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AN EMPIRICAL ACCURACY TEST ON POINT TRANSFER IN AERIAL TRIANGULATION

SUMMARY:
Controlled analytical blockadjustments with artificially marked image points are compared against results obtained with signalized points. The data refer to the Appenweier testblock, point marking on film negatives with the Zeiss point transfer device PM 1, and measurements with the Zeiss Monocomparator PK 1. Depending on the number of tie points used the comparisons demonstrate that also artificial marking and transfer of tie-points can give high accuracy blockadjustment results.

1. Introduction

The recent development of aerial triangulation has confirmed and realized the very high level of geometrical precision of aerial photographs. The obtainable limits are marked by standard deviations of image coordinates of 2 - 3 µm. Such results are made possible only by avoiding or compensating all major sources of errors: The instrumental errors of mono- or stereocomparators have become negligible; the systematic image errors are compensated to a great extent by block-adjustment with additional parameters; and errors of point identification and of point transfer are negligibly small with the use of signalized control- and tie-points. It is particularly important to note that up to now all high accuracy results have been obtained with signalized points only.

However, the majority of practical aerial triangulations do not and cannot operate with signalized tie-points. The use of artificially marked and transferred points is still standard. For such cases the operational level of precision is represented by standard errors of image-(or model-)coordinates in the order of 10 - 20 µm or larger, the point transfer being the dominant source of errors. It is of great interest, therefore, to investigate and compare the precision of point marking and point transfer, also in view of new equipment from industry for such operations. Previous investigations have suggested that the accuracy potential of point transfer and marking is higher than hitherto exploited (see [1], [2]).

In the following a report is presented about a controlled experiment on block-adjustment with artificially marked image points, the results of which are compared with signalized points.
2. The test material

For the experimental investigations the 3rd flight mission (block C, flight direction west-east) of the test block "Appenweier" was used, see [3], [4]. The essential project parameters are: Wide angle photography, Zeiss RMK 15/23, Pleogon A lens, photo scale 1:7800; 7 x 17 = 119 photographs, 60% forward-, 20% side-overlap; flat terrain, photography was flown in April 1973.

Fig. 1 shows the 2 control versions used, originating from the given network of horizontal geodetic control points, and implemented by 8 additional points for the perimeter control version. The vertical control points are irregularly distributed, hardly adequate for a test on vertical accuracy. (The Appenweier test originally aimed at horizontal accuracy only; the test implied 4 fold stereo-overlap for which case the vertical control point distribution was highly sufficient.

For the accuracy investigation additional check-points were available (78 horizontal, 58 vertical), with an estimated horizontal and vertical accuracy of about 1.5 cm.

In the test area all tie points were targeted with triple targets. Their location corresponds to the 9 standard positions in the photographs.

The original negative film was cut and directly used for the investigation.

3. Point transfer and point marking

A prototype of the new Zeiss point marking instrument PM 1 (see [5]) was made available for the investigation. It had still some shortcomings which have been remedied, in the meantime, for the serial production. In particular, switching of Dove-prisms from ortho- to pseudo-stereoscopic observation was not yet possible.

At all 9 standard image positions triplets of tie points were transferred and marked. The operations went without serious difficulties. The quality of the marked points turned out to be highly satisfactory.

A special transfer- and marking procedure was adopted, as indicated in fig. 2, according to an unpublished suggestion by Van den Hout. The procedure avoids the transfer of a previously marked point. Instead, one floating mark is kept fixed on a point in a central photo whilst this particular point is transferred into all adjacent photos and marked there. The kept point in the central photo is marked only at the end of each turn, together with its last transfer. This procedure implies forward and backward transfer of points within a strip as well as transfer into 2 or 3 photos of the next strip. It goes systematically through the central line of points of each photograph (+1 additional point in the first and the last photo of a strip), and through all photographs of a block.
The alleged advantages of the procedure are:
- no resetting on and transfer of an already marked point
- best possible stereoscopic conditions for the transfer
- transfer into an image corner point only from a central line point; advantages for easy selection and stereoscopic transfer.

There are two instrumental conditions on which this procedure is based which aims at maximum precision of point transfer:
For transferring a point from the center line of a photograph both the photograph and the floating mark on the particular point must be kept fixed whilst the adjacent photographs are exchanged and the kept point is respectively transferred. And because of forward and backward transfer of the kept point into adjacent photographs the stereoscopic observation must switch from ortho- to pseudo-stereoscopy in order to maintain constant conditions of observation.

Evidently the procedure requires repeated exchange of photographs and therefore might be time consuming. It can be kept efficient, however, if the handling of the photographs is well organized, by ordered arrangement within reach of the operator. The procedure actually proved to be highly efficient as is demonstrated by the total time required:
The point transfer for the testblock implied marking of altogether 3159 image points, grouped in triplets, in 119 photographs. The total operation required 45h 11min. This is equivalent to

51 sec/marked image point or
22,8 min/photo (for standard 9 x 3 = 27 points/photo).

Those figures refer to the operations with the PM 1. The general preparation of paper prints remains excluded.

4. Measurement of image coordinates

The measurements of image coordinates were carried out with the Zeiss mono-comparator PK 1, in connection with the program system PK - AS for interactive guidance of measurements, data checking and data editing, see '6). The hardware system contained the mono-comparator PK1 with coordinate counters Direc 1, a HP 1000 computer, interactive terminal and printer. The comparator measurements are directed by a menu, the data are checked and edited by the AS program and, after clearance, stored on a data file for subsequent off-line block-adjustment.

The block was measured twice, first for signalized points only and independently for the artificially marked points. Either set included, of course, measurement of fiducial marks and of (signalized) control- and check points. Within each set there were no double measurements.
The use of the AS program system proved advantageous. It sped up the operations and reduced the rate of erroneous data. The subsequent blockadjustment was faced with only 6 erroneous tie points and 3 erroneous control points in the data set for the signalized points. This corresponds to a rate of erroneous image point measurements (including fiducial marks) of 2.3 \( \text{o/o} \). The blockadjustment required only 2 adjustment runs.

The blockadjustment of the data set for artificially marked points discovered remaining 15 erroneous tie points and 1 erroneous control point. This corresponds to an error rate of 4.1 \( \text{o/o} \) of all measured image points. The adjustment required 3 runs for obtaining the final result.

The data set for the signalized tie-, control- and check-points included 3935 image points (476 fiducial marks included). The total measuring time was 44h 02 min. This corresponds to

\[ 40.3 \text{ sec/image point (fiducial marks counted as image points), or} \]
\[ 22.2 \text{ min/photo} \quad \text{(average 29.1 points + 4 fiducials/photo).} \]

The data set for the artificially marked points contained 3863 image points (signalized control- and check-points and 476 fiducial marks included). The total measuring time of 39h 42min gives

\[ 37.0 \text{ sec/image point (fiducial marks counted as image points), or} \]
\[ 20.0 \text{ min/photo} \quad \text{(average 28.5 points + 4 fiducials/photo).} \]

of the total measuring time of 40 sec and 38 sec, respectively, per image point about 16 sec per point (40 \%) were used up by administrative (point-numbering etc.) and checking operations.

5. Block adjustment

The block-adjustments for both sets of data (signalized points versus artificially marked points) were computed with the bundle program PAT-B after having applied the conventional image corrections for lens distortion, earth curvature, and refraction. All image coordinates were given weight 1.

Each set was treated with 2 different versions of horizontal control: 22 perimeter control points and 23 control points randomly distributed throughout the area, thus simulating 2 practical standard cases. Either case is far from ideal as fig. 1 shows: The perimeter control leaves uncontrolled gaps of 4 - 6 base lengths. The random control covers the perimeter only poorly. As previously mentioned, the vertical control points do not control the block very well, in spite of their large total number (52). The geodetic control coordinates were introduced into the bundle-adjustment as observations with 1.5 cm standard deviation.

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The bundle adjustment of each case was computed without and with additional parameters in order to show the sensitivity of the results with regard to systematic image errors. Being not the main item of the investigation only 1 set of 12 block-invariant additional parameters was applied. It is known that a more sophisticated use of additional parameters would not substantially change the results.

The main scope of the investigation refers to the different types and different numbers of tie-points. Therefore, apart from the main subdivision in signalized and artificially marked tie-points, we distinguish 3 different versions according to the number of tie points used for the computation: Version 1 represents the standard case with 1 point each in the 9 standard positions within a photograph. Version 3 represent the strong ties, implying 3 x 9 = 27 tie points per photograph, with triplets in the 9 standard positions. Version 2 represents pairs of tie-points, with the modification that in the points along the line of nadir points only single points were used, thus giving 9 x 2 - 3 = 15 points per photograph. The 3 versions can clarify the effects which strengthening of ties between photographs has on the accuracy of adjusted blocks.

The results of the various block-adjustments are summarized in table 1.

6. Discussion of results

The results as displayed in table 1 can be summarized and commented as follows, first with regard to signalized tie-points:

σ₀ values of 3,1 μm without and of 2,5 μm with additional parameters indicate in general a very high level of precision, which is not affected by different control versions. Improvement with additional parameters by a factor 1,25 (1,26) is normal; it confirms the existence of systematic image errors. σ₀ is practically not affected by the number of tie-points. The σ₀ values with additional parameters (2,4 μm resp. 2,5 μm) show that some systematic or other image errors are left, if compared with the noise level of about 2 μm or less which is expected to represent the limit of precision obtainable at present.

The absolute horizontal accuracy of the block, expressed as r.m.s. errors μₓ and μᵧ of the independent check points, is again quite good. It is interesting to note that strong ties without additional parameters, with μₓ,ᵧ values of 6,5 μm and 6,3 μm, are highly affected by systematic image errors, as indicated by ratios μₓ,ᵧ/σ₀ of 2,26 and 2,05. Strong ties propagate systematic image errors more rigidly into block deformations than weak ties. This explains why tie-point version 1 has significantly smaller μ-values than version 3 (factor 1,19). Only with additional parameters both versions give about equal accuracy, indicating remaining small systematic image errors and/or the small magnitude of noise at the
signalized points. The improvement of absolute horizontal accuracy to 4,5 μm (improvement factors between 1.51 and 1.15) again underlines the high inherent accuracy of the test block Appenweier, although the remaining ratios of 1.9 and 1.8 for \( \frac{\mu_{x,y}}{\sigma_o} \) are not yet in agreement with the theoretical ratios in case of random errors only.

Without additional parameters, the 2 control versions differ by 10% (11%), which might have been expected in the presence of systematic image errors. After correction of systematic errors both control versions agree closely within 1% (2%).

The absolute vertical accuracy with \( \mu_z \) values between 14,7 μm and 13,7 μm remains in all cases < 0,10/°h. It is quite acceptable, considering the unfavourable distribution of vertical control. It is remarkable, and confirms previous experience, that additional parameters improve the vertical accuracy very little (4%, 5%) in blocks with 20% lateral overlap and dense control. Also the number of tie points has almost no effect on vertical accuracy.

The maximum errors \( e_{max} \) remain close to 3 times the respective r.m.s. errors \( \bar{\mu} \), as expected.

With regard to artificially marked tie-points the results can be commented as follows:

The \( \sigma_0 \) values without additional parameters range between 7,4 μm and 6,6 μm. With additional parameters they are reduced to values between 6,4 μm and 5,8 μm. The improvement factors by additional parameters are between 1.13 and 1.18. Thus, they are slightly smaller than with signalized points. This can be explained by a higher noise level whilst the systematic errors remain more or less unaltered. The same explanation accounts for the fact that now the \( \sigma_0 \) values depend on the tie-point version, being larger in the case of strong ties.

The absolute horizontal accuracy of the blocks, in case of no correction of systematic errors, ranges between 8,9 μm and 12,3 μm, rather strongly depending on the tie-point version, weak ties again giving the best results. Because of the larger magnitude of random errors the ratios \( \frac{\mu_{x,y}}{\sigma_o} \) remain between 1.4 and 1.7 (1.2 and 1.8 with additional parameters).

The improvement by additional parameters is generally less marked than with signalized points (improvement factors between 1.4 and 1.1) which again is explained by a high level of random errors as compared with the systematic image errors.

The \( \mu \) values range from 7,7 μm to 10,4 μm, and the effects of strong ties now turn into the expected advantage.

It is noticeable in all cases that here the control version has quite a strong influence on the resulting accuracy. With perimeter control the results deteriorate consistently by ratios between 1.12 and 1.26, as compared with the random distribution of control.
The absolute vertical accuracy of the blocks with artificially marked tie points shows $\mu_z$ values between 14.6 $\mu$m and 26.3 $\mu$m. In general, the differences against the respective cases with signalized points remain smaller than for the horizontal accuracy. The additional parameters now improve the vertical accuracy by factors between 1.2 and 1.5. Thus the improvement is considerably more effective than with signalized points, which brings the final vertical results of both cases close together. Also the effect of the strong ties is now visible as expected.

7. Conclusion

The test has confirmed first the good economic position of preparation and data acquisition for blocks with artificially marked tie-points.

With regard to the accuracy results we refer to the case of pairs of tie points, as it is recommended in view of easy detection of gross errors:

With signalized tie-points the test results are summarized and represented by

\[
\begin{align*}
\sigma_0 &= 3.1 \, \mu m \ , \ \mu_{x,y} = 6 \, \mu m \ , \ \mu_z = 14.5 \, \mu m \ (\text{without additional parameters}) , \text{ and} \\
\sigma_0 &= 2.5 \, \mu m \ , \ \mu_{x,y} = 4.5 \, \mu m \ , \ \mu_z = 13.9 \, \mu m \ (\text{with additional parameters}).
\end{align*}
\]

Such figures are to be compared with the respective results referring to artificially marked tie-points:

\[
\begin{align*}
\sigma_0 &= 7.0 \, \mu m \ , \ \mu_{x,y} = 10.2 \, \mu m \ , \ \mu_z = 24.3 \, \mu m \ (\text{without add. parameters}) \\
\sigma_0 &= 5.9 \, \mu m \ , \ \mu_{x,y} = 9.3 \, \mu m \ , \ \mu_z = 19.1 \, \mu m \ (\text{with additional parameters}).
\end{align*}
\]

The results can be interpreted that the operation of point transfer adds independently to the $\sigma_0$ values valid for signalized points a random contribution of about 5.3 $\mu$m and a systematic component of 3.4 $\mu$m, giving a ratio of 2.4 for the $\sigma_0$ values. The systematic component is taken out by additional parameters, which leaves $\sigma_0$ for artificially marked points at a level of about 6 $\mu$m. The absolute accuracy (9.3 $\mu$m horizontal, 19 $\mu$m vertical) of the adjusted block is also inferior to the case of signalized points (horizontal factor 2.0; vertical factor 1.4). However, the ratios $\mu/\sigma_0$ (1.5 and 1.6) are comparable or better than signalized points (1.9 and 1.8) and can be used in the conventional way for estimating the absolute accuracy of a block from the $\sigma_0$ estimate.
Considering that the results refer to one experiment only, they may not be directly representative for other cases. Nevertheless, it can be stated as general conclusion: The test has shown that high precision aerial triangulation is possible with artificial tie-points although the results obtainable with signalized tie-points are not completely reached.

References:


Fig. 1: Testblock Appenweier, ground control

- △ horizontal and vertical control point
- ○ vertical control point
Fig. 2: Systematic procedure of point-transfer and point-marking
Table 1 Appenweier test block C. Results of bundle-block adjustments, signalized tie-points versus artificially marked tie-points

<table>
<thead>
<tr>
<th>case</th>
<th>tie-pts.</th>
<th>horiz. control</th>
<th>σ₀ (μm)</th>
<th>Check-points</th>
<th>max. errors</th>
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