**Block Adjustment Including GPS Observations of the Photo Stations**

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**ABSTRACT:**

The National Land Survey of Sweden has lately begun a series of attempts to use kinematic GPS navigation of the aircraft. A second step is to get coordinates of the photo stations. This paper describes the theoretical tests done to investigate what results can be expected with GPS measurements combined with blockadjustment. An existing bundle adjustment program, GENTRI, is revised to include GPS observations. Simulated flight data is used to evaluate what effects different parameter accuracies, different ground control configurations and different block configurations will have on the result.

When using only four ground control points, one in each corner of the block, combined with GPS measurements the height and plane precision in the new points will be very close to what traditional blocktriangulation without GPS measurements can perform. Two crossing strips in each end of the block will give a higher precision of the new points than two crossing height chains. If the precision in the GPS measurement can be maintained when flying on higher heights it will have a positive effect on the result. When changing the block length or the block width no significant trends concerning the new points precision have been proved.

GPS measurements can be used in combination with blocktriangulation and its advantage is mainly that it can reduce the amount of field work, both measuring points geodetically, targeting them and guarding them.

**KEY WORDS:** Aerotriangulation, Blockadjustment, NAVSTAR GPS, Additional observations

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1. **INTRODUCTION**

1.1 The NAVSTAR GPS

Since the Department of Defense, USA, started to plan the NAVSTAR GPS (NAVigation System with Time And Ranging Global Positioning System) in the 1970's a number of civilian implementations has been developed. When the system is fully developed there will be 24 satellites placed in orbits covering the earth. That means that at least five satellites will be visible above the horizon wherever on earth you are. This will naturally lead to more efficient use of the system and also to higher accuracies of the measurements.

1.2 Kinematic GPS positioning

For kinematic GPS positioning ranges to satellites are used. If the satellites orbits are known and the ranges to three different satellites are measured it is possible to determine the position in space of your receiver. However, the receiver clock error will make it necessary to make observations to four satellites to determine the clock error - pseudoranging.

The absolute precision of the coordinates you get from kinematic positioning with one receiver can reach an accuracy of a few metres. You can get a more precise absolute position if a receiver is placed on a ground control point simultaneously; single differences positioning. In that way the position of the aircraft relative the control point can be determined more precisely. The precision will increase when decreasing the duration of the measurements and when decreasing the distance between the ground point receivers and the aircraft.

Previous tests show that a precision of 0.10 metre of the GPS measurements can be reached (Friess 1990) (Jacobsen 1991) if post processing is done to model the drift. This fact tells us that GPS measurements combined with blocktriangulation can be used.

1.3 GPS and aerotriangulation

GPS in aerotriangulation has so far mostly been measurements of ground control points. Also kinematic positioning, used for navigation of the aircraft to get the exposures at the right position, is in development and some experiments in using GPS measured coordinates of the photo stations as additional observations in block adjustments have been carried out (Jacobsen 1991).

1.4 Short description of the tests

Theoretical tests of using GPS measurements of the photo stations combined with blocktriangulation have been performed. As a base, a bundle adjustment program, GENTRI (Larsson, 1983) has been used. The GPS measurements are included as additional observations of the photo stations. Six unknown GPS parameters have been used for each strip. Several block adjustments with and without GPS measurements have been carried out and the root mean squares have been calculated for plane and height coordinates of the new points.
2. TEST DESCRIPTION

2.1 The observation equation

There are systematical errors of the GPS positioning that depend on:
- displacement of the satellites predicted orbits
- disturbance of the signals between the satellites and the receiver (atmospheric influence, clock errors etc.)

They can as an approximation be modeled by six parameters, three shifts and three time dependent drift parameters. As it is difficult to keep contact with the satellites when turning the aircraft there will be one set of parameters for each strip:

\[
\begin{bmatrix}
X_{iS} \\
Y_{iS} \\
Z_{iS}
\end{bmatrix}_{GPS} =
\begin{bmatrix}
X_{pi} \\
Y_{pi} \\
Z_{pi}
\end{bmatrix} -
\begin{bmatrix}
a_0s \\
b_0s \\
c_0s
\end{bmatrix} -
\begin{bmatrix}
a_1s \\
b_1s \\
c_1s
\end{bmatrix} \cdot t_i
\]

\([X_{iS}, Y_{iS}, Z_{iS}]_{GPS}\) GPS registration (i) of the antenna position in strip (s)

\([X_{pi}, Y_{pi}, Z_{pi}]_{GPS}\) The projection centre (i)

\([a_0s, b_0s, c_0s]\) Shift parameters of strip (s)

\([a_1s, b_1s, c_1s]\) Time dependent drift parameters of strip (s).

ti Time registration (i)

This is the observation equation used. There are other parameters that can be included in the adjustment, such as the eccentricity parameters between the antenna and the camera and the transformation parameters between WGS 84 and the geodetical coordinate system.

2.2 A priori standard deviations

2.2.1 Image measurements. The chosen a priori standard deviation for the image measurements is 5 μm.

\[\sigma_x = \sigma_y = 5 \mu m\]

2.2.2 Ground control points. The chosen a priori standard deviation of the control points depends on the flying height (table 1).

<table>
<thead>
<tr>
<th>Flying height (m)</th>
<th>(\sigma_x = \sigma_y = \sigma_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>0.01</td>
</tr>
<tr>
<td>2300</td>
<td>0.02</td>
</tr>
<tr>
<td>9200</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1 A priori standard deviations of the control points depending on flying height.

2.2.3 GPS measurements. Three different a priori standard deviations are used: 0.10, 0.25 and 0.50 metre. The first one, 0.10 m, corresponds to the precision the National Land Survey of Sweden believes can be possible to achieve in the future. The last one, 0.50 m, is what they believe is possible with their system today.

\[\sigma_x = \sigma_y = \sigma_z\]

<table>
<thead>
<tr>
<th>(GPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10 m</td>
</tr>
<tr>
<td>0.25 m</td>
</tr>
<tr>
<td>0.50 m</td>
</tr>
</tbody>
</table>

Table 2 A priori standard deviations of the GPS measurements of the camera position.

2.3 Block configurations

The block sizes used are represented in table 3. The stereo coverage is 60 % and the strip overlap is 20 %.

<table>
<thead>
<tr>
<th>Blocksizes Strips * Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 * 13</td>
</tr>
<tr>
<td>6 * 13</td>
</tr>
<tr>
<td>6 * 25</td>
</tr>
</tbody>
</table>

Table 3 List over used block sizes.

The block with 6 strips, each with 13 images, will be a square and the block with 6 strips, each with 25 images, will be twice as long as wide.

2.4 Flying heights

The flying heights correspond to three of the most common ones used in Sweden for mapping (table 4).

<table>
<thead>
<tr>
<th>Flying heights (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
</tr>
<tr>
<td>2300</td>
</tr>
<tr>
<td>9200</td>
</tr>
</tbody>
</table>

Table 4 List over used flying heights.

2.5 Ground control configuration

Different ground control configurations are used as shown in table 5.
Without GPS-measurements

<table>
<thead>
<tr>
<th>Alt.</th>
<th>Plane</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>In a frame distance = 2B</td>
<td>In chains distance = 4B</td>
</tr>
</tbody>
</table>

With GPS-measurements

<table>
<thead>
<tr>
<th>Alt.</th>
<th>Plane</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>In a frame distance = 4B</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>III</td>
<td>In a frame distance = 6B</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>IV</td>
<td>One double in each corner (2*4)</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>V</td>
<td>One double in each corner (2*4)</td>
<td>In 2 chains</td>
</tr>
<tr>
<td>VI</td>
<td>One in each corner (4)</td>
<td>In 2 chains</td>
</tr>
<tr>
<td>VII</td>
<td>One double in each corner (2*4)</td>
<td>One double in each corner - two crossing strips</td>
</tr>
</tbody>
</table>

Table 5  Ground control configurations for the blocks. B = flying base.

3. RESULTS

To evaluate the method of using GPS measurements as additional observations of the photostations the co-variance matrices of each new point in the block has been calculated. The root mean square, RMS, of the standard deviations of all new points in plane and height has been calculated to show the overall accuracy of the new points in each block.

There are four parameters that differs from block to block, namely:

- a priori standard deviation of the GPS measurements
- configuration of the control points
- flying height
- block size

It is of interest not only what the result is in each block but also how it is influenced by the changes in the parameters above.

3.1 Some results in figures

Three different ground control configurations (I, IV and VII, see table 5) are used for all flying heights. Their results are shown in table 6 for the RMS of the new points in plane and table 7 for the RMS in height.

<table>
<thead>
<tr>
<th>Alt.</th>
<th>Plane</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>In a frame distance = 2B</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>IV</td>
<td>One double in each corner (2*4)</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>VII</td>
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<td>One double in each corner - two crossing strips</td>
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</table>

Table 5  Ground control configurations for the blocks. B = flying base.

Table 6  RMS of the new points in plane. Three different control configurations (I, IV and VII) and three different flying heights (750, 2300 and 9200 m). $\sigma_{GPS} = 0.10$ m and the block size is 6 strips * 25 images.

<table>
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<tr>
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<th>Plane</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>In a frame distance = 2B</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>IV</td>
<td>One double in each corner (2*4)</td>
<td>In chains distance = 4B</td>
</tr>
<tr>
<td>VII</td>
<td>One double in each corner (2*4)</td>
<td>One double in each corner - two crossing strips</td>
</tr>
</tbody>
</table>

Table 6  RMS of the new points in plane. Three different control configurations (I, IV and VII) and three different flying heights (750, 2300 and 9200 m). $\sigma_{GPS} = 0.10$ m and the block size is 6 strips * 25 images.

3.2 Changing the a priori standard deviations in the GPS measurements

When changing the a priori standard deviation of the GPS measurements the root mean square (RMS) of the new points will increase both in plane and height. For example, with four double ground control points, both in plane and height, an increase of the a priori standard deviation of the GPS measurements will lead to an increase in the RMS of new points of approximately 1.4 times in plane and 2 times in height. (Figure 1a and 1b.)

3.3 Changing the configuration of control points

When using only four control points in plane the RMS in plane of the new points will be higher than when using more control points. When using two crossing strips instead of height chains the RMS in plane is lower.

Four control points in height and crossing strips gives a higher RMS in height than when using height chains. One exception is when the a priori standard deviation of the GPS measurements is 0.10 m. (Figure 1a and 1b.)

The difference in the results between using double control points and using single control points in the corners of the block is negligible. (Figure 1a and 1b.)

3.4 Changing the flying height

When increasing the flying height the RMS of the new points gets higher. However, if GPS measure-
Figure 1a  RMS in plane of the new points. The flying height is 2300 m and the block size is 6 strips * 13 images. Two crossing strips (block type VII) give a lower RMS than two height control chains (block types V and VI) when using only four plane control points. When increasing the standard deviation of the GPS measurements five times the RMS increases approximately 1.4 times for the case with four control points in plane and crossing strips.

Figure 1b  RMS in height of the new points. The flying height is 2300 m and the block size is 6 strips * 13 images. Two crossing strips (block type VII) give a lower RMS than two height control chains (block types V and VI) for the lowest standard deviation of the GPS measurements. When increasing the standard deviation of the GPS measurements five times the RMS increases approximately two times for the case with four control points in height and crossing strips.

Figure 2a  Normalized RMS (divided by the standard error of unit weight, $\sigma_0$, of the image measurements and the image scale factor, $s$) in plane of the new points. $\sigma_{GPS} = 0.10$ m. The block size is 6 strips * 13 images. The blocks with GPS measurements will give better results relative the block without GPS measurements on higher flying heights.

Figure 2b  Normalized RMS (divided by the standard error of unit weight, $\sigma_0$, of the image measurements and the image scale factor, $s$) in height of the new points. $\sigma_{GPS} = 0.10$ m. The block size is 6 strips * 13 images. The blocks with GPS measurements will give better results relative the block without GPS measurements on higher flying heights.
Figure 3a RMS in plane of the new points. The flying height is 2300 m and \( \sigma_{GPS} = 0.10 \) m. There is no significant trend in the RMS when changing the block size.

Figure 3b RMS in height of the new points. The flying height is 2300 m and \( \sigma_{GPS} = 0.10 \) m. There is no significant trend in the RMS when changing the block size.

3.5 Changing the block size

There are no significant differences in the results between different block sizes. (Figure 3a and 3b.)

4. DISCUSSION

The results show that GPS measurements of the photo stations are of most interest when flying on higher heights. This is due to the fact that the precision of the measurements is rather low (0.10 m) compared to the precision of the ground control points on the lower flying heights (0.01 and 0.02 m).

When using two crossing strips the result is better than using two height chains both in plane and height. This is because there will be more observations in plane and that the additional image observations will improve the stability of the block. The loss of height ground control points is compensated for by the additional GPS observations in height.

These tests have shown that it is possible to use GPS observations of the photo stations and few ground control points instead of only image observations and many ground control points, especially on higher flying heights.

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REFERENCES

