XIV CONGRESS OF THE INTERNATIONAL SOCIETY OF PHOTOGRAMMETRY
Hamburg, 1980

Commission III - Presented Paper

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# INDEPENDENT MODELS WITH PROJECTION CENTERS CONNECTED ONLY IN HEIGHT

Abstract:
A simple procedure is proposed for independent models aerial triangulation, in which the connexion of the projection centers is imposed only on the Z coordinate. After some iterations the results appear well comparable with those obtained by conventional procedures; a comparison is reported with some ones of them in a small block of strips.

Rome, April 1980

Independent Models with Projection Centers connected only in height

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1. - It is well known that the main source of unaccuracy in independent models serial triangulation is the instability of the projection centers (PCs) along the observations.

When using analogical plotters, this instability is particularly felt in the planimetric $X_0Y_0$ coordinates, while the $Z_0$ coordinate seems to be less influenced by the variations in the camera's attitude. In some experiments which we present in another paper [1] , we show that in a Santoni Simplex II C plotter the variations in $X_0Y_0$ are almost 3 to 10 times bigger than in $Z_0$, depending on the magnitude of the $\varphi\ \omega\ \varkappa$ variations imposed to the camera.

This situation is implicitely aknowledged in some independent models procedures, where the planimetric connexion of the PCs is given a weight much smaller than the altimetric one. We believe that the results of the above said experiments may be extended to many other analogical plotters, excepted those - like the Kern PG2 - where the PCs' coordinates are measured in each model (and here also the measure of $X_0Y_0$ is less accurate than of $Z_0$).

A completely different speech is to be done for analytical plotters, where the PCs' coordinates are computed in each model by space resection. But if we analyze the causes of unaccuracy in their computation - like too big variations in the $b_y$ $b_z$ components and in the $\varphi\ \omega\ \varkappa$ attitude in bad flights - we still find that most of their effects are worse on the $X_0Y_0$ coordinates than on the $Z_0$ one.

As a general conclusion, we may say that the planimetric coordinates of the PCs are generally worse than the altimetric ones.

2. - From these considerations naturally comes out the question, if we can avoid the use of the planimetric $X_0Y_0$ coordinates for the bridging of independent models, and limit the PCs' connexion to the $Z_0$ coordinate.

In fact, if no connexion of the PCs is considered, one only degree of freedom - approximately a $\varphi$ rotation - remains undefined; therefore one only equation is sufficient to complete the full orientation of the second model on the first one. Now, there is no doubt that the connexion on the three $X_0Y_0Z_0$ coordinates is much stronger than in $Z_0$ only; this is mainly due to the fact that the $X_0$ connexion gives a 1st order tie, while the $Y_0$ and $Z_0$ ones give a 2nd order tie.

However, if there are no contrary causes, also a 2nd order constraint is enough to correct an imperfect attitude; it is something like the equilibrium of a bicycle, which is kept by the very weak couple given by the wheels' rotation. Now, in our problem not only there are no contrary stresses, but there are very frequent favourable conditions - given by transversal tie points, ground controls, zenithal angles, etc. - which help the $Z_0$ connexion to fix the correct $\varphi$ attitude, despite its weakness.

We presume therefore that a block computation by independent models can be done introducing only the heights of the PCs, and that the general accuracy of the aerial triangulation may have some advantage from this approach, or at least a negligible loss. This should be particularly true in analogical aerial triangulation, where the instability of the PCs' planimetric coordinates is really dangerous; and is certainly true in block triangulation with uniformly distributed controls.

3. - It would be extremely complex to give an analytical full demonstration of what above, and maybe it isn't worth while. We have preferred to set up an experiment, which should at least empirically show that it is possible to use only height connexions on the PCs in blocks with uniformly distributed control points, without significant loss of accuracy in the final results.

The experiment is set up as follows:

i) - observation of a little block of strips with the independent models technique. The block (see Annex 1) is derived from a larger one employed for the 1:5,000 technical map of the Regione Toscana (flights 1977, Zeiss RMK 23 A camera, 6" focal length, relative height ~2200 m, photo scale ~ 1:13,000; 4 short strips, each one of 4-5 models, for a total amount of 17 models; 24 control point almost uniformly distributed on the whole surface of the block; observations done at an OMI AP/C, one pass, with "independent models" program; 6 pass-points, and 2-4 transversal tie-points in each model);

ii) - adjustment and computation of the whole block performed with 5 different procedures:

A. - Ackermann procedure [2] ;

B. - King procedure [3] ;

C. - Schut procedure [4] ;

D. - TABLO procedure (rigid bridged models) [5] ;

E. - TAMI 1 procedure (rigid independent models with connexion of the PCs alternatively in height and planimetry in 6 successive iterations);

F. - TAMI 2 procedure (rigid independent models with connexion of the PCs only in height in 6 successive iterations);

iii) - comparison of the coordinates of each computed point obtained with the above 6 procedures. The differences are reported on synoptic tables, separately for ground points (full con_ trols, single heights), and for the pass- and tie-points (see Annex 2; due to space shortage, only a sample table is reported here). Their mean absolute values are reported in the following table:

MEAN ABSOLUTE RESIDUALS ON THE ".1 POINTS"

(control and ground points)

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| M (X) | .26 | .36 | .21 | .41 | .36 | .37 |
| M (Y) | .22 | .25 | .22 | .29 | .37 | .30 |
| M (Z) | .33 | .41 | .31 | .44 | .39 | .32 |

MEAN ABSOLUTE DIFFERENCES ON THE ".2 POINTS"

(pass and tie points)

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| M (X) | 0 | .13 | .27 | .47 | .50 | .51 |
| M (Y) | 0 | .22 | .28 | .30 | .56 | .56 |
| M (Z) | 0 | .17 | .34 | .35 | .41 | .40 |

iv) - a synthetic comparison of the results obtained in the heights definition - the most important ones - is reported in Annex 3, where the differences with the ground and with the Ackermann heights in the above procedures are described by contours. Due to space shortage, here also only two contour maps - the TAMI 2 vs. ground heights, and the TAMI 2 vs. Ackermann heights - are reported (the remaining ones may be issued on request).

4. - From the above results the following conclusi_ ons may be drawn:

a) - the differences obtained by independent models with con_ nexion of the PCs only in height, and the remaining procedures as specified in 3, ii), are unsignificant for practical carto_ graphic purposes;

b) - similarly unsignificant are the differences in the co_ ordinates obtained from each one of the six methods specified in 3, ii). We may say that for cartographic purposes anyone of these methods - and any good modern method - is equally good.

Acknowledgements

The observations of the whole block at the AP/C, the computations with the D, E, F procedures, and all the ta_ bles and drawings were prepared by Mr. G. Giorgetti, for his graduation thesis as Engineer at the University of Ancona. We express him our gratitude.

We thank also Prof. M. Cunietti, Geom. L. Luchini and Geom. G.L. Pelacani, who kindly performed the block computation with the A, B, C procedures.

References

[1] - BIRARDI, G. - Computing and checking the projection centers in analogical plotters - Presented paper, XIV Congress of the ISP, Commission III, Hamburg, 1980.

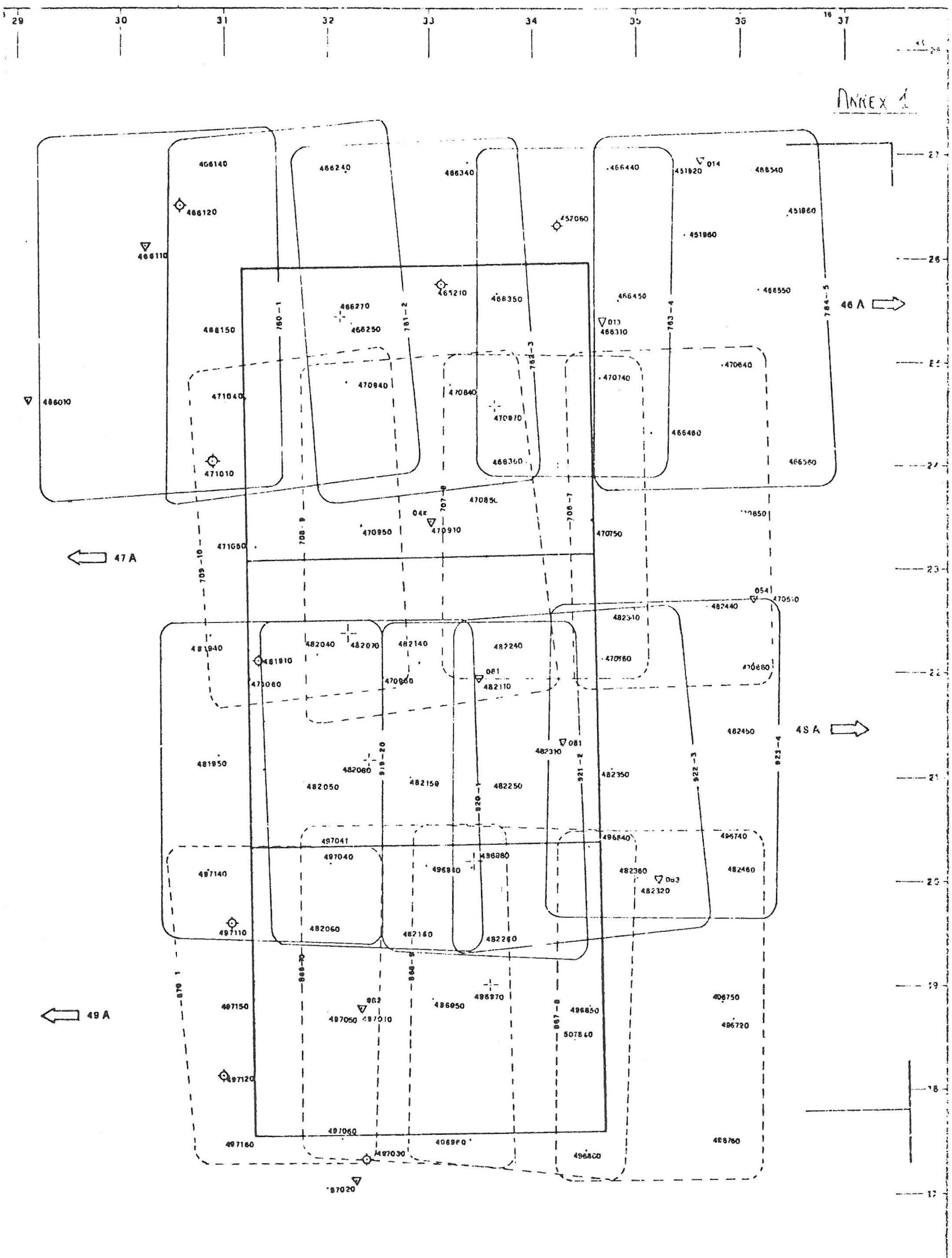
[2] - ACKERMANN, F. et A. - Block triangulation with indepen- dent models - Photogrammetric Engineering, 1973, pag. 967 sgg.

[3] - KING, C.W.B. - Programming considerations for adjustment of aerial triangulation - Photogrammetria, 23, 1968.

[4] - SCHUT, C.H. - A Fortran program for the adjustment of strips and blocks by polynomial transformation - N.R.C. of Cana_ da, AP-PR 33, 1966.

[5] - BIRARDI, G. - Calculation of a block of strips based on uniformly distributed control points - Presented paper, X Congress of the ISP, Lisboa, 1964.

Annex 1



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Annex 2

punto numero: 482460.1

| | X 1630.000 | Y 4820.000 | Z | | Δx | Δy | Δz | note |
|---|---|---|---|---|---|---|---|---|
| A | 5880.65 | 61.97 | 118.92 | | | | | |
| B | 5880.50 | 62.02 | 119.15 | A-B | 0.15 | -0.05 | -0.23 | |
| C | 5880.60 | 62.14 | 118.99 | A-C | 0.5 | -0.17 | -0.07 | |
| D | 5880.53 | 62.59 | 119.48 | A-D | 0.12 | -0.62 | -0.50 | |
| E | 5880.48 | 61.96 | 119.10 | A-E | 0.17 | 0.01 | -0.18 | |
| F | 5880.31 | 61.67 | 119.19 | A-F | 0.34 | 0.3 | -0.27 | |
| G | 5880.03 | 62.78 | 119.22 | A-G | 0.62 | -0.81 | -0.30 | |

punto numero: 497123.1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | 1005.44 | 8142.58 | 312.88 | | Δx | Δy | Δz | |
| B | 1005.15 | 8142.70 | 312.69 | A-B | 0.29 | -0.12 | 0.29 | |
| C | 1005.11 | 8142.63 | 312.93 | A-C | 0.33 | -0.05 | 0.05 | |
| D | 1005.41 | 8142.82 | 312.81 | A-D | 0.03 | -0.24 | 0.17 | |
| E | 1005.27 | 8142.19 | 313.40 | A-E | 0.17 | 0.39 | -0.42 | |
| F | 1005.30 | 8142.54 | 312.74 | A-F | 0.14 | 0.04 | 0.24 | |
| G | 1005.24 | 8142.95 | 312.58 | A-G | 0.20 | -0.37 | 0.4 | |

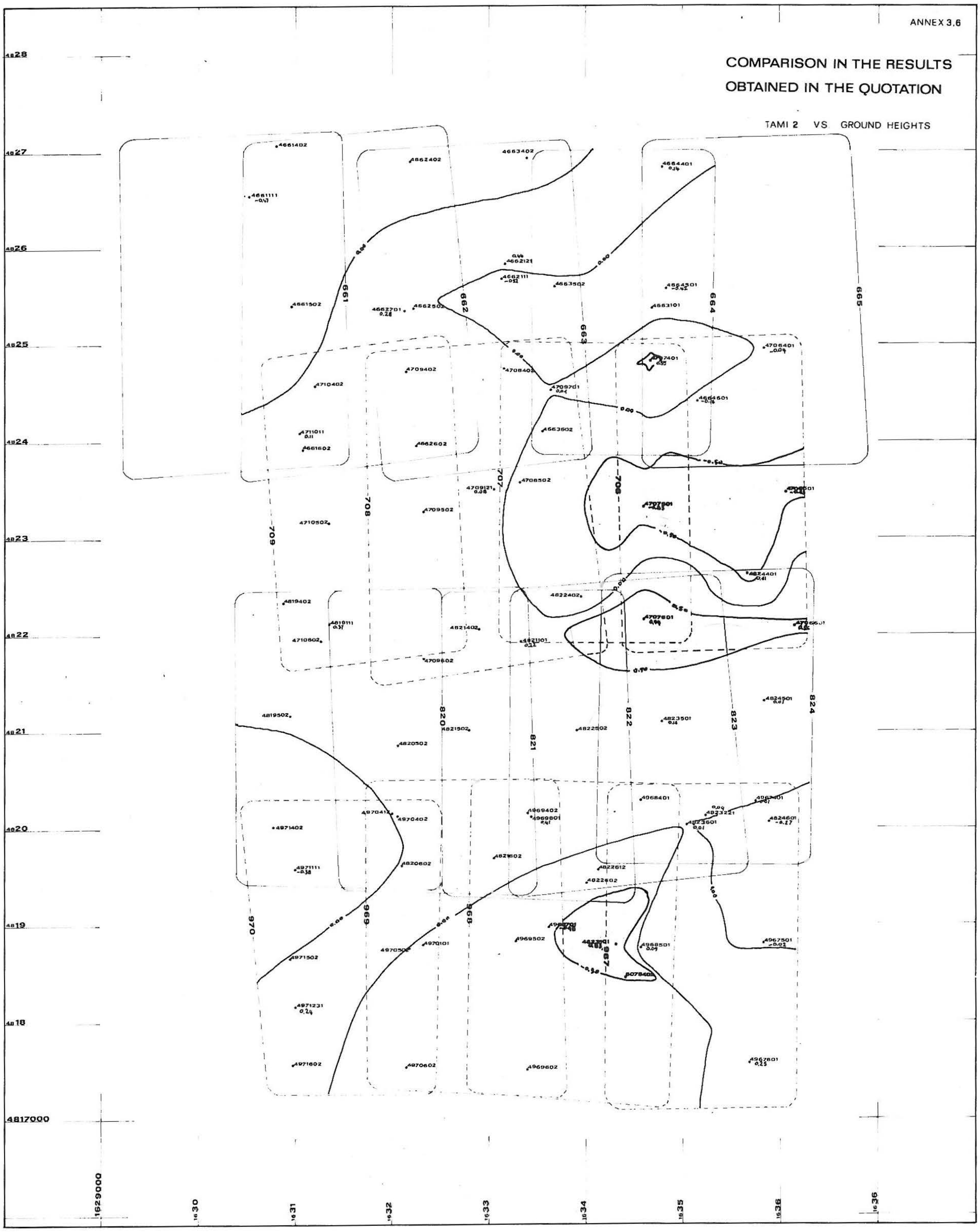
punto numero: 497122.1

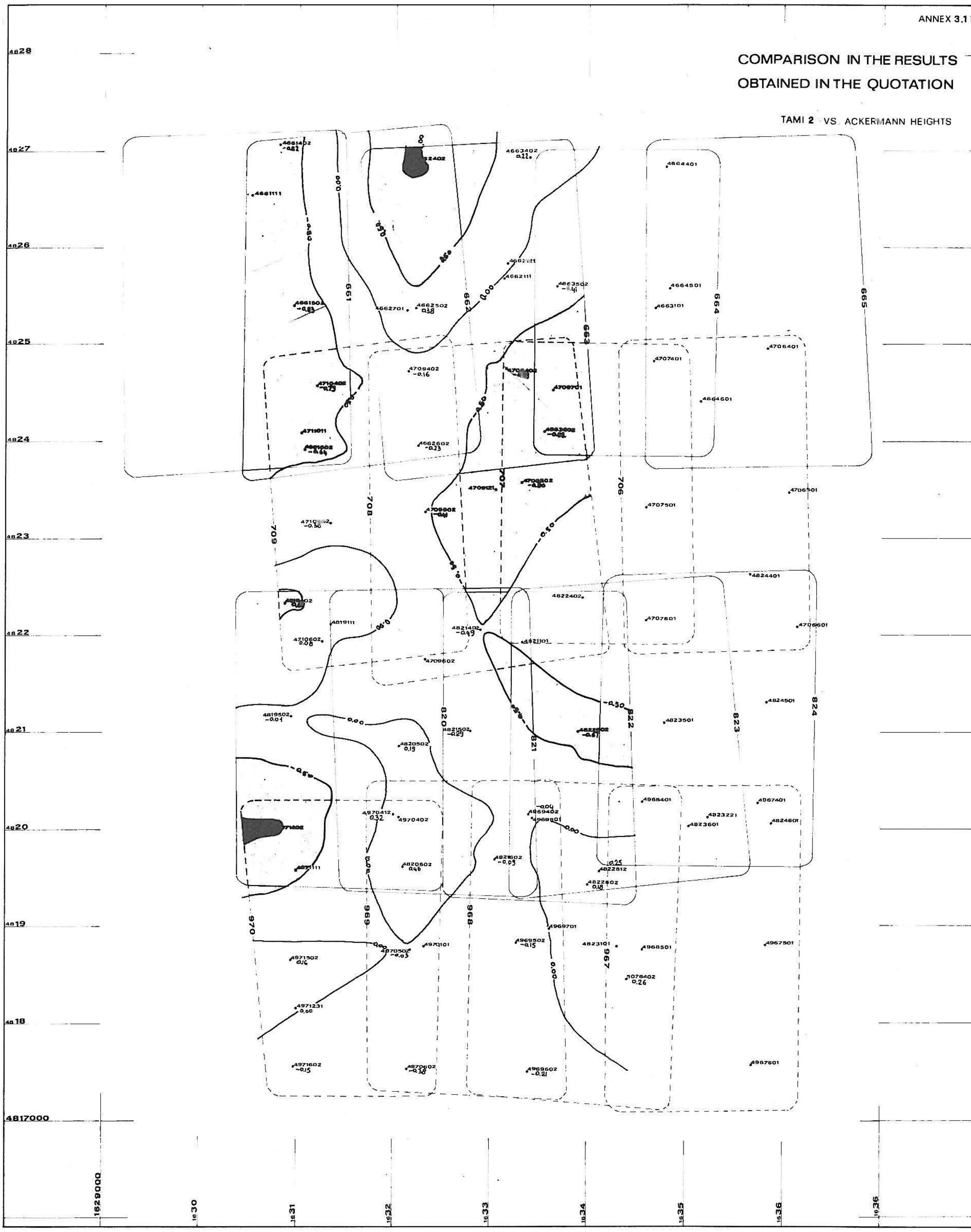
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | 978.20 | 8168.62 | 312.60 | | Δx | Δy | Δz | |
| B | 977.86 | 8168.25 | 312.95 | A-B | 0.34 | 0.37 | -0.35 | |
| C | 977.82 | 8168.15 | 313.10 | A-C | 0.38 | 0.47 | -0.5 | |
| D | 978.10 | 8168.92 | 313.01 | A-D | 0.10 | 0.30 | -0.41 | |
| E | 977.98 | 8167.91 | 312.88 | A-E | 0.22 | 0.71 | -0.28 | |
| F | 977.97 | 8168.10 | 312.99 | A-F | 0.23 | 0.52 | -0.39 | |
| G | 977.92 | 8168.54 | 312.85 | A-G | 0.28 | 0.11 | -0.25 | |

ANNEX 3.6

## COMPARISON IN THE RESULTS OBTAINED IN THE QUOTATION

TAMI 2 VS GROUND HEIGHTS




ANNEX 3.11
COMPARISON IN THE RESULTS OBTAINED IN THE QUOTATION
TAMI 2 VS. ACKERMANN HEIGHTS
4817000
1629000