REDUCTION OF SYSTEMATIC ERRORS IN GPS-BASED PHOTOGRAMMETRY BY FAST AMBIGUITY RESOLUTION TECHNIQUES

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0. ABSTRACT

A technique to resolve the integer cycle ambiguities of kinematic GPS-Phase observations in a kinematic environment is examined. Further the statistical background and the practical implementation of the used algorithm is presented. The preconditions to resolve the correct set of ambiguities are analyzed. It is shown that a positioning accuracy of 10 cm can be achieved under most conditions, if the presented algorithm is used. The accuracy and reliability of the algorithm is assessed by examining kinematic data of a photogrammetric testflight, a Laser-Profiling flight and static data which was processed as if it were kinematic. Further the impact of such an accurate positioning capability is shown on selected examples of daily photogrammetric work.

Keywords: GPS, Algorithm, Sensor

1. INTRODUCTION

Recently kinematic position determination with NAVSTAR/ Global Positioning System (GPS) has gained great attention for photogrammetric purposes. The application of GPS in aerial- (or space) photogrammetry can be mainly distinguished in 3 seperate tasks:

- a.) Precise survey flight navigation
- b.) GPS camera positioning for aerial triangulation
- c.) GPS positioning of other photogrammetric sensors

As the requirements for survey flight navigation (task a.) are usually not very stringent, in most cases the positions can be determined with pseudorange observations and a standard real-time navigation algorithm (e.g. Wells [1986]). In contrary, the applications b.) and c.) are more demanding with respect to the needed positioning accuracy. Depending on the photoscale or the type of photogrammetric sensor used, the accuracy requirements for the positions, at the time exposure, can range from a few centimeters to a few meters (Table 1).

map scale	accuracy X,Y	accuracy Z
1:50000	15 m	8 m
1:25000	5 m	4 m
1:10000	1.6 m	0.7 m
1:5000	0.8 m	0.35 m
1:1000	0.4 m	0.15 m
Laser-Profiling	0.1 m	0.1 m

Tab.1 Required positioning accuracy in photogrammetry

The table shows that highly accurate positions are needed for some specific photogrammetric applications. To achieve the needed decimeter positioning accuracy the use of GPS phase observable is absolutely necessary. In Equation 1 the standard observation equation for the between station, between satellite double-differenced carrier phase observation is formulated.

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla N + \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \Delta\nabla d_{Orbit} + \Delta\nabla d_{noise} \qquad (1)$$

If the seperation between the monitor and the remote receiver is not very large (< 50km) the terms $\Delta\nabla d_{ion}$, $\Delta\nabla d_{trop}$, $\Delta\nabla d_{orbit}$ and $\Delta\nabla d_{noise}$ are usually small. For the sake of simplicity, these terms are neglected in the further considerations.

In order to exploit the high accuracy of the phase observations the carrier phase ambiguities $\Delta\nabla N$ have to be determined. As long as no cycle slip[1)] or loss of phase lock occurs, the ambiguities remain constant integer values. In principle, they can be estimated and fixed at the beginning of a continous sequence of observations in a static initialization, but due to banking angles in flight turns and the highly kinematic environment losses of phase lock and cycle slips are frequent in photogrammetric applications. Hence, in most cases there is a need to reinitialize the ambiguities while the aircraft is moving. Therefore, an approach to resolve the correct cycle ambiguities on the fly for each satellite-station pair seems to be essential.

Prior to discussing the details of ambiguity resolution on the fly, it should be demonstrated what kind of error effects are to be expected if the ambiguities are fixed to the wrong values. Figure 1 shows the position error components in WGS - X,Y,Z direction if the cycle ambiguities are falsified by 2 cycles each.

The GPS data which was used to demonstrate the error effects was gathered on a 2 m static baseline on March 15, 1992. The originally static data was processed as if it were kinematic. For each observation epoch a set of coordinates was computed.

Analyzing the first figure it can be seen that the effects

[1)] Discontinuity in the integer number of cycles in the measured carrier beat phase (Wells et al. [1986]).

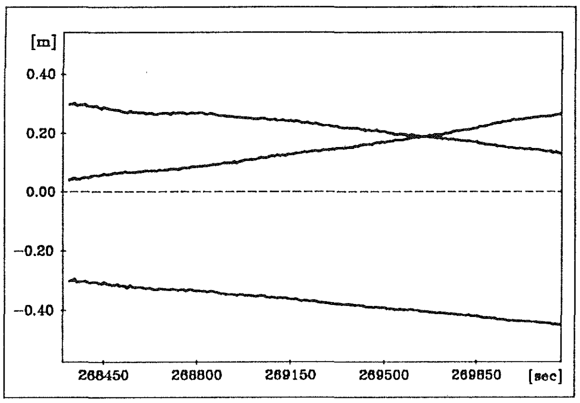

Fig. 1 Systematic Error Effects Due to Incorrect Ambiguities

of incorrect cycle ambiguities over shorter periods of time are in first approximation straight lines. A similar behaviour could be observed in several different data sets, if the cycle ambiguities were falsified by a certain amount. The magnitude of the offset and the drift compared to the true values can be considerable, depending on the satellite geometry and the size of the ambiguity error.

2. DETERMINATION OF CARRIER PHASE CYCLE AMBIGUITIES IN A KINEMATIC ENVIRONMENT

The correct determination of the cycle ambiguities is still one of the most critical tasks in GPS based photogrammetry. Until recently photogrammetrists were dealing with this problem in different ways:

- A survey flight is flown very carefully with flat turns, so that at least 4 satellites remain trackable during the entire mission. If a loss of phase lock occured the ambiguities were determined via the observations of the continously tracked satellites. The drawbacks of this approach can be summarized as: not very reliable, because the entire survey flight must be repeated if only 3 satellites (or 4 satellites with weak geometry) are available for a short period of time; the flight path is unnecessarily longer compared to the standard case.
- Additional sensors are used to bridge a loss of phase lock, to detect cycle slips or to aid positioning in times of weak satellite geometry. In principle this integrated sensor approach would satisfy the requirements for a reliable and accurate positioning system, but aiding sensors (e.g. INS) which provide a comparable positioning accuracy to GPS are usually very expensive.
- A further method which is also based on an additional sensor, but which is very interesting from the photogrammtrists point of view, is the combined block adjustment of photogrammetric and GPS data. This method which was developed at Stuttgart University (Frieß [1991], Ackermann[1990]) models the error effects of incorret cycle ambiguities in the photogrammmetric bundle block adjustment. For each photogrammetric strip 6 additional unknowns (3 Offsets, 3 Driftparameters) control the propagation of the mentioned systematic errors. Although more unknowns have to be estimated in the aerial triangulation, the photogrammetric block is stabilized via the additional GPS observations. It is even possible to reduce the number of ground control points to a minimum of 4 points in the block corners, if two additional cross strips are flown (Ackermann [1990]). From the practical point of view this method is very suitable for aerial triangulation applications, due to the extensive reduction of ground control and no limitations for the survey flight (steep turns may be flown, continous phase lock is just required within a photo strip, no static initialization is required). Figure 2 shows the basic observation equation that is used to introduce the GPS Positions in the aerial triangulation.

Principal observation equations for GPS data in combined block adjustment

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}_{GPS} = \begin{bmatrix} X_U \\ Y_U \\ Z_U \end{bmatrix} + (1+\Delta m) \cdot R_D \cdot \left[\begin{bmatrix} X_{PZ} \\ Y_{PZ} \\ Z_{PZ} \end{bmatrix}_{AT} + R_B \cdot \begin{bmatrix} x^A_{PZ} \\ y^A_{PZ} \\ z^A_{PZ} \end{bmatrix}_B \right] + \begin{bmatrix} a_X \\ a_Y \\ a_Z \end{bmatrix} + \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix} \cdot t$$

GPS antenna coordinates — datum transformation — exterior orientation — GPS antenna eccentricity — GPS drift parameters

Fig. 2 Observation Equation for GPS Positions in the Aerial Triangulation

The only disadvantage of this method is, that the idea of correcting the GPS positions with photogrammetric data is not easily transferable to non-imaging remote sensing applications (e.g. Laser-Profiler, Laser-Scanner, Multi-Spectral-Scanner). Further it would be beneficial if the GPS positions could be directly used in the aerial-triangulation without the need to estimate 6 additional unknowns for each photogrammetric strip.

Recent development of new methods and algorithms to obtain the correct set of carrier phase cycle ambiguities in a moving environment might overcome the stated problems. In the following chapter the theoretical and statistical background of the implemented algorithm to resolve the ambiguities on the fly is presented.

3. AMBIGUITY RESOLUTION APPROACH

Several methods to resolve the GPS carrier phase ambiguities have already been proposed by different authors (e.g. Hatch [1990], Frei/Beutler [1990], Counselman/Gourevitch [1981]). The mentioned authors developed algorithms which share some basic principles to distinguish between the correct set of cycle ambiguities and the incorrect ones:

- The accuracy of the double differenced carrier phase observation should be below 1.5-2 cm. The observation accuracy is used in statistical tests to determine wether a set of cycle ambiguities is potentially correct or false.
- The compatibility between carrier phase observations and pseudorange observations must be guaranteed. Usually an adjusted pseudorange position and its associated covariance matrix are taken to check wether a set of cycle ambiguities is potentially correct or not.
- If more than 4 satellites are observed only a subset of 4

cycle ambiguities (primary satellites) are independent of each other. If 4 correct ambiguities are known a unique position can be determined. Using the accurate and unique position all additional ambiguities can be computed.
- The most probable set of primary cycle ambiguities is the one which yields the smallest σ_0 of all observations in all epochs under consideration, providing no cycle slip or loss of phase lock occured on the primary satellites.
- Observing dual frequency data (widelane ambiguities) aids the ambiguity search considerably.

The algorithm which is presented here, works with a combination of the above mentioned features. The statistical tests which are used to distinguish between the correct and the false ambiguity sets form 4 rejection criteria:

Test 1: Compatibility of code and carrier measurements: The position computed from phase oservations using a potential solution must be within the probability region of the associated pseudorange position. Mathematically this rejection criteria can be defined as follows.

$$\sqrt{(X_{Phase}-X_{Code})^2 + (Y_{Phase}-Y_{Code})^2 + (Z_{Phase}-Z_{Code})^2} \leq const \sqrt{\sigma_X^2+\sigma_Y^2+\sigma_Z^2} \qquad (2)$$

$\sigma_x,\sigma_y,\sigma_z$ are the a posteriori variances of the pseudorange solution. With the constant the statistical properties of the rejection criteria can be chosen.(e.g. const = 3 gives a statistical probability of 99.9%)

Test 2: Compatibility of a priori and a posteriori variance of a single epoch: If the a posteriori variance of the phase observations of the actual epoch is not compatible with the known a priori variance of the double differenced phase observations (e.g. < 1.5cm), the set of ambiguities has to be rejected. To assess the compatibility a χ^2 - Test is used.

$$\frac{(n-1)\ \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_{(df,1-\alpha)} \qquad (3)$$

Test 3: Compatibility of a single observation with the a priori variance factor: With this test the single observation is tested. It has to be emphasized here, that the residual has to be standardized before it can be assessed. The size of the residual of a phase observation can change significantly if the satellite geometry is changed. Statistically the standardized residual is normally distributed. Hence the boundary values of the Gauss-distribution can be taken as a test-value.

$$\frac{|v|}{\sigma_v} \sim N_{1-\alpha} \qquad (4)$$

Test 4: Compatibility of a priori and a posteriori variance of all epochs so far: Test 4 and Test 2 are absolutely similar but in Test 4 all observations which are gathered up to the actual point in time are taken into account. Here also a χ^2 distribution is used as rejection criteria.

A further test to assess the reliability with which the potentially best ambiguity set has been determined was proposed by Frei/Beutler [1990]. The ambiguity set with the minimal variance factor using all observations is tested against the set with the second smallest variance factor. This test gives an indication whether the potentially best solution is significantly better as all others. If the test value in equation 5 is larger than the associated F-Test value, the user can assume that the selected solution can be used for position computations.

$$\frac{\hat{\sigma}_2 - \hat{\sigma}_1}{\sqrt{\dfrac{\hat{\sigma}_2^2}{2n_2} + \dfrac{\hat{\sigma}_1^2}{2n_2}}} \sim F_{n_1,n_2,1-\alpha} \qquad (5)$$

The flowchart in figure 3 shows the actual implementation of the ambiguity resolution algorithm.

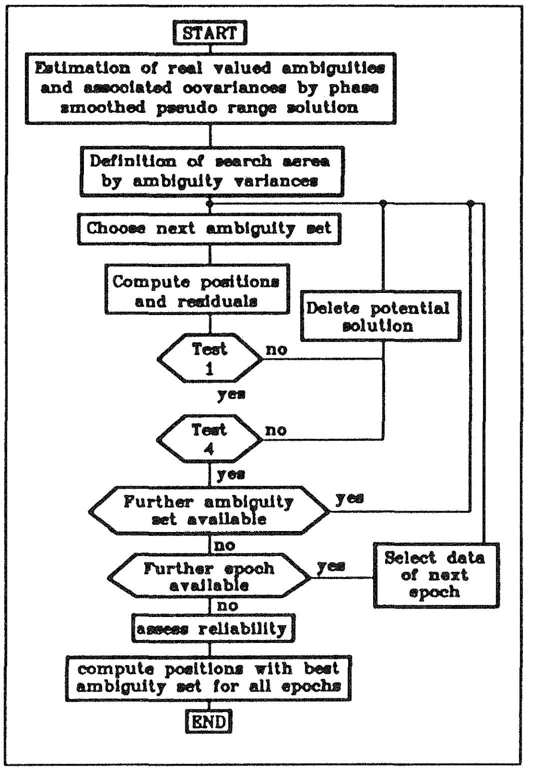

Fig. 3 Implementation of the ambiguity resolution

4. EMPIRICAL RESULTS WITH THE AMBIGUITY RESOLUTION ON THE FLY

The algorithm which was introduced in chapter 3 was tested with several sets of real GPS data. Intentionally only L1 / C/A Code data was used for the analysis, to show that the algorithm is also working in a worst case scenario. Table 2 summarizes the data sets in more detail.

Although the data sets 1 - 3 are originally static, they were processed as if they were kinematic to show the properties of the algorithm. The performance criteria which were analyzed in more detail for each data set are the convergence behaviour, the power of tests and the reliability of the correct estimation.

Data sets 1,2,3:
The 3 static data sets were mainly used, to have well controlled test conditions with different baseline lengths. The data for all three baselines was gathered with ASHTECH L-XII receivers.

Nr	Date	rec. inter [sec]	Nr. of epochs	line length [km]	Obs.
1	7.1.1992	1	2349	0.002	L1,C/A
2	4.4.1991	10	111	1.8	L1,C/A
3	10.10.1991	20	70	3.4	L1,C/A
4	13.3.1992	0.6	3416	kinem.	L1,C/A
5	5.8.1991	0.5	7000	kinem.	L1,C/A

Tab.2 Description of data sets

The reference positions were computed using standard, commercial software for baseline computations (ASHTECH Program Linecomp). In all analyzed cases the algorithm under consideration was able to resolve the correct set of cycle ambiguities. Figure 4 shows the differences (3km baseline) between the results obtained by ASHTECH'S Linecomp and the ambiguity resolution algorithm. Further the figure shows the convergence behaviour using some standard statistical assumptions for the rejection criteria (Variance of double differenced carrier phase: 1.5cm ; confidence region for code solutions 99.9%).

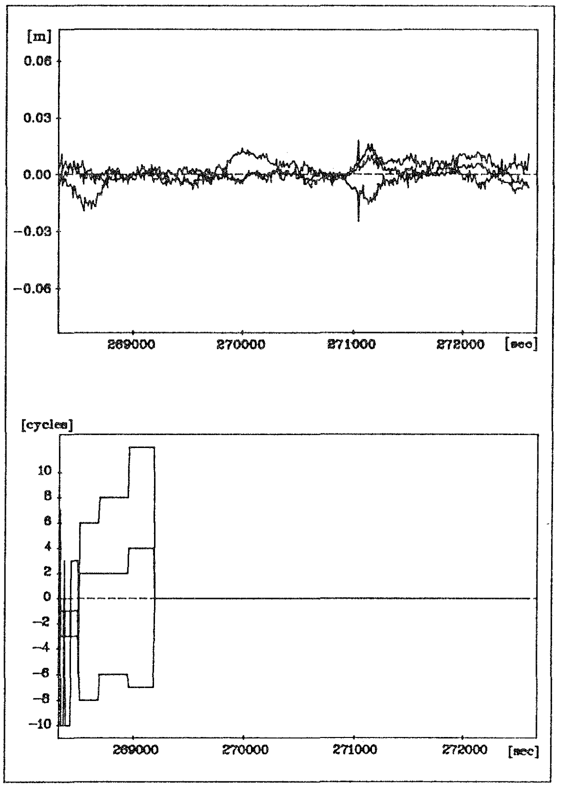

Fig. 4 Accuracy and Convergence Behaviour of Ambiguity Resolution

Although the accuracy of the algorithm seems to be very high, the convergence behaviour was not very satisfying. Providing that there should not be any restrictions to the flight behaviour during survey flights, the aim for photogrammetric applications must be, that the cycle ambiguities can be resolved within the length of a single photogrammetric strip. Under special conditions the convergence of the algorithm can be accelerated by decreasing the test values for the rejection criteria in tests 1-4, but there would be no guarantee that the correct ambiguity solution is not rejected. The performance will always be a trade-off between convergence speed and reliability, and the user has to make his own decision what is more important to him. In the tests which are presented here the emphasis was laid on the reliable estimation of the correct carrier phase ambiguities. The reliability analysis for the three processed baselines indicated significant differences between the potentially best and the second best solution. Table 3 summarizes the Fisher-Test values and the a posteriori variance factors for the tested data sets.

Nr.	F-Test	Sigma 0
1	98.2	0.2 cm
2	42.3	0.5 cm
3	6.6	0.4 cm

Tab.3 Reliabilty of estimation

Data set 4:
This kinematic data set was observed in a airborne Laser-Profiling mission to derive a digital elevation model in forest aereas from laser measurements (Lindenberger [1991]). 10-channel Sercel receivers were used in the aircraft and on the reference station. Altogether 3416 epochs with a 0.6 sec data rate were observed (35 min). 6 Satellites were tracked continuously. As in Laser-Profiling missions no direct control for the GPS derived positions can be made, the data set was divided in two subsets each of approximately 15 minutes length. The two data sets were processed with the described algorithm. Providing no cycle slips occured on the primary satellites the program should converge to the identical sets of carrier phase cycle ambiguities. Although the identity of the ambiguities was achieved with the algorithm, the best set of cyle ambiguities was not significantly better than the second best (see Table 4.). Hence, without any exterior control it can not be reliably distinguished which one of the ambiguity sets is correct.

best solution	-3608143 4636099 2721870	sigma 0 1.38 cm	F-Test Value
second best solution	-3608143 4636098 2721871	sigma 0 1.39 cm	1.10

Tab.4 Estimated Ambiguity Sets and Reliability Values

Data Set 5:
A more complete control for the performance of the algorithm with a true kinematic data set could be made with the well controlled aerial triangulation testflight GLANDORF 1991. The coordinates of the airborne GPS antenna, at the time of exposure, were determined with a standard aerial triangulation. The block, with image scale 1:8000, consisted of 14 strips

which were aligned crosswise (Frieß [1992]). Totally 140 images with 17 horizontal and 43 vertical control points were processed. The photogrammetric measurements reached a theoretical standard deviation of 0.12 m for the antenna phase center coordinates. To assess the absolute GPS positioning accuracy in a reference coordinate system, the photogrammetrically determined positions were directly compared with the results of the ambiguity resolution algorithm. From the entire data set a subset of 5 strips was chosen in which 4 satellites were observed continously, and a maximum of 6 satellites for more than 35 minutes was reached. Entirely the analyzed data set had 7000 epochs with a 2 Hertz update rate. On purpose the rejection criteria were chosen very weak, so that a total of 170000 potential solutions were processed. Figure 5 shows the differnces between the GPS- and the photogrammetric antenna coordinates in Gauss-Krüger X,Y,Z coordinates.

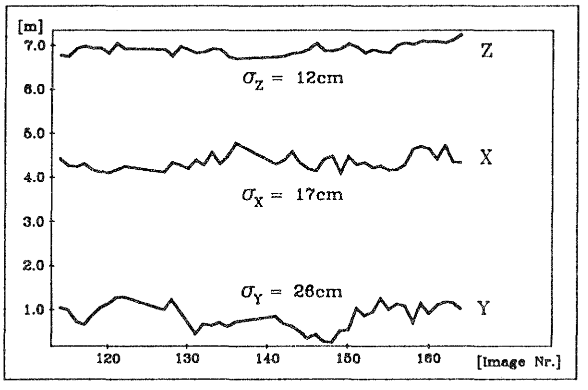

Fig. 5 Coordinate Differences between aerial triangulation and ambiguity resolution algorithm

Remarkable are the fairly big offsets in all three coordinate directions (X~4m,Y~1m,Z~7m). Further it can be seen that no time dependent drift errors are included in the computed differences. Although the offsets exist, it is believed that the estimated ambiguity set is the correct one. If the cycle ambiguities would be incorrect, a drift similar to the one in figure 1 would be unavoidable. A possible explanation is, that the reference position to which all positioning computations are tied, is incorrect by the observed offsets. Most likely the differences result from the transformation between the two coordinate systems which were used for the computations (aerial triangulation = Gauß-Krüger , GPS = WGS 84). To determine the 7 transformation parameters only a small geodetic network was measured, therefore the adjusted datum shift was only weakly determined. If the offsets are removed from the observed coordinate components, a variance of 10-20 cm remains. After removing the standard deviation of the photogrammetric control measurements an accuracy of 10 cm results for the GPS positions.

5. CONCLUSIONS AND IMPACT OF HIGH ACCURATE GPS-POSITIONS ON PHOTOGRAMMTERY

In the previous chapter it was demonstrated with several examples, how GPS can be used as a highly precise positioning tool. Under standard operating conditions 3 of the 6 exterior orientation elements for photogrammetric sensors can be observed with an accuracy of 10 cm. Several authors (Frieß [1990] , Ackermann [1990]) showed already that these orientation elements can be used advantageously. The directly measured camera projection centers can reduce the need for ground control extensively. The impact of directly measured projection center coordinates is best demonstrated using the GPS-observations of the controlled testflight GLANDORF (data set 5). The computed positions were introduced in an aerial triangulation with only 4 ground control points in the block corners. Only six additional unknowns have been estimated to account for an additional datum transformation (compare Fig. 5). The results of this minimal ground control block configuration was compared to a standard aerial triangulation. Empirical accuracies could be computed by analyzing the horizontal and vertical control points which were not used in the GPS - block. From Fig. 6 it can be seen that the combined bundle block adjustment with a minimal ground control configuration achieves a comparable accuracy to the conventional case.

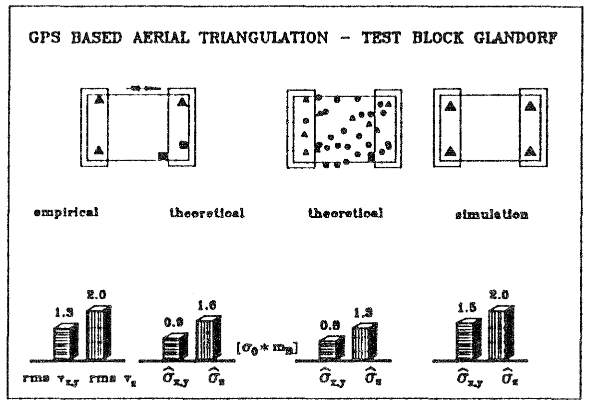

Fig. 6 Comparison of standard aerial traingulation with GPS based aerial triangulation

Not only for aerial triangulation, but also for the direct exterior orientation of new sensors (e.g. Laser Profiler) the GPS positioning capability has been used with a lot of success. With the Laser Profiler it is possible to measure directly a digital elevation model in aereas where conventional photogrammetry fails (dense woods, coastal regions). A more detailed description of the results with this new photogrammetric sensor system can be found in Ackermann/Lindenberger/Schade [1992].

In conclusion it has been shown that GPS Positioning for photogrammetric sensors is possible with an accuracy of 10-20 cm under conditions which mostly apply. GPS opens new applications in photogrammetry or makes existing applications more cost effective. The economic benefit of GPS by reducing ground control is extensive. The software for relative kinematic GPS positioning and combined block adjustment as well as cheaper GPS hardware and an improving satellite geometry makes the NAVSTAR/Global Positioning System ready for practical application in photogrammetry.

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