IMPORTANCE OF MODERN MATHEMATICAL METHODS FOR THE DIGITAL PHOTOGRAMMETRY

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ABSTRACT

From a mathematical point of view modern photogrammetric tasks are to define characteristic transformations in suitable function spaces. In such an uniform model both image space and feature space are realisations for the digital photogrammetry and the GIS-technology integrated also the cartography. Two examples – the scene correspondence and an abstract description of the map updating – are presented with the help of functional analytical means like measures, norms, and scalar products as tools of an abstract characterization.

Keys: theory, system integration, algorithm

1. INTRODUCTION

The state of the art in modern geoinformatics is characterized by the integration of classic disciplines like photogrammetry and cartography with remote sensing and GIS-techniques. The separate development in modelling must be integrated in one model (see Foerstner 1991).

A mathematical model demonstrating a functional analytical description in geoscience especially remote sensing, photogrammetry, and cartography in one calculus was developed (see Pross 1990 a,b, 1991 a).

The disciplines induce several views on the same data and information processing with emphasis on remote sensing – data collection photogrammetry – information derivation and structuring cartography – information storage and presentation.

In section 2 we will define the image and the feature space and also the characteristic transformations in this frame and in the section 3 we formulate two tasks – the scene correspondence and the updating of maps – in such a model. An outview completes the paper.

2. SPACE DEFINITION

From the mathematical point of view it is necessary to define characteristic operations in suitable function spaces. Two kinds of spaces are modeled – the image space and the feature space.

In correspondence with Leberls Image Cubes (see Leberl 1991) the image space is more-dimensional and the first two coordinates are the \( x - y \) – geometry (in the small-scale information systems the altitude is a separate level and not a complete coordinate axis) added by time or spectral coordinates.

If the time is discrete or other thematic levels are contained the image space can be interpreted as slides.

The feature space is a generalized description of thematic information. In cartography respectively in GIS-technologies these spaces are vector-oriented containing the objects, their geometry, and relations between them organized as a data base (see Figure 1).

![Figure 1: Feature and Image Space](image)

On the base of such defined spaces specific thematic processes are represented as transformations. In addition to Figure 1 in Figure 2 the operator activities are shown.
The two kinds of spaces permit a classification of the process-dependent operations. Such a generalized point of view leads to ideas of algorithmization as

- finding of suitable projections $T_{Pr}$ and integrations $T_{In}$ with suitable domains and weights within the image space,
- introduction of a continuous time coordinate and the time operator
  \[ T_{t}(f(x,y)) = f_t(x,y) \],
- upgrading the problem from the image space to the feature space by hierarchic transformations $T_{H}$.

By using the functional analytical especially the Hilbert-Space-Methods the transformations can be designed and analyzed by a scalar product $\langle \cdot, \cdot \rangle$ realized by integrals or series in the sense of $L^2$- and $l^2$-Hilbert-Spaces. Then the image space transformations $T_{I}$ are being modeled

\[ T_{I} = \langle \cdot, \rho \rangle \text{a} \]

\[ T_{Pr} = \langle \cdot, \delta \rangle \text{b} \]

$\rho$ is a weight-function corresponding to an area and $\delta$ is the Dirac-Distribution depending on the projection direction (see also Pross 1991 b).

Because the feature space is organized as a data system the transformations $T_{F}$ within the feature space are designed as a generalized matrix.

The feature space exists not only in one qualitative level. Table 1 illustrates this fact.

Table 1: Feature Space Characterization

<table>
<thead>
<tr>
<th>Qualitative Level</th>
<th>low</th>
<th>middle</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>geometry</td>
<td>*</td>
<td>*</td>
<td>+</td>
</tr>
<tr>
<td>objects</td>
<td></td>
<td>(+)</td>
<td></td>
</tr>
<tr>
<td>relations</td>
<td>(+)</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td>characteristics</td>
<td>geometry</td>
<td>geometry</td>
<td>geometry</td>
</tr>
<tr>
<td></td>
<td>(topology)</td>
<td>(topology)</td>
<td>(topology)</td>
</tr>
<tr>
<td></td>
<td>semantics</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 2 the time characterization is compared with mathematical methods.

<table>
<thead>
<tr>
<th>Time</th>
<th>Mathematical Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>point sequence</td>
<td>least square means</td>
</tr>
<tr>
<td>continuum</td>
<td>variation methods</td>
</tr>
<tr>
<td></td>
<td>functional analysis</td>
</tr>
</tbody>
</table>

Table 2: Characterization of Time

Depending on the kind of problems and on the data using we act in the scheme of Figure 3.

Figure 3: Mathematical Levels
3. PROCESSES

In this section we will give two examples demonstrating the philosophy of an abstract description of photogrammetric tasks.

3.1 Scene Correspondence

The task of scene correspondence is to define the geometric relations between two or more images to the same domain. Thereby the time is running continuously and typically the time points are discrete on the time axis in the image space. Consequently these images also correspond to discrete levels in the image space.

The correspondence problem is solved with the aid of image information (grey values, image frequencies, textural features etc.) by using the variation calculus and defining functionals to be minimized. The minimizing problem leads to the solution of EULER–Equations (systems of partial differential equations).

Because a sequence of images is a set of discrete planes in the image space one direction of generalization is the change over to the time continuum and to get a complete image space. In this model it is possible to define termini as image flow or trajectories of objects.

The digital photogrammetry transfers both the spaces and their transformations in a digital world and leads also to new notions as softcopy photogrammetry (see LEBERL 1991). The transformations are from Type $T_T$ acting nearly exclusive in the image space. As results of the discretization of the image space we get sequences of points. The correspondence is defined by the solution of attached difference equations being also an analogon in the digital sense to the differential equations. By using an iterative scheme it can be formulated as a complete digital photogrammetric task (see HAHN, PROSS 1992). Further image processes can be designed by finding attached transformations $T_T$ within the image space especially by separation and qualified algorithmization in geometry and time.

New kinds of mathematical means and methods for the representation like stochastic (see BUSCH, KÖCH 1990) and functional analytical methods — see PROSS 1990 b, 1991 a for the use of HILBERT–Space–Methods — are the theoretical background.

Figure 4 shows the differences between the discrete and the continuous image space.

3.2 Updating of Maps

By using digital image processing methods in photogrammetry and cartography the problem of map updating is rising from the images and symbolized description to an abstract desymbolized structure. The definition of characterized structures leads to a correspondence problem in such structures added by difference measures in structure and geometry.

Figure 5 shows the action of the hierarchical transformation $T_H$.

The transformation $T_H$ creates

$$ m = T_H(f_j^{(m)}(x, y)) = \{s_k\} $$ (4)

with

$$ s_k = (g_x, o_x, r_k|d) = (s_k|d) $$ (5)
The nonsymbolic description \( \delta_k \) contains the geometry \( g_k \) of the objects \( o_k \) and the relations \( r_k \) between the objects \( o_k \) added by the set of symbols of the map \( d \).

The updating operator \( U \) can be designed as

\[
\begin{align*}
 m^{(n+1)} &= U(m^{(n)}) \\
 s^{(n+1)}_j &= U(\{s^{(n)}_j\}, \{f_j(x,y)\}) .
\end{align*}
\]  

(6) (7)

Updating is a result of comparing the (cartographic) objects with topical structured image data. Comparing uses a difference operator \( D \)

\[
D = (S,G)
\]  

(8)

separated into two parts: \( S \) acting in the structure, \( G \) in the geometry. The difference operator \( D \) is a functional and weighted the structure differences by \( S \) and compares the geometry differences by \( G \).

Based on the three kinds of cartographic objects points, lines, and areas the structure operator \( S \) forms a symbolic 4 by 3 matrix describing the several possibilities of transmission between the cartographic object groups (see Figure 6). The principle diagonal of \( S \) contains the structure invariant part. The upper right part of \( S \) is the generalization part corresponding to the dimension reduction in the structure and the upgrading in the object hierarchy. The last row in \( S \) is the zero-space corresponding with such objects deleted during the generalization.

After a geometric transformation to a reference geometry the structure is in coincidence. A difference measure \( d(s_i, s_j) \) between the structures \( s_i \) and \( s_j \) is defined by

\[
d_C(s_i, s_j) = \sum_{k=1}^{3} \alpha_k d^k_C(s_i^k, s_j^k) .
\]  

(9)

\( d^k_C \) is the point coincidence measure, \( d^k_C \) that for lines, and \( d^k_C \) that for areas; \( \alpha_k \) are weights. A lot of difference functionals \( d \) is possible. The simplest ones are the point distances, the differences of line lengths and of square measures.

In classic maps line objects dominate. Therefore it is also adequate to integrate about the square differences of normalized length.

\[
d^2(t_1, t_2) = \|t_1 - t_2\|^2 = \int ((x(t_1) - x(t_2))^2 + (y(t_1) - y(t_2))^2) dt
\]  

(10)

with

\[
t_i = \frac{t_i - t_i(P_1)}{t_i(P_2) - t_i(P_1)}
\]  

(11)

\( t_i \) is the normalized length of the lines.

In summary a correspondence measure of two structures \( s_i \) and \( s_j \) is defined by

\[
D(s_i, s_j) = \alpha d_C(s_i, s_j) + \beta d_S(s_i, s_j) .
\]  

(12)

The structure measure \( d_C \) is derived from the structure matrix \( S \). The weights \( \alpha, \beta \), and \( \alpha_k \) estimate the significance of the several parts.

4. OUTVIEW

From a generalized point of view a structuring of processes and definition of suitable spaces and operators is very important.

By using a connection-symbol of operators \( \circ \)

\[
(A \circ B)(x) = B(A(x))
\]  

(13)

we can formulate a thematic processing with an operator sequence acting in the image and the feature space (see also Figure 2)

\[
\{T^i_k\}_i \circ \{T^h_k\}_k \circ \{T^h_{-1}\}.
\]  

(14)

For a concrete task the question is to formulate a target functional \( F \) that must be extremally and depends on the sequences of operators

\[
F = F(i, j, h, h^{-1}).
\]  

(15)
In the case of map updating (section 3.2) the operator sequence contains

\[ T_i \] – image enhancement, map scanning

\[ T_h \] – generation of a structured map representation from images and maps

\[ T_j \] – comparison of structure, deletion and creation of objects

\[ T_{h-1} \] – symbolization of a map structure.

By definition of suitable target functionals an automatic structure generation can be added by the computation of difference measures controlled by an operator sequence.

After the first practical steps it is necessary to separate the techniques into an interactive and an automatic part. The analysis of data structures leads to suitable interfaces in the data and information flows.

REFERENCES


