$

The exterior orientation can also easily be determined from equations (2.3)

$$\begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix}_i = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_5 & a_6 & a_7 \\ a_9 & a_{10} & a_{11} \end{bmatrix}^{-1} \begin{bmatrix} -a_4 \\ -a_8 \\ -1 \end{bmatrix}_i$$

(3.12)

$$T_i = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix}_i = \begin{bmatrix} u_6/u & -u_5/u & u_4/u \\ -u_3/u' & u_2/u' & -u_1/u' \\ -a_9/\sqrt{Q_3} & -a_{10}/\sqrt{Q_3} & -a_{11}/\sqrt{Q_3} \end{bmatrix}_i$$

4. TRANSFORMATION TO TOWER COORDINATE SYSTEM

The X_j, Y_j, Z_j coordinates resulting from the simultaneous calibration and block triangulation, constitute coordinates of a model of the tower in an arbitrarily defined camera coordinate system . The model can be properly scaled if the true distance between two points in the model is known, or alternatively if the true distance between the two cameras is known. Since the civil engineering is interested primarily in the deviation of the set of triangulated points from a "best fitting" hyperboloid of revolution, the reduction must proceed beyond the photogrammetric triangulation. We assume that the tower is indeed a true hyperboloid of revolution, defined in the tower coordinate system by the equation:

(4.1) $\frac{x^2}{r^2} + \frac{y^2}{r^2} - \frac{z^2}{c^2} - 1 = 0$, which may also be written as:

(4.2) $x^2+y^2+z^2= r^2+(1+r^2/c^2)z^2$, or in matrix notation:

(4.3) $s^Ts=r^2+(1+r^2/c^2)z^2$, in which $s^T=\begin{bmatrix} x & y & z \end{bmatrix}$

The relation between the x,y,z and X,Y,Z systems may be written as:

(4.4) $s = T\,S$, where:

(4.5) $S = \begin{bmatrix} X-X_o & Y-Y_o & Z-Z_o \end{bmatrix}^T$; $\quad T = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix}$

$$T=\begin{bmatrix} \cos\alpha\cos\varkappa & \cos\omega\sin\varkappa+\sin\alpha\sin\omega\cos\varkappa & \sin\omega\sin\varkappa-\sin\alpha\cos\omega\cos\varkappa \\ -\cos\alpha\sin\varkappa & \cos\omega\cos\varkappa-\sin\alpha\sin\omega\sin\varkappa & \sin\omega\cos\varkappa+\sin\alpha\cos\omega\sin\varkappa \\ \sin\alpha & -\cos\alpha\sin\omega & \cos\alpha\cos\omega \end{bmatrix}$$

and X_o, Y_o, Z_o denote the coordinates of the origin of the tower coordinate system.

Substitution of (4.4) into (4.3) and because $T^TT=1$, and from the third row of T: $z=D(X-X_o)+E(Y-Y_o)+F(Z-Z_o)$, equation (4.3) can be reduced to:

(4.6) $e=(X-X_o)^2+(Y-Y_o)^2+(Z-Z_o)^2-r^2-(1+r^2/c^2)z^2=0$

This represents the equation of the hyperboloid in the camera coordinate system. The equation involves a total of seven unknowns, three translations X_o, Y_o, Z_o, two rotations, α, ω, ($\varkappa=0$, as the hyperboloid is a surface of revolution) and the parameters r and c. Seven distinct points on the hyperboloid suffice to generate seven equations of the form (4.6), which could be solved for the seven parameters of the transformation. The best fitting hyperboloid may be defined as that for which the sum of the squares of the distances from the surface of the observed points is minimum:

(4.7) $$\sum_{j=1}^{n} v_j^T W_j v_j \longrightarrow \text{minimum}$$

in which $\bar{X}_j, \bar{Y}_j, \bar{Z}_j$ denote the point on the surface of the hyperboloid corresponding to the observed point X_j, Y_j, Z_j and

W_j is the covariance matrix of X_j, Y_j, Z_j computed in the analytical stereotriangulation. According to (4.6), the linearized condition equation arising from the j-th point is:

$$\underset{(1.3)}{B_j}\;\underset{(3.1)}{v_j} + \underset{(1.7)}{A_j}\;\underset{(7.1)}{\Delta} = \underset{(1.1)}{E_j} \qquad (4.8)$$

where:

$$\Delta = [\Delta X_o \;\; \Delta Y_o \;\; \Delta Z_o \;\; \Delta r \;\; \Delta c \;\; \Delta\alpha \;\; \Delta\omega]^T; \quad A_j = \frac{\partial e_j^o}{\partial(X_o, Y_o, Z_o, r, c, \alpha, \omega)^o} \qquad (4.9)$$

$$E_j = -e_j^o; \quad B_j = \frac{\partial e_j^o}{\partial(X_j, Y_j, Z_j)^o}; \quad v_j = [v_X \;\; v_Y \;\; v_Z]^T \text{ or } v = W_j^{-1} B_j \lambda_j$$

The primary matrices defined above may be used to define the set of condition equations arising from all n triangulated points. The matrix representation of this ensemble is:

$$\underset{(n.3n)}{B}\;\underset{(3n.1)}{v} + \underset{(n.7)}{A}\;\underset{(7.1)}{\Delta} = \underset{(n.1)}{E} \quad \text{where:} \qquad (4.10)$$

$$\underset{(3n.1)}{v} = \underset{(3n.3n)}{W^{-1}}\;\underset{(3n.3)}{B^T}\;\underset{(n.1)}{\lambda}, \quad \lambda = \text{Lagrange multipliers}$$

The general system of condition equations may be expressed in matrix form as:

$$\begin{bmatrix} B\,W^{-1}B^T & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \Delta \end{bmatrix} = \begin{bmatrix} E \\ 0 \end{bmatrix} \qquad (4.11)$$

The solution of the normal equations are thus provided by:

$$\Delta = [A^T G^{-1} A]^{-1} A^T G^{-1} E, \quad \lambda = G^{-1}(E - A\Delta) \qquad (4.12)$$

where $G = B\,W^{-1}B^T$

Since each point is processed independently, then matrices N and C can be formed by the cumulative addition of N_j and C_j where:

$$N = \sum_{j=1}^{n} N_j; \qquad C = \sum_{j=1}^{n} C_j \quad \text{and}$$

$$N_j = A_j^T G_j^{-1} A_j; \quad C_j = A_j^T G_j^{-1} E_j; \quad G_j = B_j W^{-1} B_j^T; \quad \lambda_j = G_j^{-1}(E_j - A_j\Delta) \qquad (4.13)$$

By virtue of this fact, the internal storage required by the computer is essentially independent of the number of carried points. The adjustment has been programmed on a FELIX-256 computer. The output of the program CONFORM provides not only the parameters of the transformation and their covariance matrix, but also the x,y,z, components of the deviations of the triangulated points from the best fitting hyperboloid(3). The perpendicular distance from each point to the best fitting hyperboloid is computed by:

$$d_j = (v_x^2 + v_y^2 + v_z^2)^{1/2} \qquad (4.14)$$

5. PRACTICAL RESULTS

The above method was performed under operational conditions by photogrammetric surface conformity measurements of a cooling tower of the CET - Iassy (Fig.1). In the absence of a

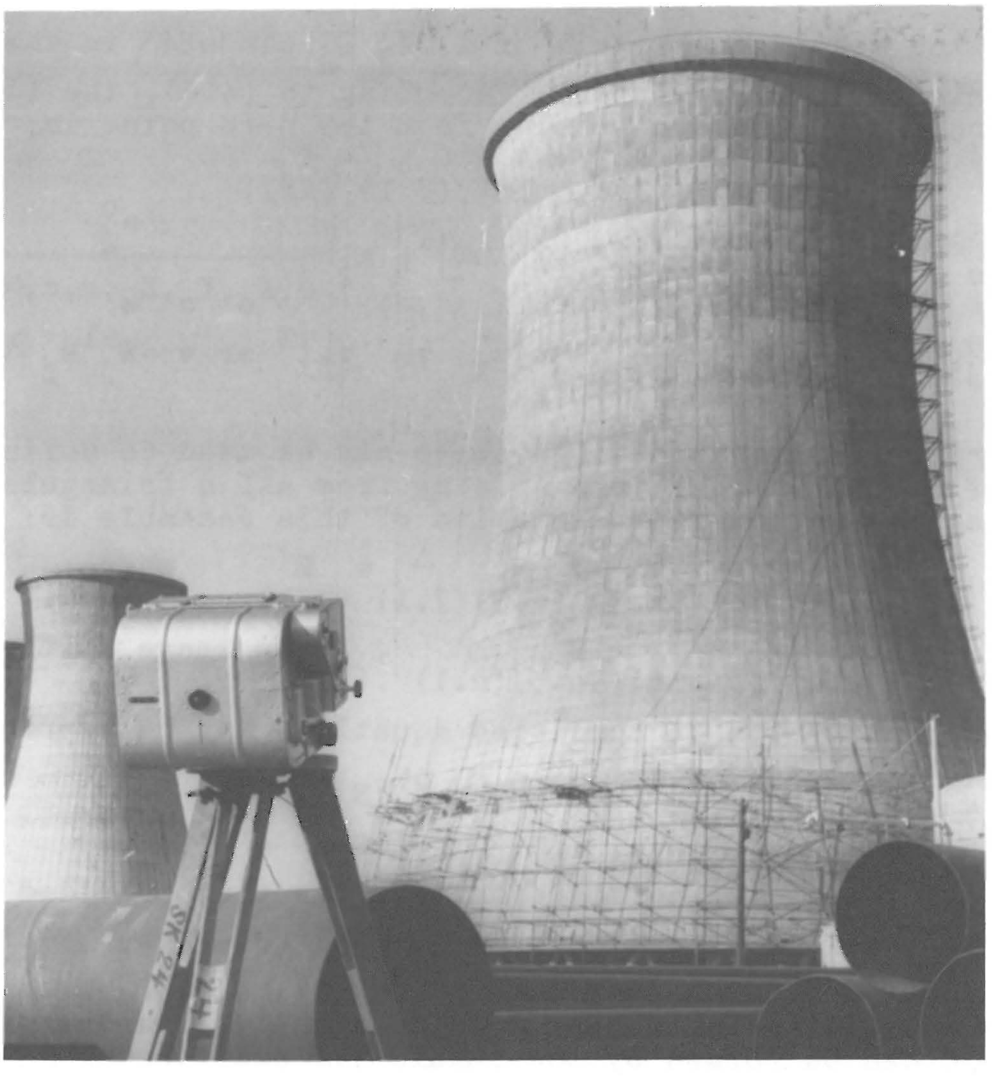

Fig. 1. The hyperbolic cooling tower.

field of target points, an array of artificial targets was used instead. The camera used in this project was a TELEMAR 130x180 mm with a focal length of 400 mm. The camera was refocused to operate at a distance of aproximately 120 m. In order to obtain adequate depth of field at this distance, it was nwcessary to stop the lens down to f/22. Eight camera sites were established on a circle approximately 120 m from the tower. The plates were measured on a liniar spectrophotometric IZA-2 comparator and the plate coordinates with their weight matrices were obtained by the least squares solution using a trilaterated photocoordinates method. The entire set of observations for 932 points was processed through the least squares adjustment. Four iterations were necessary before the adjustment converged to a sufficiently stable solution. The root mean square error of the residuals from the final solution was 18.6 µm and the residual vectors were random from point to point. The minimum variance transformation from camera coordinate system to hyperbolic tower coordinate system was applied to all 932 points. Detailed informations on this research project are given in (3) and (4).

6. CONCLUSIONS

The analytical photogrammetric surface conformity measurements of structures offer an economical alternative to the classical field survey. Among the advantages of the structural measurements are the following:

- All points on the surface are determined to essentially the same accuracy.
- All precise measurings are accomplished on a liniar spectrophotommetric comparator and the image coordinates are obtained by the least squares using a trilaterated photocoordinate method.
- Coordinates of other points external to the surface of the tower, can readily be determined to the same accuracy as points on the surface itself.
- In the absence of a field of targeted points, an array of artificial targets may be used instead.
- Based on the general linear transformation, an exact solution for the orientation and calibration is derived, which leads to a simultaneous least squares adjustment of Projective Transformation Parameters and of relative control point coordinates.

REFERENCES

1. Barbalata C.I.,"Study of the Deformations on Models by Photogrammetric Methods",Buletinul Stiinţific Inst.Politehnic Iaşi, Vol.16 (20), fasc. 1-2, 1970, pg.37-42.

2. Barbălată C.I.,"Stereophotogrammetric Measurements on Soil Erosion",Buletinul Stiinţific Inst.Politehnic Iaşi,Vol.18 fasc.3-4, 1972, pg. 103-109.

3. Barbalata C.I.,"Contributions to Monitoring Deformations of Structures by Analytical Photogrammetric Methods",Ph.D. Thesis,Institute of Civil Engineering,Bucharest,198 pg.

4. Barbalata C.I.,"An Analytical Stereotriangulation Method in Close-Range.I.",BuletinulStiinţific Institutul Politehnic Iaşi, Vol.25, fasc.3-4, 1979, pg.85-96.

5. Bopp H.,Krauss H.,"An Orientation and Calibration Method for Non-Topographic Application",P.E.& R.S.,Vol.44(9), 1978, pg.1191-1196.

6. Brown D.C.,"A Solution to the General Problem of Multiple Station Analytical Stereotriangulation",Technical Report No.58-8 (1958), 53 pg.

7. Brown D.C.,"Precise Calibration of Surfaces of Large Radio Reflectors by Means of Analytical Photogrammetric Triangulation", Technical Report No. 10, 1962.

8. Brown D.C.,"Advanced Methods for the Calibration of Metric Cameras, Final Report part 1, 1968, 120 pg.