NEURON NETWORKS WITH NONLINEAR INTERCONNECTIONS FOR ANALYSIS OF DATA.

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Abstract - Structure of neuron networks with different types of interconnections is described. Analytical method, based on nonlinear spectral representation of optical information is used to classify real signals and objects corresponding to them. The important idea of time evolution of spectral parameters is then introduced and it is shown that the problem of parallel signal processing can be solved with the aid of large number of identical elements with nonlinear interconnections. Problems, connected with distortion and signal correction as well qualitatively discussed. It is investigated influence of losses along network on signal propagation. Main characteristics of nonlinear neuron network were marked out. 2D neuron networks with nonlinear interconnections for analysis of data were investigated as well. Models of real neuron-like elements were suggested. Their regimes were investigated, simulating neurons with the aid of digital computer. Experimental results enabled to make a conclusion of significant possibilities which arise when using networks with nonlinear interconnections. It is marked some problems and applications inherent to these networks.

KEY WORDS: Neuron, network, soliton, spectrum, filtration.

1. INTRODUCTION

In the systems of remote sensing investigator often meets with problems of preliminary signal processing. Having read signal from receiving matrix it is necessary to realize its transformation to form admissible for digital computer. Latter operates with high dimension matrix of data using algorithms which depend on concrete task. Sometimes that sequence of actions is inconvenient. Time, requested for computer, and registering processes velocity may come into contradiction. Besides, distribution of operations between analog and digital units is far from symmetry. And problem of transference of processing center for whole system to analog unit arises.

It has become possible to come to this conclusion after investigations connected with analysis of pictures through atmosphere. Discussed in /1/, acoustooptical device required preliminary analog signal processing directly at the optical matrix of photodetectors. Furthermore in perspective it is necessary to organize parallel functioning of all elements. That is why direction of investigations was concentrated to biological objects such as retina. It was determined that real neural system may be envisaged as a model of complex technical system for effective transformation of information.

Human brain consists of a great number of unique elements - neurons. Much time has been spent investigating structure, interconnections and operations of them. It can be said that now we have several lines of development those systems which are often called neuron networks. Although the term "neuron" may be taken as undefined it must be restricted to avoid ambiguities. In this paper when we speak of "network" we should be referring to collections of physical elements. Their interconnections provide all system with new functions. The nature of these interconnections may be different.

It is a poor idea to imitate the behaviour of neurons. It's more productive to use some principals inherent to neurons. Among them flexibility, access to a large amount of data and of course, opportunity to operate with digital and especially analog signal that most investigators find possible to neglect.

Almost all applications of neuron networks in optical systems are based on ability to recognize data. Performing some mathematical operations, whole neuron system is divided into several layers. They have interconnections which may be called vertical, i.e. between different layers (see Fig.1a). In most situations each element of network realizes addition and substraction. More complex operations are performed by network as a whole. Thus a role of interconnections comes to a simple transportation of data.

Imagine another situation when each neuron in the layer is connected with the neighbour one. Such structure may be called as network with horizontal or parallel interconnections in layer (see Fig.1b).

Graph of all interconnections defines its topology. Idea of symmetry plays here an extraordinary role.
Nevertheless it is important first to examine line network where each element has only two interconnections. Layer should be desintegrated into several pieces or sets of neurons which can operate independently.

Functions of connections between neurons in linear parallel networks are significantly more rich than in vertical ones. The main goal of this paper is to show real possibility of application such neural networks for 2D optical signal treatment.

2.1 Functioning in general

Examination of human eye promises not only clarification in idea of its layer structure but as well enables to mark out several cells between layers which bind certain other neurons into a network whole. These specific cells are called horizontal. They fulfill actions similar to ones of springs between metal spheres. Each element of retina receives optical signals and if horizontal interconnections are linear, then all information will equilibrates. Consequently, relations between neurons should be considered as non linear elements.

The impetus for the investigations was a theory of nonlinear lattices in accordance with which whole network can be represented as a set of oscillating elements. Specific feature of this structure is definition of external optical field, influencing on network as potential, correspondingly to Toda or Korteveg-de-Vries equations at initial moment. The profound theory of those systems one can find in 1/3, 1/4. It is necessary to recall that in the finite dimensional case the problem consists in finding n × n matrices and known as a Lax pair, such, that equations of motion are equivalent to the matrix equation

\[ \frac{dA}{dt} = [R, A] \]

where \( R \) - Lie brackets.

That this equation implies the eigenvalues of the matrix R are conserved in time. It defines discrete spectrum for the corresponding well known Schodinger equation. Hence each new optical image has own collection of spectral components. Single component in its turn defines non linear stable wave-soliton. Solitons move along network with different velocities. Their complete characteristics are amplitudes, location at network. Most interesting parameter is phase shift between two solitons which appears after their collisions.

2.2 Information processes in networks

From this starting point we will try to show possibilities of networks with non linear interconnections to analyze information, containing in the optical images. Imagine a linear lattice of photodetectors with exponential links. The simplest example of this links one can find in 1/4. Determine a moment of time when all detectors receive simultaneously signals. Then influence of them ceases and picture of moving solitons begin to form. If network is infinite, then there are discrete and continuous spectrums. Continuous component is analog of spectrum for linear network. Thus for every neuronalike element its output is a solution of Toda's equation

\[ \frac{d\psi}{dt} = e^{\eta \psi} - e^{\eta \psi} \sum_{n=1}^{\infty} -2 e^{\eta \psi} \]

where \( \eta \) is a normalized value of charge at the n-th output (1/5). Define set \( \{y_{1}, y_{2}, ..., y_{N}, ..., y_{n}\} \) at moment \( t=0 \) as an initial conditions. Let's find all spectral components when quantity of neurons is great enough to reproduce optical signal with expecting accuracy. Values and locations of solitons determine two Jest solutions \( \psi_{1}(x), \psi_{2}(x) \) which can be found in accordance with

\[
\begin{align*}
\psi_{1}(x) &= \exp(\imath kx) - \frac{1}{2} \int \tilde{y}(x', k) \exp[\imath \eta (x-x')] dx', \\
\psi_{2}(x) &= \exp(-\imath kx) + \frac{1}{2} \int \tilde{y}(x', k) \exp[-\imath \eta (x-x')] dx'.
\end{align*}
\]

These equations describe process of wave propagation trough potential

\[ K(x, x') = y(x') \sin[k(-x')] \]

In the theory of light scattering this function is sometimes called transition function. Thus, solving scattering problem for a wave, which propagates to (exp(\imath kx)) it is necessary to investigate influence of \( K(x, x') \) on \( \psi(x) \). \( K(x, x') \) is defined by optical image forming at the detector plane. We can achieve a result, using Monte-Karlo method. Assume, that wave propagates through plane parallel nonhomogeneous media. The scattering properties are defined by \( K(x, x') \). Then express \( \psi(x) \) as

\[ \psi(x, k) = \sum_{n=1}^{\infty} \exp(\imath kx) \sum_{m=1}^{\infty} \delta_{mn} \phi(x') \exp(-\imath kx') dx' \]

This sum consists of components which can be called components of i-th order of scattering. This property enables to classify all optical images from the viewpoint of their discrete spectrum. Amplitudes of each component of sum are different, but decrease when i has tend to increase. If we substitute sum (2) with finite series with unusual then it is necessary to choose \( n_{i} \) specific for each signal. To achieve the purpose of classification and analyze extent of correspondence between spectrum of signals and collection of their discrete components numerical experiment was made. As in the scattering theory we can express \( K(x, x') \) in the form

\[ K(x, x') = \phi(x, \Theta) \exp[-\imath \eta (x-x')] \]

where \( \phi(x, \Theta) \) - scattering and full cross sections, \( \Theta \) - optical length and g(\cos \Theta) denotes the first component (F11) of general transformation matrix by Van de Hulst (1/6/1). \( \Theta \) scattering angle. We discuss only situation when \( k \in \eta \) and

\[ K(x, x') = \frac{1}{\eta} g(x', \Theta) \frac{\eta}{\eta} k(x'-x) \]

For linear network we consider only \( \Theta = 0 \). It should be noted that solution of eq.1 can be found by two ways. First suppose changes \( \Theta(x) \) in accordance with \( y(x) \), and the second change of \( g(\cos \Theta) \). Sometimes it is more convenient to define \( g(\cos \Theta) \) in every point of network than \( \Theta \). Function \( y(x) \) can be found using well known weight method (1/7/1), averaging

\[ y(x) = \frac{1}{\eta} \int_{\eta}^{\eta} g(x, \Theta) \frac{\eta (\Theta - x)}{\eta} \]

where \( x \) is determined by Monte-Karlo chain.

For this numerical experiment \( g(\cos \Theta) \) was calculated. It has been done for characterization of real optical field with collection of parameters which enables to obtain suitable illustrative material and supplementary information for building 2 D network. To achieve this goal, the diffraction task for spherical particles was solved. G(\cos \Theta)
was calculated for homogenous sphere and also for two and tree concentric spheres with different complex indeces. Fig.2 illustrates some results.

Figure 2. Function, characterizing "scattering" properties of neurons

Utilization of different spheres enables to establish group of parameters which suffered influence of external signal. This parametrization is very useful for building of two dimensional network. The number of them may be chosen basing on model of sphere (number of its layers and indices).

Figure 3. Discrete representation of initial conditions at the detector plane

\[ b(k, s) = \frac{1}{\pi} \frac{e^{ik\varphi}}{(k^2 + s^2)^{0.5}} + \frac{i}{\pi} \frac{e^{-ik\varphi}}{(k^2 + s^2)^{0.5}} \]

where \( s_k = \pm s \pm k \) and we have as well recurrent expression

\[ a_n = \frac{1}{\pi} \left[ A_{n-1} e^{i(s \pm k)n} + B_{n-1} e^{-i(s \pm k)n} \right] \]

\[ b_n = \frac{1}{\pi} \left[ A_{n-1} e^{-i(s \pm k)n} + B_{n-1} e^{i(s \pm k)n} \right] \]

This recurrence enables to obtain general formula for \( a(k) \) and estimate influence of high frequency part of spectrum on discrete components of spectrum. General form of \( a(k) \) is obtained from (6) and (7).

\[ a(k) = \frac{1}{2} \left[ e^{-ik\varphi} \left( \left\{ \sum_{s=-k}^{k} \frac{e^{is\varphi}}{(s^2 + k^2)^{0.5}} \right\} \right)^2 \right] \]

where \( \varphi \), \( \gamma \) are sign functions. Equating \( a(k) = 0 \) we get

\[ a(k) = \frac{1}{2} e^{-ik\varphi} \left( \left\{ \sum_{s=-k}^{k} \frac{e^{is\varphi}}{(s^2 + k^2)^{0.5}} \right\} \right)^2 \]

In other words the left part of (8) defines components of following type

\[ \Delta \Gamma \gamma_{n-1} \rightarrow 0 \]

Solution of (8) defines only \( A\) and \( B\). Those objects are equivalent to light source with the rectangular boundaries of brightness at the detector plane. But it's not means that middle and high part of spectrum have not any correspondence with discrete components of nonlinear spectrum. Weak change of illuminance defines collection of solitons with low amplitudes and small velocities. Using theory of amplitude spectral analysis to pick out some part of spectrum it is necessary to build optical system with great accuracy. With the aid of neuron networks this task can be solved significantly easier without analog to digit transform.
2.4 Determination of spectral components

Let's discuss methods of obtaining information about optical picture making use of neuron networks. Envisage some neuron network with nonlinear interconnections. Assume that optical signal influences on each neuronlike element. Besides, magnitude of electrical signal in network is enough that all nonlinearities would be able to reveal itself. Suppose that only part of all quantity of elements are lighted up. This group located at the beginning of network. Another part of neurons have no influence of light and connected with lighted section only electrically. Each element can find in \( \Omega \). There non-linearity is realized using capacity. After exposition a set of solitons forms along the network and besides of this there appears a component of continuous spectrum. After a while solitons, moving along the network, reach to its darked part. And continuous component forms as linear interaction between neurons. Hence it distributes along whole line, decreasing own amplitude. Consequently we can suggest that main part of information contains in solitons which move with different velocities. Center of soliton is at

\[
x = \delta_i^\gamma - c \left( \frac{\eta^2 - \eta_i^2}{\eta_i} \right) t,
\]

where \( \delta_i^\gamma = \text{Const} \). Its velocity is equal to \( c \left( \frac{\eta^2 - \eta_i^2}{\eta} \right) \). More strong soliton moves quicker than weak one.

With the aid of neuron system it is possible to devide weak solitons which correspond to high frequency component from spectrum which characterize low frequency components of spectrum. All discussed solitons have different velocities. Hence at the darkened end of network strong pulses appear first. There soliton characteristics determination. Most important are amplitude and width. But they have one-to-one correspondence.

Let's find derivative of signal with respect to time interval between appearance of solitons at first and second neurons. Geometrical distance one can choose taking into account information about brightness of source and its extent. Thus at darkened end of network all solitons are identified sequently.

Because we envisage situation when process has become settled, it is possible to realize identification by control soliton which moves to meet information ones. It can be received from other networks as well. During collision of two solitons they acquire phase shifts which depend on components of discrete spectrum corresponding to them. For this purpose, these phase shifts may be stated as following relation

\[
\delta_k = - \frac{1}{\eta} \log \left| \frac{\eta^2 - \eta^2_k}{\eta^2_k - \eta^2} \right| \quad \delta_k = \frac{1}{\eta} \log \left| \frac{\eta^2 + \eta^2_k}{\eta^2_k + \eta^2} \right|
\]

Introduce additional function \( F = \arctan \left( \frac{\eta^2}{\eta^2 - \eta^2_k} \right) \).

After collision due to phase shifts function \( F \) suffers jump, characterizing by big value of time derivative. Counting up all those jumps along network and knowing in advance amplitude of control soliton, set of discrete components is determined without fails.

Besides, signal measurement in a single network the same operation is possible using information, containing in several networks. Imagine two identical networks, for example, two neighbour ones. First, light distribution at every of them is identical too. Let us fix two pairs of neurons at each network with the same number. And choose those pairs at a short distance. Begin to calculate function \( F \) using signals from every pair for this operation. If illuminations are identical as supposed above, then we will obtain as a result \( F = 45 \) in every moment. The same signal will be observed at the next input, connected with the second pair. Examination of pair with another number, different from those, shows that there are not change of initial picture. This result shows as well coincidence of signals at both networks.

Imagine situation when at the same network initial picture has small differences from first one. Then, as we have seen above, discrete spectrum will suffer changes too. Several solitons will increase or decrease their amplitudes and accordingly their velocities. That pair of neurons which number is the least, will acquire insignificant change. The more number of next pair is different from initial one, the more function \( F \) acquires change. It can be explained by different connections of two solitons in some moment. It is advisable to introduce such parameter as length of identity. It can be defined taking into consideration permissible level of difference \( F \) from 45.

One and the same 2 D network can have different discrete spectrum structure depending on topology of all subnetworks. Calculation of function \( F \) will enable to produce optimal topology. Choose minimal of equivalent length as a criterion of identity. In that situation discrete spectrum is most rich. Let us place several pairs of neurons for measurement of \( F \) at 2 D photodetector plane. And envisage several standard topologies to choose optimal type. This task was solved by aid of digital computer. It was examined following types of possible networks: horizontal lines, vertical lines, rectangulars and spirals. For different light distribution optimal topology should be determined.

Algorithm, represented above, evidently is suboptimal one, because we examined only standard types of networks. More complex task is to form network by changing this network through several steps, leading criterion to minimum. Several values, corresponding to discrete spectrum, characterize received optical signal, its parameters. Comparison with the standard image will enable to estimate difference in their spectrums. If it is necessary one can rehabilitate initial image making use the method of inverse scattering transform (1/10).

2.5 Signal distortion

Special question of influence of different distortions of initial image on discrete spectrum components should be discussed. Assume that light distribution at the detector plane suffered a small increment \( \delta' \). Then after substitution of it into Schrodinger equation and making some transformations we get

\[
\sum \delta k \eta_k = - \frac{\Delta}{2i \eta_k} \int \delta' (y) \psi_k^2 \, dx.
\]

Hence increments of discrete spectrum component depend on type of optical signal, i.e. how many elements there are in the series corresponding to this signal. Coefficients \( \psi_k \) are time independent.

In other situations it is possible to correct initial signal with the aid of spectral correction.
This task may be solved with the aid of inverse scattering method.

If there are many neurons in the nonlinear network then continuous component of spectrum plays more important role. It can be characterized by scattering coefficient $R'(t)$. But this coefficient can be found from the initial light distribution at the detector plane, because this distribution is Fourier transform of $R'(t)$. Then it is necessary to use Helfand-Levitan equation.

2.6 Losses in the networks

Basic philosophy adopted in our discussion is absence of attenuation in network. Within some time period this statement is correct. But the length of network is proportional of losses. As it is determined in ([11]), spectral components suffer changes with the time

$$\eta(t) = \eta(0) \exp(-\frac{t}{\tau}) \int_{0}^{t} \eta(t') dt'$$

It means that amplitude and velocity of soliton reduce with course of time. This effect may be used while adjusting losses at some sections of network. Discrete components of spectrum suffer changes due to attenuation too.

3. MODEL OF NEURON

Examine laboratory prototype of neuron. Each of them has receiving element. For this it is possible to choose capacity of photodetector. It has constant component $C_0$ and nonlinear one. Then all equivalent scheme can be represented as long wire, where inductor is an element of connection and neuron is nonlinear itself (see Fig. 4).

- **a)**

- **b)**

![Figure 4. Electrical model of neuronlike network](image)

As one can see main element their is nonlinear capacity. This system with the aid of differential amplifiers produces nonlinear capacity through logarithmic and exponential transforms. The impedance of system consisting of amplifiers has capacity character. Inductance may be formed by converters. Extent of nonlinearity was chosen in two types:

$$C = C_0 \ln(1 + \frac{U}{U_0}) \quad C = C_0 (1 + \frac{U}{U_0})^m$$

Corresponding circuits are represented at Fig. 5 a,b. One can show, that for circuit, represented at Fig. 5

$$C = C_0 (1 + A)$$

where $A$ is realized as function of $U$.

In the case a)

$$A = -i + \ln (1 + \frac{U}{U_0})$$

and in case b)

$$A = \left( \frac{R_0}{R_b} - L \right)$$

Potential $U_0$ shifts constant charges in capacitors and enables to regulate the extent of nonlinearity. Electrical network consisting of such neuronlike elements was simulated using CAD system DISPS (for micro VAX type computer) to obtain real nonlinear waves.

When signal is small and $U_0$ is close to zero, network functionates as a linear one. The second type of nonlinearity leads to different but important results. There are more strong losses. Initial conditions were defined as potentials at all knots. This procedure is equivalent to carrying in nonlinear capacities of initial charges in addition to constant charges. Boundary conditions were chosen as fixed at the ends of line.

First type of nonlinearities produces as well continuous component but less than in second type. Simulation has shown that if nonlinearity declines from logarithmic form, contribution of discrete spectrum becomes less. If neuron is perfect and network consists of finite quantity of neurons then this network has only discrete components.

Level of initial potential $U_0$ is important parameter of network. There are worked out method of regulating this value making use information from another networks.

This idea may be realized with the aid of two neighbour networks. First should be network discussed above and second is one with elements which are connected by linear links. This system will average out charge $Q_0$ at all elements. Charge $Q_0$ determines potential $U_0$. Furthermore let us place near every nonlinear network, network for production $Q_0$ and connect all them together. We obtain

$$Q_0 = \int_{-\infty}^{\infty} \frac{Q_0}{n}$$

where $n$ - number of neurons.

Then whole system acquire property to estimate integral influence of receiving matrix. For pictures which produce almost homogeneous light distribution at the detector plane constant level of $U_0$ will transform optical field to high part of spectrum. If there are bright points at the receiving matrix, then they will be transformed to strong solitons. This method enables to pick out bright points. However, if there are many of those points, then efficiency of average charge method for all network decreases. In that situation it is necessary to devote linear networks into several groups. And every of them will have own average charge. It would enable to pick out isolated bright points and even lines (envisaging network as a whole).

Third method consists in accidental connections of different groups of linear networks. But this case supposes analysis of picture using several sequences. Consequently it may find application only when change of external illumiance happens slowly.

It should be stressed that charge $Q_0$ changes initial conditions.

If boundary conditions are cyclic then solitons move round the cycle and after some period of time initial picture restores. These experiments have shown that theoretical suggestions were close to characteristics of real network.

4. EXPERIMENTAL INVESTIGATIONS

Experimental investigations had for an object
Figure 5. Models of neurons
qualitatively to show opportunity of signal processing with the aid of neuron networks using non linear interconnections. Because of it technical realization is elements and their interaction was rather different from one discussed above. It was made so for more convenience to set initial conditions and to obtain results.

Network, based on long line, was assembled making use of analog computer. Nonlinearities there were build up using internal units inherent to these computers. Thus simulations were carried out with 12 elements in network. It is convenient to form necessary initial charges at all knots. A schematic diagram of experimental system is shown in Fig. 6.

Output of each neuron has been taken from corresponding knot. Received signals moved to acousto-optical cell which had 5 parallel channels through frequency transformers. System of transformers could form 36 different channels (6 at one acousto-optical element). Each channel has own attenuator. In that way it was possible to regulate weights of outputs. Collection of moving plates had functional transparency along its length. All optical elements provide necessary characteristics of light beams at all points. Group of photodetectors registered signals which then were transformed to digital code. This experimental system enabled to realize discrete spectral filtration. As an initial image a rectangular with distorted plane top (symmetrically with respect to plane top) moving different functionally transparent plates, and treating signals at outputs, were obtained two different results. First signal is undistorted rectangular with suppressed oscillations of plane top and second one is high frequency component with suppressed initial rectangular.

It was proposed to fix time of back calculation but it was considered only as first approximation and the finish result was fixed, using qualitative determination of expected signal. Real network and signal processing system may be formed on the base of VLSI structures, which will provide opportunities of operating with complicated images using many neuronlike elements.

5. NETWORKS OF HIGH DIMENSION

Previous discussion was concerned only linear networks. Nevertheless networks of more than first dimension can be devised into two classes. First is n-dimensional network formed by only linear interconnections. There are mathematical literature where 2D solitons are investigated. They are called lumps. Lumps are solitons of Kadomtsev-Petviashvili equation. It is a problem to find a suitable model of this mathematical object. Spectral theory of 2D networks with non linear interconnections is not enough clear to build on the base of it system for signal processing. However principle of scattering in that theory can be used.

Stream of information through neuron oscillations interacts with the set of centers scattering part of this information and producing two components of signal spectrum. Representation of 2D signals as a superposition of some set of centers and their scattering part may be as for linear networks. It should be stressed that task of scattering acquires thus a new character. Using idea of absence of continuous spectrum component one can obtain instead of real image systems of moving lumps. One can suppose that parameters of lumps have as well one-to-one correspondence with the Fourier spectrum of initial image. Then nonlinear spectral filtration would enable to pick out necessary elements from realizations of elements of network is more than two then it is difficult to find fruitful concept of interaction of neurons. Toda (1/12) has represented several matrices corresponding to differential equations of more high order than Korteweg-de-Vries one. Exact technical realization of these mathematical objects is impossible, but may be approximation would enable to build new interesting network.

The second type of 2D network can be built organizing links between several ordinal networks. This problem is closely depends on the set of neuron parameters, introduced while describing scattering process. Fig. 2 gives picture of distribution of several signals, directed to different network sections. Control solitons are formed not at all neurons. Two neurons, having the same scattering properties at $\theta = 0$, can differ at any different angles. In polar coordinates angle corresponds to solitons phase or its position at network. Radius defines normalized amplitude of soliton. However it is a special question.

6. CONCLUSION

The most important conclusion of this study is that it is possible to use neuron network with nonlinear interconnections for signal processing. Ordinal vertical networks usually play role of some mathematical model and technical realization by digital computer translates it into reality. Term "neuron" in those networks has as well mathematical meaning. Neuron in parallel networks has its own technical implementation as analog and digital elements.

The perspectives of neuron networks utilization are in parallel structures for signal processing. Whole 2D set of photodetectors is divided into several line networks. Signal analysis is carried out separately for each network. Simplest way to devide all photodetectors into subsets is to pick out strings or lines at matrix. Definition of organization method of neurons into groups is a special problem which depends on concrete conditions and task.

As a whole networks with nonlinear interconnection may be used in systems for signal processing, to transform initial 2D optical field into different type of information field which should be convenient for further transformation and utilization.

REFERENCES


Figure 6. Schematic diagram of experimental system


