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WHAT SHOULD A FUTURE PHOTOGRAMMETRIST KNOW ABOUT STATISTICS AND ADJUSTMENT?

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What should a future photogrammetrist know about statistics and adjustment?

Modern research on the adjustment of aerotriangulation blocks is very much directed to the analysis of the reliability of the photogrammetric point determination. It is very likely that in the future the precision of this point determination will get new attention. This research will result in techniques for quality control of aerotriangulation and for groundcontrol. What should a future photogrammetrist know of statistics and adjustment to be able to work with these evaluation techniques, to understand the meaning of boundary values, of S-transformations and criterion matrices? At which level should he be educated for this? This paper is ment as a discussion paper and therefore tries to sketch the problems of keeping education of photogrammetrists up to date with research on the items mentioned.

Derzeitige Untersuchungen im Bereich der Ausgleichung von Aerotriangulationen richten sich hauptsächlich auf die Zuverlässigkeitsanalyse der photogrammetrischen Punktbestimmung. Es is sehr warscheinlich dass künftig die Genauigkeit der Punktbestimmung neue Aufmerksamkeit erfährt. Derartige Untersuchungen werden in Techniken zur Qualitätskontrolle von Aerotriangulationen und Festpunkten resultieren. Was sollte ein zukünftiger Photogrammetrist von Statistik und Ausgleichsrechnung wissen, um mit diesen Techniken arbeiten zu können, um die Bedeutung von Grenzwerten, S-Transformationen und Kriterium Matrizen zu verstehen? Diese Aufsatz ist als Diskusionsbeitrag gedacht und trachtet die Probleme aufzuzeigen, die Ausbildung von Photogrammetern auf den Hand der Forschung im erwähnten Bereich zu bringen.

La recherche moderne sur la compensation de blocs d'aérotriangulation est tres orientée vers l'analyse de la sûreté de détermination photogrammétrique de points. Il est très probable que dans le futur cette détermination de point attirera une attention nouvelle. Cette recherche aboutira à des techniques pour le contrôle de la qualité de l'aérotriangulation et du canevas terrestre. Que devrait connaître un futur photogrammètre en statistiques et compensation pour être capable de travailler avec ces techniques d'évaluation pour comprendre le sens de valeurs limites, de transformations "S-systèmes" et matrices-critère? A quel niveau doit se situer l'enseignement sur ces sujets? Cet article vise à ouvrir la discussion et pour cela essaie d'esquisser les problèmes pour maintenir la formation des photogrammètres à jour avec la recherche sur les sujets mentionnés.

What should a future photogrammetrist know about statistics and adjustment?

1. On the future task of a photogrammetrist

Before we answer the question in the title of this paper, firstly the tasks which a future photogrammetrist should be given will be described. Here we meet two problems, i.e. first, a description is required of the activities of photogrammetrists nowadays and second, one should be able to forecast how his field of activities will develop in the near future. These two problems are difficult or even impossible to overcome, and therefore the answer to the question cannot be found and we could conclude this paper with the statement : "We don't know".
Yet the author would like to endeavour a more positive answer, but only when restricting himself to photogrammetrists involved in aerotriangulation, or rather photogrammetric point determination. In this field much effort has been made during the last two decades to solve the numerical problems which one had to face when using computers for the adjustment of observations. Programmes are available now for the adjustment of independent model - or bundle-blocks. The photogrammetrist in practice only has to decide which programme fits best to his aims and facilities.
This means that in the future he will be able (and has) to shift his attention to other problems which we will group under the heading "quality control".
Two aspects of this quality control for aerotriangulation are considered here as the most important, these are the "reliability" and the "precision" of the photogrammetric point determination. The term reliability is used here in relation to the search for gross data errors and, eventually, systematic errors. In fact it expresses the probability of finding errors of a certain magnitude. A proper understanding can be best based on a good knowledge of the statistical theory of testing. This subject will be elaborated in section 2.2.
When discussing the precision of coordinates, we talk in fact about their variance-covariance matrix. There one always meets the problem of how to decide whether the precision of a coordinate field can be considered as good or not. In modern research one tries to establish criteria for this decision. Section 2.3 will explain the theoretical background which is required to understand properly the formulation of these criteria.

2. Theoretical background

2.1. General

The need for quality control, as mentioned in the previous section, is caused by the fact that the observations in photogrammetry and geodesy are stochastic. We are lucky enough, however, that their stochastic behaviour can be described, with a sufficient degree of accuracy, by means of the normal distribution. The same fact is true for (linear) functions of the observations. So criteria for quality control can be developed starting from a good understanding of the characteristics of the multivariate normal distribution and its derived distributions, i.e.: Chi-square- and Fisher distribution.
Within this framework it can be made clear why a least squares adjustment is preferable above other methods, as it leads to "best linear unbiased estimators", i.e.: minimum variance- or maximum likelyhood estimators.
A nice didactical aspect of normally distributed variates is that they can easily be interpreted as a set of coordinates in a linear space, from which the geometry of the coordinate base is given by the inverse of the covariance matrix as a metric tensor [6] , [17] , [18] . In this interpret-

ation, linear functions of the observations can be considered as coordinate transformations. Least squares corrections to the observations then give the orthogonal projection of a point in the observation space into a subspace defined by the unknown parameters of the adjustment. This interpretation is a useful tool especially if one wants to understand the meaning of null- and alternative hypothesis and the formation of best tests and the related reliability studies.

2.2. Reliability studies

In modern literature on aerotriangulation the expression "reliability" is almost exclusively used in relation to data-snooping, i.e. the detection of gross observational errors. Baarda originally introduced this term, however, in the field of geodesy within the much wider context of hypothesis testing [1][2] . Moreover, reliability studies cannot be basically understood without a good knowledge of the statistical theory of hypothesis testing. The concept of boundary values is directly based on the concept of type I and type II errors. Let me explain this in some more detail.
Let an adjustment be based on a set of condition equations formulated for the mathematical expertation of a set of observations $\underline{x}$. In fact these condition equations

$$0 = \tilde{t} = t \; (\; \ldots\ldots \; \tilde{x} \ldots\ldots)$$

are an assertion about the expectation $E\{ \underline{x} \} = \tilde{x}$ of the observations. Such an assertion is called a null hypothesis Ho. One can never be sure, however, whether this Ho is true. So a test has to be formulated to decide whether the actual observations $\underline{x}$ are in agreement with Ho or not.
Now two possible situations may occur : either one suspects a specific disturbing factor to cause conflict between the observations and Ho, or no special reason for such a conflict can be found. The suspicion that something is wrong we call an alternative hypothesis Ha. An example of the first situation is the data snooping technique, which is based on a series of tests from which each is a test of Ho versus an alternative hypothesis in which one of the observations is suspected of being erroneous. Another example is found in tests for systematic errors [12] . In cases like these we talk about simple alternative hypotheses.
In the second situation where no special alternative hypothesis has been formulated we talk about a composite alternative hypothesis.
In most cases a test of Ho versus Ha will make use of a function of the observations, say $\underline{W}_i = W_i \; (\ldots x \ldots)$ for which the expectation under Ho is known i.e.:

$$E \{ \underline{W}_i \mid Ho \} = \tilde{W}_i$$

Of course, any such a function can be used, like the condition equations mentioned before, but not everyone of these leads to an effective test. In fact we should search for a function which gives a "best test". But then the concept of a best test should be defined first. This can be done as follows : If the expectation $E\{ \underline{W}_i \mid Ho \} = \tilde{W}_i$ is known and we know also that $E \{ \underline{W}_i \mid Ha \} > \tilde{W}_i$, then a test of Ho versus Ha is formulated as :
reject Ho if $\underline{W}_i = W_i (\ldots x \ldots) \geq C_i$, accept Ho if $\underline{W}_i < C_i$, where C_i is called the critical value of this test.
From the distribution functions of the observations $\underline{x}$, the distribution function of $\underline{W}_i$ can be derived. From the latter we find the probability for $W_i \geq C_i$, for the case where Ho is true, say $\Pr (\underline{W}_i \geq C_i \mid Ho) = \alpha$.
In general there is a unique relationship between α and C_i which is normally

used in the reverse way, that is : first α is specified and then C_i is computed. α is the probability of a "type I error", i.e. the rejection of Ho as false for the case where it is true. α is the significance level of the test. For a specified alternative hypothesis, Ha, the expectation of $\underline{W}_i$ is $E\{\underline{W}_i \mid Ha\} = \widetilde{W}_i + \nabla \widetilde{W}_i$ and under this hypothesis we find

$Pr(\underline{W}_i \geqslant C_i \mid Ha) = \beta_i$. This probability is the power of the test, i.e. it is the probability that Ha will be found for the case where it is true, while $1 - \beta_i$ is the probability that the test may fail to find Ha for the case where it is true, this is a "type II error".
Now a best test for Ho versus Ha is given by that function W_p for which $Pr(\underline{W}_p \geqslant C_p \mid Ho) = \alpha_0$ and $Pr(W_p \geqslant C_p \mid Ha) = \beta_p$ with $\beta_p \geqslant \beta_i$ for all i. This means that a test based on $\underline{W}_p$ with significance level α has a power which is larger than or equal to the power of tests based on any other function W_i. So a best test here is identical to a most powerful test. [10][15] .
For the case where no Ha has been specified, one will search for a function W_p which gives a best test for any Ha which could possibly be formulated. If such a function exists it leads to a "uniform most powerful test". An example is the testing of the estimator for the variance factor computed after adjustment.
The concept of most powerful tests is essential in the search for optimal testing procedures, especially when large sets of observations are involved. (see i.e.: [12]).
For a good understanding of the whole technique of hypothesis testing, after adjustment a student should see how a test for an individual alternative hypothesis, Ha, is related to the global test based on the estimator for the variance factor. When a geometric interpretation of least squares adjustment is given according to section 2.1, the variance estimator can be seen as the norm of the vector of the corrections to the original observations. The value of the test function for an Ha is then the length of the orthogonal projection of the vector of corrections onto another vector defined by the Ha ([2] p.13). With this approach it is simple to understand why some tests are powerful and others are not.
The concept of a type I error and a type II error are basic for the definition of the reliability of a testing procedure in terms of a "boundary value" and a power β . The "boundary value" is the difference between the expectation of a statistic under Ho and its expectation under Ha, where the test of Ho versus Ha has a power β and significance level α [1] . A best test can now also be defined as a test for which the boundary value is less than, or at most, equal to the boundary value of any other test of Ho versus Ha with a level of significance = α_0 and power = β_0. In fact, Ha is now a composite alternative hypothesis. The geometric interpretation will be helpful again to show the relationship among the tail regions of several tests, and to show how their boundary values are related.

2.3. The precision of geodetic pointfields

The second aspect of quality control mentioned in section 1 of this paper has been known much longer than the reliability studies. The precision of coordinates in various types of pointfields, photogrammetric or geodetic, has been investigated by means of their covariance matrices and standard ellipses. It was not before 1973, however, that a consistent criterion theory for the precision of planimetric and height coordinates was published [3] . This publication was a further elaboration of directives for the reconnaissance of cadastral survey networks in the Netherlands. The basic idea is that one should not always try to reach the best possible precision

in a network by making the standard ellipses as small as possible, but one should set a criterion which is an upperbound for the precision to be obtained. Then a network can be designed so that the variances of the coordinates and of any function of the coordinates are always less than or at most equal to this upperbound.
In his publication, Baarda realised that size and shape of point and relative ellipses and thus the covariance matrix of a network is dependent not only on the structure of the network but also on the choice of the coordinate system in which the points are computed. As this latter choice gives no information about the strength of a network, a measure for precision should be found which is independent of the coordinate system. Such an approach is possible if one refers to the precision of elements which are invariant under certain coordinate transformations, here similarity transformations. These invariant elements are called form-elements. In a planimetric network these are angles and ratios of length between sides. Using these, one can define geodetic pointfields which are ideal with respect to precision.
In an ideal network the precision of angles and length ratios depends only on the size and shape of triangle from which they are taken and not on the actual position of the triangle in the pointfield.
The expression "homogeneous and isotropic inner precision" has been used for this situation in [14] . A covariance matrix for heights and one for planimetry satisfying this constraint have been developed in [3] . The real covariance matrix of a network can be compared now with such an artificial, or, "criterion" matrix. To eliminate the effect of the choice of the coordinate system on this comparison, the concept of S-systems was introduced in [3] . The real and the criterion matrix should be computed in the same S-system. The concept of S-systems and a sketch of how the comparison is made is treated in [13] (a presented paper to comm. III at this congress).
Coordinates computed in an S-system are functions of only the form-elements in a network and therefore their covariance matrix is a mere transformation of the covariance matrix of these form-elements. A comparison of this matrix with the criterion matrix by means of the generalised eigen-value problem (see [13] § 2.2) gives then results in terms of eigen-values, which are invariant with respect to transformations of the S-system.
The link between Baarda's invariant elements and the statistical theory of unbiasedly estimable quantities was made in [9] and in [14] . This made it simple to understand Baarda's approach from a statistical point of view and to develop his theory so that it could be applied in cases which were less transparant than the complex plane. In [14] the formulation of the criterion theory for the precision of geodetic pointfields, including the design of a criterion matrix in three-dimensional space, has been based on the concept of unbiasedly estimable quantities. Grafarend studied homogeneous and isotropic random fields and arrived for euclidian coordinates at a covariance matrix which had great similarities with the matrix of Baarda. There, one might find a link between the criterion theory for precision and the theory of random fields and may be collocation techniques.
Research in this field only started recently but one can expect that some results will be applied in practice rather soon. As for the criterion theory for precision of planimetric coordinates, quite a lot of experience has been obtained during the last decade in the Netherlands. The result was a method for finding optimal network structures and for the classification of pointfields with respect to precision. A crucial point here is the choice of a covariance function for the criterion matrix, experience must show which functions fit best to give a description of the precision of several types of pointfields.

3. The education of photogrammetrists

After this outline of what is involved in quality control for photogrammetric (or, in general, geodetic) point-determination, it is time to draw conclusions for the education of photogrammetrists. Although this paper only referred to photogrammetry, the same story is true for any method of geodetic point-determination. Moreover, the conclusions drawn here are equally valid for the training of geodesists in general.

When discussing the education of photogrammetrists we will refer to two levels:

At the first level (say B.Sc. level) a student should learn to compare survey and mapping methods and evaluate their output by means of quality control as explained in this paper. He should be able to make optimal use of existing methods. This requires some knowledge of the statistical theory of hypothesis testing, sufficient for the understanding of techniques for gross error detection, i.e. data snooping and the related reliability studies. For the analysis of the precision of point-fields he should know the meaning of covariance matrices and he should have a global idea of what criterion matrices are. The knowledge of point and relative standard ellipses will be very helpful to make him understand the generalised eigen-value problem for comparing a real covariance matrix with a criterion matrix. One should not require a full understanding of this problem, but only some knowledge of how it is applied in practice with a given criterion matrix, that is a matrix with a specified covariance function. So a student at this level should be able to apply evaluation techniques as i.e. described in [5] (pres. paper comm. III at the congress).

At the second level (say M.Sc. level) a student should be able to perform the same task as given for the first level. But in addition to that he should be able to face new problems which cannot be solved with well formulated given techniques. In that case he must find his own solution using his scientific knowledge, which means that he has to design experiments, and measuring and testing methods. Therefore, a full understanding of the theory of hypothesis testing as explained in section 2.2 is required. This should, of course, be based on a profound knowledge of statistical distributions (mainly the normal-, Chi-square- and Fisher distribution in the field of photogrammetry) and the theory of estimation, with emphasis on least squares adjustment.

Besides the theory of testing, the student should pay attention to the analysis of covariance matrices in the sense of section 2.3. The meaning of criterion matrices and S-transformations can best be understood if the student has sufficient knowledge of the theory of unbiasedly estimable quantities. When criterion matrices are used to classify types of point-fields, he should know how the choice of covariance functions, used in these matrices, is related to the structure of real covariance matrices. The study of many of these subjects will be facilitated if the student has sufficient knowledge of linear algebra, if he is able to operate with vector bases and their matric tensors, linear spaces and their sub-spaces, the orthogonalisation of spaces, eigen-values (also in the generalised eigen-value problem) and eigen-vectors. The geometric interpretation of algebraic procedures is here very important.

Although in modern education in statistics and adjustment, the emphasis should be transferred from the actual solution of the adjustment problem to an evaluation of the final results, a thorough knowledge of adjustment procedures as such (i.e. the five standard problems of Tienstra) is indispensable. If we require less skill in the solution of adjustment problems, then we should require a better understanding of the theory.

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