A RIGOROUS CALIBRATION METHOD FOR DIGITAL CAMERAS

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ABSTRACT:

In this paper, the features of a digital camera and the error sources are analyzed. Attention is focused on finding a rigorous mathematical model for the calibration of digital cameras. We employ the DLT method for removing the deformation of CCD elements. In order to circumvent the high correlation between the DLT parameters and the camera distortion parameters, A sequential adjustment approach method is applied. Experimental results demonstrate the feasibility of our approach and the accuracies achieved. The tests prove that the mathematical model, originally developed for the EIKONIX camera, is suited for other digital cameras.

KEY WORLDS: Digital cameras, Image, Pixels,Distortions, Calibration,

INTRODUCTION

In recent years,digital photogrammetry has been developing well. High precision image matching and other digital processing methods render subpixel accuracy that may exceed one-tenth of a pixel. High accuracy in data processing may be limited by the scanning device for the digitized image however. Special attention must be given to the process of image data acquisition. Generally, there are two ways to get digital images during data acquisition: One is a direct method,that is, to obtain a digital image about an object using the digital camera directly. The other is an indirect way — first a photograph is taken and then the photograph is scanned by a digital camera. In both cases, the digital camera serves as an important instrument for photogrammetric data acquisition. The digital camera must be calibrated to obtain the correct relationship between the digital image and its corresponding digitized photograph (or object) before the digital image can be used for precision photogrammetric work.

In this paper, the features of digital cameras are discussed and the probable errors are analyzed. A mathematical model for calibration of digital cameras is presented. A sequential adjustment procedure, alternating two groups of unknowns, is employed in order to circumvent the high correlation between the DLT parameters and the camera distortion parameters. Experiences show that the mathematical model can be used not only for the calibration of a linear array camera but also for a 2-D array cameras and other similar digital cameras.

1. MATHEMATICAL MODEL FOR DIGITAL CAMERA CALIBRATION

1.1 Error Sources of a Digital Camera

In the following discussion, we will focus our attention on the features of the EIKONIX, a commonly used linear array digital camera in order to get a rigorous mathematical model for digital camera calibration.

As shown in Figure 1, the digital camera lens focuses an an object photograph on a movable stage. A linear array, mounted on the movable stage, is composed of 4096 elements. The array is driven by a step motor and scans the image on the stage to obtain the digitized image (digital image) of 4096 X 4096 pixels. Obviously, there are some errors during the imaging procedure. The errors mainly come from three sources: digitized image deformation,lens distortion and linear movement of the CCD array.

The digitized image deformation is equivalent to film deformation during optical projection. It is the deformation between the digitized array image and the optical image formed by the lens. It has the feature of affine deformation and is caused by two factors. First, because the array may not be parallel to the J axis of the scanned image a rotation is caused. Second, due to the different sizes of each array element a nonuniform deformation may be yielded.

The lens distortions consist of two parts: radial distortion and decentering distortion [4]. The radial distortion is symmetric with reference to the principal point of the optical image on the stage. The decentering distortion is the error caused by the inconsistency between the optical center and the geometric center of the lens. The decentering distortion includes both a radial and tangential component.

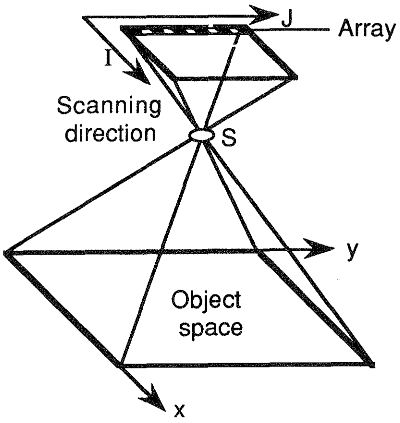

Figure 1 Imaging procedure of a digital camera

The linear movement is the error due to array driven by motor in scanning direction. The error is caused by a combination of various factors, such as power and step motor instability, and electronic noise.

1.2 Mathematical Model for Digital Camera Calibration

In order to calibrate the digital camera, the transformation between digitized array coordinates and object space (photograph) coordinates must taken. Generally, there are two steps: (1) to transform the digitized array coordinates (digitized image coordinates) into the optical image (imaged on the stage) coordinates, (2) to transform the image coordinates into the photograph coordinates. The transformation from digitized array coordinates to image coordinates can be expressed as:

$$\left.\begin{aligned} \overline{DI} &= e_1 + e_2\,DI + e_3\,DJ \\ \overline{DJ} &= e_4 + e_5\,DI + e_6\,DJ \end{aligned}\right\} \qquad (1)$$

$$DI = I - I_O$$

$$DJ = J - J_O$$

Where, $\overline{DI},\overline{DJ}$ represent transformed image coordinates; I and J express digitized image coordinates obtained from the display; e_1, e_2, e_3, e_4, e_5, e_6 express transformation coefficients; I_O, J_O are the coordinates of origin of the digital image corresponding to the optical image on the stage. When the lens distortion and the linear movement are not considered, collinearity equations can be written as:

$$\overline{DI} = -f_i\,\frac{\overline{X}}{\overline{Z}}$$

$$\overline{DJ} = -f_j\,\frac{\overline{Y}}{\overline{Z}}$$

i.e.

$$\left.\begin{aligned} e_1 + e_2DI + e_3DJ &= -f_i\,\frac{a_{11}(x-x_o)+a_{12}(y-y_o)+a_{13}(z-z_o)}{a_{31}(x-x_o)+a_{32}(y-y_o)+a_{33}(z-z_o)} \\ e_4 + e_5DI + e_6DJ &= -f_j\,\frac{a_{21}(x-x_o)+a_{22}(y-y_o)+a_{23}(z-z_o)}{a_{31}(x-x_o)+a_{32}(y-y_o)+a_{33}(z-z_o)} \end{aligned}\right\} \qquad (2)$$

where x, y, z are the coordinates of the photograph point in object space, a_{ij} represent the elements of the rotation matrix.

A nonmetric camera is used in the EIKONIX system. Owing to the absence of fiducial marks on the stage, the coefficients e_1 -- e_6 cannot be computed directly. A direct linear transformation (DLT) is employed so that digitized image coordinates are transformed directly into object space coordinates.

Eliminating DJ,DI from equations (2) and combining f_i, f_j and the elements of exterior orientation we obtain the DLT equations:

$$\left.\begin{aligned} DI &= \frac{L_1\,x + L_2\,y + L_3\,z + L_4}{L_9\,x + L_{10}\,y + L_{11}\,z + 1} \\ DJ &= \frac{L_5\,x + L_6\,y + L_7\,z + L_8}{L_9\,x + L_{10}\,y + L_{11}\,z + 1} \end{aligned}\right\} \qquad (3)$$

in which, $L_1,\ldots,L_{11}$ are the parameters for transformation between the digitized image and the photograph coordinates.

In this paper, the DLT parameters are used to determine the geometric transformation relationship between a photograph in object space and the corresponding digitized image, since the object space is a 2-D plane. In this case, the z coordinates are equal to zero for all the object control points. So, we obtain from equations (3):

$$\left.\begin{aligned} DI &= \frac{L_1 x + L_2 y + L_4}{L_9 x + L_{10} y + 1} \\ DJ &= \frac{L_5 x + L_6 y + L_8}{L_9 x + L_{10} y + 1} \end{aligned}\right\} \quad (4)$$

In order to obtain rigorous collinearity equations, the lens distortions and linear distortion of the CCD array must be considered. The distortions (δ_i,δ_j) can be expressed as:

$$\left.\begin{aligned} \delta_i &= \delta_{r\,i} + \delta_{d\,i} + \delta_m \\ \delta_j &= \delta_{r\,j} + \delta_{d\,j} \end{aligned}\right\} \quad (5)$$

Here, δ_{ri} and δ_{rj} represent radial lens distortions in x directions, δ_{di} and δ_{dj} express decentering distortions, δ_m is the linear movement in the scanning direction.

Generally, the radial lens distortion can be written as:

$$\delta r = k_1 r^3 + k_2 r^5 + k_3 r^7 \quad (6)$$

where, $k_1,\ldots,k_3$ are coefficients of radial distortion, and r is the radial distance referred to the image center:

$$r = \sqrt{(I - I_0)^2 + (J - J_0)^2}$$

The components of the distortion in x, y directions can be given by:

$$\left.\begin{aligned} \delta_{r\,i} &= \frac{DI}{r}\delta_r = DI\,(k_1 r^2 + k_2 r^4 + k_3 r^6) \\ \delta_{rj} &= \frac{DJ}{r}\delta_r = DJ\,(k_1 r^2 + k_2 r^4 + k_3 r^6) \end{aligned}\right\} \quad (7)$$

The decentering distortion can be expressed by:

$$\left.\begin{aligned} \delta_{d\,i} &= [1 + P'_3 r^2]\,[p'_1 (r^2 + 2DI^2) + 2p'_2\, DI\, DJ] \\ \delta_{d\,j} &= [1 + p'_3 r^2]\,[2p'_1 DI\, DJ + p'_2 (r^2 + 2DJ^2)] \end{aligned}\right\} \quad (8)$$

where, p'_1, p'_2 and p'_3 represent the coefficients of the decentering distortion correction.

Owing to a lack of detailed information of the linear CCD array movement with the EIKONIX camera, an analysis method is employed in order to remove the linear distortion of the array. First, we did not consider the linear distortion. Then, the residual errors were approached by a polynomial. A proper model was found:

$$\delta_m = m_1 I + m_2 I^3 + m_3 I^5 + m_4 I^7 \quad (9)$$

According to the above discussion, the totally rigorous mathematical model for digital camera calibration can be expressed as:

$$\left.\begin{aligned} DI - \delta_i &= \frac{L_1 x + L_2 y + L_4}{L_9 x + L_{10} y + 1} \\ DJ - \delta_j &= \frac{L_5 x + L_6 y + L_8}{L_9 x + L_{10} y + 1} \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} \delta_i = {} & k_1 (DI r^2) + k_2 (DI r^4) + k_3 (DI r^6) + \\ & p_1 (r^2 + 2DI^2) + p_2 (2DIDJ) + \\ & p_3 (2r^2 DIDJ) + p_4 (r^2 (r^2 + 2DI^2)) + \\ & m_1 (DI) + m_2 (DI^3) + m_3 (DI^5) + m_4 (DI^7) \\ \delta_j = {} & k_1 (DJ r^2) + k_2 (DJ r^4) + k_3 (DJ r^6) + \\ & p_1 (2DIDJ) + p_2 (r^2 + 2DJ^2) + \\ & p_3 (r^2 (r^2 + 2DJ^2)) + p_4 (2r^2 DIDJ) \end{aligned}\right\} \quad (11)$$

Comparing equation (11) with (8), we have:

$$p_1 = p'_1 \qquad p_2 = p'_2$$

$$p_3 = p'_2 . p'_3 \qquad p_4 = p'_1 . p'_3$$

In the discussion below, we define $L_1, \cdots L_{11}$ as the parameters of direct linear transformation (DLTP) and k_1, k_2, k_3, p_1, p_2, p_3, p_4, m_1, m_2, m_3, m_4 as the parameters of distortion correction (DSTP)

2. METHOD OF SOLUTION ABOUT THE MODEL

Analyzing equations (10) and (11), we find a high correlation between DLTP and DSTP. When no constraints are available in the adjustment system, it is impossible to obtain a correct solution. A sequential adjustment procedure, alternating two groups of unknowns, can be employed in order to circumvent the high correlation between DLTP and DSTP and to get a reliable solution.

Supposing the DSTP and the position (I_0, J_0) of the digital image center are known, then the observation equations for DLTP can be written as:

$$\left.\begin{aligned} DI - \delta i &= A/C \\ DJ - \delta j &= B/C \end{aligned}\right\} \tag{12}$$

Where,

$$\left.\begin{aligned} A &= L_1 x + L_2 y + L_4 \\ B &= L_5 x + L_6 y + L_8 \\ C &= L_9 x + L_{10} y + 1 \end{aligned}\right\} \tag{12-1}$$

The corresponding error equations in the matrix can be expressed as:

$$\mathbf{v}_l = \mathbf{G}_l \mathbf{u}_l + \mathbf{k}_l = \mathbf{0} \tag{13}$$

Similarly,the observation equations for DSTP can be given from formula (12) after obtaining DLTP.

$$\left.\begin{aligned} I - A/C &= I_0 + \delta_{ri} + \delta_{dj} + \delta_m \\ J - B/C &= J_0 + \delta_{rj} + \delta_{dj} \end{aligned}\right\} \tag{14}$$

As described in section 1.2, (I_0, J_0) is the center of the digital image and should be consistent with the center of the optical image on the stage. It is necessary to solute I_0, J_0 in equation (14) in order to calculate the distortions. The error equations in matrix form corresponding to (14) are represented as:

$$\mathbf{v}_d = \mathbf{G}_d \mathbf{u}_d + \mathbf{k}_d = \mathbf{0} \tag{15}$$

Alternating the above two groups of computations repeatedly until the variations of all the corrections for DLTP and DSTP become very small, we can get correct solutions for DLTP and DSTP.

Obviously, equations (13) are nonlinear. The solution by least squares must be a process of iteration.

For a single point, corresponding vectors in equation (13) can be expressed as:

$$\mathbf{G}_l = \begin{bmatrix} \frac{x}{C} & \frac{y}{C} & \frac{1}{C} & 0 & 0 & 0 & -x\frac{A}{C^2} & -y\frac{A}{C^2} \\ 0 & 0 & 0 & \frac{x}{C} & \frac{y}{C} & \frac{1}{C} & -x\frac{B}{C^2} & -y\frac{B}{C^2} \end{bmatrix} \tag{13-1}$$

$$\mathbf{u}_l = \left[L_1 \,......\, L_8 \right]^t \tag{13-2}$$

$$\mathbf{k}_l = \left[(DI - \delta_i) - \frac{A}{C} \quad (DJ - \delta_j) - \frac{B}{C} \right]^t \tag{13-3}$$

In equations (15),

$$\mathbf{G}_d = \left[\mathbf{R}_d \quad \mathbf{D}_d \quad \mathbf{M}_d \quad \mathbf{O}_d \right] \tag{15-1}$$

$$\mathbf{R}_d = \begin{bmatrix} DI\,r^2 & DI\,r^4 & DI\,r^6 \\ DI\,r^2 & DI\,r^4 & DI\,r^6 \end{bmatrix} \tag{15-1-1}$$

$$\mathbf{D}_d = \begin{bmatrix} r^2+2DI^2 & 2DIDJ & r^2(r^2+2DI^2) & 2r^2DIDJ \\ 2DIDJ & r^2+2DJ^2 & 2r^2DIDJ & r^2(r^2+2DJ^2) \end{bmatrix} \tag{15-1-2}$$

$$\mathbf{M}_d = \begin{bmatrix} I & I^3 & I^5 & I^7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{15-1-3}$$

Because k_1, k_2, k_3, p_1, p_2, p_3 and p_4 are very small, $\mathbf{O}_d$ can expressed as:

$$\mathbf{O}_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15-1-4}$$

The total solutions of the calibration model for a digital camera consist of equations (13) and (15) and their submatrices. Equations (13) includes an affine transformation and can be used for calibration of a scanner. Combining equations (13) and (15), the above model can serve as calibration of linear array camera and 2-D array camera. The procedure for the solution is shown in Figure 2.

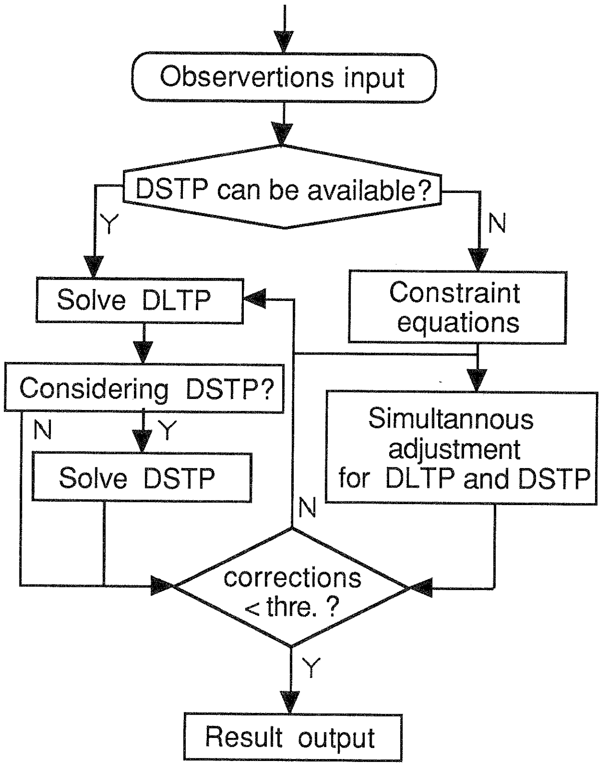

Figure 2. Procedure of calibration

3. TESTS AND ANALYSIS

According to the above mathematical model, several tests are made for three kinds of digital cameras. There are two purposes for the tests. First is to prove the correctness of the principle and method described in this paper. Second is to discuss the compatibility of the mathematical model .

3.1 Subpixel Location for Image Point

It is apparent that the accuracy of the calibration parameters is determined by the accuracy of digital image location. Because the resolution is limited, the subpixel location of the center of the image point is important in order to obtain more accurate image coordinates. For points larger than 1 pixel, a weighted method is used to determine the coordinates of the center of image point because an image point consists of n X m pixel. First, the histogram for each part of the digital image is calculated. Second, the average grey value A_g, B_g of all image points and the background are obtained. Then the subpixel location (I_c, J_c) is obtained:

$$\left.\begin{aligned} I_c &= \sum_{i=1}^{m} W_p P_i \\ J_c &= \sum_{j=1}^{n} W_p P_j \end{aligned}\right\} \qquad (16)$$

$$W_p = \frac{G_p}{A_g - B_g} \qquad (16\text{-}1)$$

where, P_i, P_j are the coordinates of a pixel with grey value G_p larger than a threshold value T; W_p represents the weight of the pixel(P_i, P_j); T is determined by A_g and B_g. Generally, $T = 0.5(A_g + B_g)$.

For a normal photograph, only a few control points are needed; the image coordinates can be measured directly from the display . For calibration of the grid plate, the image coordinates of points may be obtained by image matching.

The coordinates of all photograph points can be obtained from analysis plotter.

3.2 Test 1, EIKONIX Camera

Test 1 includes four photographies and a grid plate. The corresponding digital images are from an EIKONIX digital camera. In Table 1 calibration model I expresses that no distortion and no linear movement are considered during calibration. Model II the distortions and linear movement are considered. The calibration accuracy is checked by individual check points. Tests show that the EIKONIX camera may have distortions of one to two pixels. Obviously, it cannot satisfy the high accuracies required for digital photogrammetry. The calibration model in this paper provide subpixel accuracies that exceed one-tenth of a pixel. If the position of the EIKONIX is kept, the parameters of distortion and linear movement from the grid plate can serve as the accurate calibration parameters for the digitized photograph. In this case, only fiducial points may be used to determine DLT parameters.

3.3 Test 2, PHOTO SCAN Scanner

Test 2 indicates that the mathematical model in this paper can serve as scanner calibration. In this case, no camera distortions are introduced, and the digital image is related to the photograph as an affine transformation. The test shows that calibration for scanner using equations (4) renders subpixel accuracy that may exceed one-fourth of a pixel.

Table 1 EIKONIX camera calibration

Photo No.	Number of points		Calib. model	Error	Photo to image (pixel)				Image to photo (µm)				δ_0
					CP		CHP		CP		CHP		
	CP	CHP			I	J	I	J	x	y	x	y	
181	13	19	I	RMSE	0.70	0.35	0.73	0.36	42.7	21.6	31.8	21.6	0.071
				Max.	1.31	0.52	1.06	0.60	79.2	32.5	60.3	36.0	
			II	RMSE	0.08	0.09	0.08	0.09	4.7	5.3	4.8	5.4	
				Max.	0.13	0.16	0.15	0.18	8.3	9.6	9.0	10.7	
183	13	16	I	RMSE	0.64	0.56	0.76	0.71	39.3	34.1	45.5	42.5	0.072
				Max.	1.08	1.15	2.00	1.45	66.4	69.9	120.0	87.0	
			II	RMSE	0.08	0.09	0.07	0.11	4.8	5.5	4.2	6.6	
				Max.	0.15	0.15	0.17	0.22	9.6	9.1	10.2	13.2	
43	13		I	RMSE	1.02	1.00			61.7	60.4			0.086
				Max.	2.00	1.56			120.0	93.0			
			II	RMSE	0.09	0.11			5.8	6.5			
				Max.	0.15	0.20			9.0	11.9			
44	14		I	RMSE	1.07	1.12			65.0	68.1			0.081
				Max.	1.68	1.59			102.3	95.0			
			II	RMSE	0.09	0.10			5.8	5.9			
				Max.	0.19	0.18			11.5	11.3			
Grid stage	24	383	I	RMSE	1.15	0.74	1.04	1.18	69.8	45.0	62.1	69.5	0.053
				Max.	2.42	2.05	2.72	2.47	146.2	123.6	163.2	148.1	
			II	RMSE	0.06	0.07	0.09	0.11	4.1	4.3	5.4	6.5	
				Max.	0.12	0.16	0.34	0.26	7.6	10.2	20.4	15.6	

Table 2 Scanner calibration

Photo No.	Number of points		Calib. model	Error	Photo to image (pixel)				Image to photo (µm)				δ_0
					CP		CHP		CP		CHP		
	CP	CHP			I	J	I	J	x	y	x	y	
193	13	22	I	RMSE	0.12	0.18	0.19	0.24	7.0	10.6	11.3	14.2	0.125
				Max.	0.25	0.31	0.37	0.52	15.5	18.5	22.0	31.1	
195	12	30	I	RMSE	0.18	0.23	0.19	0.25	10.6	13.8	11.2	14.9	0.176
				Max.	0.32	0.39	0.45	0.58	19.6	22.3	27.0	34.5	

The parameters from tests 1 and 2 serve as a translation between the digital image and photograph to generate an epipolar image. The results show that the y parallax of epipolar image with 4kX4k pixels renders subpixel accuracy.

3.4 Test 3 AQUATV Camera

AQUATV is a 2-D array camera with 512 X 512 pixels. The size of a pixel in I and J directions is 11.4µm and 7.8µm respectively. A grid plate with 1m X 1m serves as the digitized object. Test results show that the parameters for linear movement are near zero. That confirms the 2-D array cameras have no error caused by linear movement. We obtained subpixel accuracy better than one-tenth of a pixel.

4. CONCLUSIONS

1. The experimental results demonstrate the feasibility of the mathematical model presented in the paper. They can tolerate high precision digital processing in photogrammetry.

Table 3 AQUATV camera calibration

Object No.	Number of points	Calib. model	Error	Object to image (pixel)		Image to Object (mm)		δ_o (pixel)
				I	J	x	y	
TA2	13	I	RMSE	0.07	0.10	0.265	0.281	0.026
			Max.	0.15	0.17	0.565	0.545	
		II	RMSE	0.03	0.03	0.121	0.107	
			Max.	0.08	0.06	0.356	0.214	
TA3	13	I	RMSE	0.08	0.06	0.430	0.249	0.025
			Max.	0.17	0.12	0.945	0.465	
		II	RMSE	0.03	0.03	0.134	0.124	
			Max.	0.06	0.05	0.285	0.172	
TA4	13	I	RMSE	0.08	0.09	0.422	0.324	0.026
			Max.	0.16	0.16	0.786	0.559	
		II	RMSE	0.03	0.03	0.162	0.096	
			Max.	0.07	0.06	0.354	0.212	

2. The sequential approach of altenating two groups of unknowns lends itself into a reliable, stable solution.. The method can also reduce the number of control points needed to calibrate. Generally, a satisfactory result can be obtained using 13 control points distributed uniformly.

3. Test results indicate that the method can be used to calibrate different cameras, for example, linear array, cameras, 2-D array cameras, and scanners.

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