GROSS ERRORS LOCATION BY TWO STEP ITERATIONS METHOD

Prof. Wang Renxiang

Xian Research Institute of Surveying And Mapping, China , Communssion III

ABSTRICT

In consideration of the capability and relibility about localizing gross errors are decreased by correlation of residuals seriously. From the strategical point view, the iterated weight least squares method is developed to so-called ' Two step iterations method '. In the first step of iterations, the observational weight is calculated by selected weight function in an usual way. In the second step, we start with statistical test and analysis of residual correlation. Based on convergence in the first step, obtain the possible gross error observation(s) and weighted zero to its. Then the second step iteration is performed. After that, gross errors localization is done by regirous statistical test according to the standardized residuals and with due regard for the magnitude of so-called ' weighted zero residual'. The capability and relibility of localizing gross errors are improved by two step iterations method.

The paper give some examples with simulated data for comparasion of the results about gross errors location by different step iterations methods.

KEYWORDS: Gross Error Location, Standardized residual, Weighted zero residual, Qvv.P matrix.

INTRODUCTION

Gross errors localizing by iterated weight least squares method has been investigated for a long time. One of the key problems of this method is to select weight function . There are many weight functions proposed by different authors in present application. Every weight function has its own properties. Among these functions, the types of function, the parameters and the statistical quantities as well as the critical values are somewhath different to each other. However a common property is that the function is an inverse measure of residual in absolute. Therefore the magnitute of main diagonal element relating the observation with rather large residual in absolute in Qvv.p matrix will be increased, aftre iteration with the weight function and will be capable of localizing gross errors (Wang Renxiang,1986a)

The present author points out in (Wang Renxiang, 1988b, appendix)that gross errors localization is unreliable by iteration with weight function, when some residuals are of strong correlation. The paper has proposed an idea so-called ' Two step iteration method ' in order to improve the capability and reliability about localizing gross errors.

1. THE TWO STEP ITERATIONS METHOD

The two step iterations method is proposed based on the properties of so-called weighted zero residual (appendix--3.) and the 'cheking correlation of residual programm '(Wang Renxiang 1998b). From the strategical point of review, the iterated weight least squares method has been contrived in two step iterations. The first step is to perform least squares iterations with weight function untill convergence. The second step is to analyse the correlation of residuals in which the standardized value is large than the critical value. Because of the first step iterated convergence, the searching areas of gross error observations are limlted in the observations in which the standardized residual is of large or strong correlation with another large standardized residual.

1.1 The First Step Iterations

In a general way, all the weight functions used in iterated least squares method or robust estimate method can be taken in the first step iterations. However the present author emphasizes that standardized residuals have to be used in every iteration at least in the last one for statistical test. Therfore Qvv.P matrix should be calculated in every iterations. The papers (Shan Jie 1988,Wang Renxiang 1990 in Chinese, appendix) have give the fast recurive algorithm for computation of Qvv.P matrix. The time comsuming for calculatng Qvv.P matrix have been overcome. As an experiment in this paper, the author gives a weight function modified from (Wang Renxiang 1989c) and used in this step iterations as fellows

$$p = \begin{cases} 1, & \dot{\lambda}_i \text{ or } \bar{\lambda}_i < C \\ 1/\lambda_i^a , & \dot{\lambda}_i < C \quad \text{for 1,2 iteration} \\ 1/\lambda_i^b , & \bar{\lambda}_i > C \quad \text{after 2 iterations} \end{cases}$$

where

$$\lambda_i = \begin{cases} \dfrac{|V_i|}{\delta_0} , & \dfrac{|V_i|}{\delta_0} > \bar{\lambda}_i \\ \bar{\lambda}_i , & \text{eleswhere} \end{cases}$$

$$\dot{\lambda} = \frac{|V_i|}{\delta_0 . K_1} = \text{enlarged residual}$$

$$\bar{\lambda}_i = \frac{|V_i|}{\delta_0 K_2} = \text{standardized residual}$$

$$K_1 = \sqrt{g_{ii}} = \text{enlarged factor}$$

$$K_2 = \sqrt{\sum_{j=1}^{m} g_{ij}^2} = \text{standardized factor}$$

$$C = 2.0, \quad a = 2.5, \quad b = 3.0\text{--}4.0$$

1.2 The Second Step Iterations

In the first step, the mistakes of localizing gross errors are from two major circumstances. The first is the gross error that can not be detected by statistical test with the standard critical value, because the magnitute of main diagonal element related to gross error observation in Qvv.P matrix and correspondene to the main component coefficient of the standardized residual MCCV (appendix----3.2) are too small. It is impossible to overcome by any iterationl method determined by the design matrix. The second, gross error revealed in the standardized residual is dispersed by the correlation coefficient of residuals and make wrong decision with statistical test. This problem can possibly be overcome by disassembling the correlation of residuals. The properties of weighted zero residual (appendix----3) as fellows plays important part in this discussion.

1.2.1 Gross error can be revealed in its weighted zero residual completly.

1.2.2 After iteration, any two observations are weighted zero, the correlation coefficient of the two observational residuals must be zero. ie. the two residuals are no longer correlated.

1.2.3 For any two observations where residuals are of strong correlation and weighted zero, the value of main component coefficient of the standardized residuals will be decreased evidently.

In the second step, the observation(s) weighted zero will be decided according to the comprehensive decison which include the analysed correlation of residuals, statistical test using standardized residuals and refered weighted zero residuals.

1.3 The Programm for the Second Step Iterations

1.3.1 Type A observation After the first step iterations, the observation where standardized residual is larger than the critical value which is 2.5 in this paper would be a possible gross error observation and called Type A observation. The correlation coefficient of residuals will be calculated and make analysis as fellows

$$\rho_{i,k} = \frac{q_{i,k}}{\sqrt{q_{ii}\, q_{kk}}} \qquad , k = 1, m \quad k \neq i$$

where i = the number of the type A observation
k = the number of another observation

For Type A observations,

if $\rho_{i,k} > 0.7$, recored $\rho_{i,k}$ and k
if $\rho_{i,k} < 0.7$, then the observation i to be decided as contained gross error and always weighted zero in sequential iterations.

1.3.2 Type B Observation Type B observation is determined by two factors. The first is the frequency of correlation coefficient of which value is larger than 0.7. The second is the magnitude of correlation coefficient.

1.3.3 Assigning Zero Weight to the Pair Observation of Type A and Type B It is allowable that more one pair observation of Type A and Type B to be assigned zero weight in an iteration, if the redundante number of the adjustment system is large enough.

1.3.4 Transferring the Weighted Zero Residuals to the Stndardized Residuals We have to transfer weighted zero residual to standardized residual, calculate the main component coefficient of the standardized residuals as well as make statistical test. In consideration of the properties of weighted zero residual, We take 1.5 as the critical value in the experiment, when the main component coefficient is smaller than 0.5.

1.4 The Factors About Comprehensive Decisions

There are five factors have to be considered in the comprehensive decisions.

1.4.1 Whether the standardized residual is larger than the critical value.

1.4.2 Checking the correlation of residuals.

1.4.3 Checking the magnitude of the main component coefficient of standardized residual.

1.4.4 Checking the magnitude of the weighted zero residual.

1.4.5 If necessary, one have to refer to the data about the previous iterations.

2. EXAMPLES ABOUT GROSS ERROR LOCATION BY THE TWO STEP ITERATIONS METHOD

We take the calculation of photo relative orientation parameters with simulated data as an example for the discussions.

design matrix A

.0	.0	1.0	.0	-1.0
.0	.0	1.0	-1.0	.0
.0	-1.0	2.0	.0	-1.0
-1.0	.0	2.0	-1.0	.0
.0	1.0	2.0	.0	-1.0
1.0	.0	2.0	-1.0	.0
-.8	-.2	2.0	-.8	-.2
.0	.8	1.64	.0	-1.0
-.04	-.16	1.04	-.2	-.8
.16	.04	1.04	-.8	-.2

Qvv.P matrix (as P = I)

.64	-.13	-.15	.11	-.06	.16	.06	-.16	.31	-.16
	.60	.18	-.12	.13	-.12	-.06	.03	-.17	-.34
		.17	.02	.06	-.13	-.15	.06	-.16	.11
			.43	-.06	.07	-.45	.01	.05	-.03
				.41	-.10	-.04	-.45	.03	.07
					.13	.03	.02	.11	-.15
	symmetry					.61	.01	.01	.01
							.62	-.08	-.02
								.71	-.18
									.70

The simulated observationl error vector of vertical parallax is

E =(-.87 -.39 1.5 -.51 .44 .08 -.87 -.75 2.11 -.75)

2.1 The Capacity of gross Error Location by First Step Iterations

It is assumed that the observations contain only one gross error and use the weight function proposed by the present author. After five times of iterations, the minimum of gross error which can be located by first step iterations is listed in Table 1.

TABLE 1 THE MINIMUM LOCATED GROSS ERROR

Point	1	2	3	4	5	6	7	8	9	10
(+)	5	4	12	8	4	11	$23 > \nabla_7 > 3$ $\nabla_7 > 36$	5	4	4
(-)	-4	-5	-9	-5	-5	-17	-4	-6	-4	-4
q_{ii}	.64	.60	.17	.43	.41	.13	.61	.62	.71	.70

The condition of this adjustment system is pretty good for gross error location, because the average value of main diagonal element of Qvv.P matrix is egual to 0.5. The main diagonal elements of Qvv.P matrix related to observation 1, 2, 8, 9 and 10 are rather big and small gross error can be located correctly. However the main component coefficient of Qvv.P matrix related to observation 6 is relatively small and only large gross error can be located. In observation 7 and 4, residuals are of strong correlation($\rho_{4,7}$ = 0.88), both observation 7 and observation 4 are decided as containing gross error. When point 7 contain gross error in the interval of 24σ -- 35σ .

2.2 The Comparasion of the Capability of the Two Step Iterations

Table 2 gives an example only observation 4 contained gross error which can not be located by first step but can be by second one.

TABLE 2 ONE GROSS ERROR LOCATION BY TWO STEP ITERATIONS METHOD

Point			1	2	3	4	5	6	7	8	9	10	Noted in comprehehsive decision
First Step	it=6	$\bar{v}$	.	.	.	5.1	.	.	5.1	.	2.5	.	error vector = E
		$\mathring{v}$	.	.	.	7.7	.	.	- 6.5	.	3.0	.	gross error ∇_4= 7
		$\rho_{4,1}$	.	.	.	.	.	.	.9	.	.	.	point 4, 7 and 9 decided
		$\rho_{8,1}$	.	.	.	.	.	.	.	.	.	.	as Type A observation.
		MCCV	.	.	.	.65	.	.	.78	.	.84	.	
Second Step	it=1	$\bar{v}$	.	.	.	1.2	.	.	1.2	.	2.3	.	point 4, 7, 9 weighted zero
		$\mathring{v}$	.	.	.	4.1	.	.	-3.4	.	2.7	.	point 9 with good observation
		MCCV	.	.	.	.29	.	.	.35	.	.82	.	point 4, 7 need further detection
	it=2	$\bar{v}$	.	.	.	1.7	.	.	.8	.	2.3	.	point 4, 7 weighted zero
		$\mathring{v}$	.	.	.	5.8	.	.	-2.1	.	2.7	.	point 7 with good observation
		MCCV	.	.	.	.30	.	.	.35	.	.82	.	point 4 contained gross error
	it=3	$\bar{v}$	.	.	.	5.3	.	.	.8	.	2.1	.	point 4 weighted zero
		$\mathring{v}$	.	.	.	8.1	.	.	-2.0	.	2.4	.	gross error complete reveal
		MCCV	.	.	.	.65	.	.	.35	.	.82	.	in the residual

Noted symbol in Table 2 and 4 :

it = sequent number of the iteration,
when it = 1, taking P = I
$\bar{v}$ = standardized residual
$\mathring{v}$ = weighted zero residual
ρ = correlation coefficient of residuals
MCCV = main component coefficient of v
. = the value is small nothing for decisions

Noted in the comprehensive decisions :

(1) The first step iterations, it = 6
(2) The second step iterations, when MCCV > 0.5, take critical value = 2.5 when 0.5 > MCCV > 0.3, take critical value = 1.5

Table 3 gives examples about the differences of capacity of localizing two gross errors by two step iterations method. In comparision with the first step iterations, some mistakes decided in first step can be corrected in second one.

TABLE 3 THE COMPARISION OF THE CAPACITY OF LOCATED GROSS ERRORS BY TWO METHODS

Correlation coefficient	Two Gross errors located by two steps	Two Gross errors located by first step
$\rho_{5,8}$ = 0.88	∇_5 = 6, ∇_8 = -6	point 5 correct point 8, 9 wrong
$\rho_{4,7}$ = 0.88	∇_4 = -12, ∇_7 = -12	point 7 correct point 4, 1 wrong
$\rho_{1,6}$ = 0.55	∇_1 = -15, ∇_6 = 15	point 1 correct point 6, 9 wrong
$\rho_{1,8}$ = -0.25	∇_1 = -8, ∇_8 = -8	point 1 correct point 8, 5, 9 wrong
$\rho_{4,10}$ = -0.05	∇_1 = -5, ∇_{10} = -5	point 1, 10 correct

2.3 The Example about Gross Errors Location by Two Step Iterations Method

We give a brief note in Table 4 about gross errors localization.

TABLE 4 GROSS ERRORS LOCATION BY TWO STEP ITERATIONS METHOD

Point			1	2	3	4	5	6	7	8	9	10	Noted in comprehehsive desions
First Step	it=6	$\bar{v}$	.	.	.	.	8.1	.	.	.	2.4	.	observational error vector = E
		$\mathring{v}$	.	.	.	.	12.5	.	.	.	2.8	.	gross error $\nabla_5 = 6$, $\nabla_8 = -6$
			.	.	.	.	.	.	.	.	.	.	point 5 as Type A observation
		$\rho_{5,i}$	.	.	.	.	.	0.8	.	3.0	.	.	point 8 as Type B observation
		$\rho_{9,i}$	.	.	.	.	.	.	.	.	.	.	point 5, 8 strong correlation
													point 9 with good observation
Second Step	it=1	$\bar{v}$	.	.	.	.	2.4	.	.	1.6	.	.	point 5, 8 weighted zero
		$\mathring{v}$	.	.	.	.	8.0	.	.	-4.4	.	.	point 5, 8 contained gross error
		MCCV	.	.	.	.	.30	.	.	.37	.	.	
First Step													observational error vector = E
	it=1	$\bar{v}$	3.9	2.9	3.6	.	2.7	3.4	2.7	.	.	.	gross error $\nabla_4 = -12$, $\nabla_7 = -12$
		$\mathring{v}$	-4.8	3.7	8.4	.	4.1	8.9	-3.4	.	.	.	point 1, 2, 3, 5, 6 and 7 are
													Type A observations
	it=6												after five iterations only
		$\bar{v}$	3.7	.	.	.	2.2	.	2.3	.	.	.	point 1 as Type A observation
		$\mathring{v}$	-4.6	.	.	.	3.3	.	-3.0	.	.	.	point 5, 7 near Type A obs.
		$\rho_{1,i}$	.	.	.	.9	.	.9	.	.	.	.	residual of point 4 is strong
		$\rho_{7,i}$	.	.	.7	3.0	.	.	.	.	.	.	correlation with point 1, 7
													point 4 as Type B observation
Second Step	it=1	$\bar{v}$	1.9	.	.	1.1	1.3	.	1.7	.	.	.	point1,4, 5, 7 weighted zero
		$\mathring{v}$	3.1	.	.	-5.4	2.4	.	-7.0	.	.	.	point 4, 7 need further
		MCCV	0.6	.	.	0.2	.55	.	0.24	.	.	.	detection
	it=1	$\bar{v}$	.	.	.	3.9	.	.	4.7	.	.	.	point 4, 7 weighted zero
		$\mathring{v}$	.	.	.	-13.0	.	.	-13.1	.	.	.	point 4, 7 contained gross error
		MCCV	.	.	.	0.30	.	.	0.35	.	.	.	

From Table 3 and Table 4, we find that when weighted zero is assigned a pair observation in which residuals are of strong correlation after iteration, the correlation of residuals have been disspated and have made convenient condition for decision of gross errors localization, because the magnitude of main component coefficient of standardized residual is decreased.

Comparasion of the observations in which residuals are not correlated shows that the capability of localizing gross errors would decrease even if the critical value is 1.5 instead of 2.5.

conclusions

Gross error location, especially for more one gross error, is a problem that has not been completely solved in adjustment. From the strategical point review, to develop the iterated weighted least squares method to two step iterations method is a powerful way to improve the capability and relibility for gross errors location. After the first step iterations, the searching gross error observations is in a comparatively limited area.

The experiment proved that the second step iterations play an important part in correcting the mistakes of decision about gross error observation(s) in the first step iterations. In the second step, the decision about gross error observation(s) are concerned with the magnitude of of standardized residuals and weighted zero residuals, the correlation of residuals as well as the main component of standardized residual MCCV. When the value of MCCV is very small, the comprehensive decisions will be particulary difficult. One has to further investigate in gross errors locationin order to get more knowledges about comprehensive decisions.

APPENDIX

MATHERMATICAL ANALYSIS ABOUT Qvv.P MATRIX

ABSTRICT:

The increment of Qvv.P matrix due to the variation of weight matrix P can be exponded by using Neumann's series and obtained both approximat and regirous expressions, which can be applicated in discussing the problams about the capability and the relibility of gross errors location.

The appendix emphasizes in discussing the proparties of co-called ' weighted zero residual ' and the fast recursive algorithm for calculating Qvv.P matrix and its limitations. Serval examples with simulated data have been computed for the discussions.

KEYWORDS: Qvv.P matrix, Standardized residual, Gross error location, Weighted zero residual.

INTRODUCTION

up to now, there are many weight functions used for localizing gross error by iterated weight least squares method and robust estimation. There is no unitized theory in use. However the characteristics of matrix Qvv.P and the variability of the relationship between Qvv.p and weight matrix p can be used for discussing the problems of localizing gross errors in a general way.

The appendix is based on the papers(Wang Renxiang,1986a, 1988b) and makes further development. The results would be benefilted for the investigation of gross errors localization.

1. THE RELATIONSHIP BETWEEN Qvv.P MATRIX AND THE INCREMENT OF MATRIX P

1.1 The Expanded Qvv.p Matrix

According to least squares method, the residuals of observation will be

$$V = -G \cdot E \tag{1}$$

$$G = Q_{vv} \cdot p = I - A \cdot (A^TPA)^{-1} \cdot A^T \cdot P \tag{2}$$

where A = the design matrix
Qvv = the cofactor matrix of residuals
P = the weight matrix
E = the vector of observational errors

let $N = A^TPA \qquad R = AN^{-1}A, \qquad u = R \cdot P$

then $G = I - U \tag{3}$

to simply, we take matrix P is a diagonal one, ie.

$$P = \mathrm{diag}(p_1 \cdots P_i \cdots p_m$$

if ΔP is the increment of P, ie. $\dot{P} = P + \Delta P$
where

$$\Delta P = \mathrm{diag}\ \{\delta p_1 \cdots \delta p_i \cdots \delta p_m\} = \sum_{i=1}^{m} p_i$$

$$\Delta P_i = \mathrm{diag}\ \{0 \cdots \delta\ p_i \cdots 0\}$$

δp_i = the increment of P_i

According to the least squares method we get

$$\dot{G} = I - \dot{U}\ ,\quad \dot{U} = \dot{R} \cdot \dot{P} = A\dot{N}^{-1}A^T \cdot \dot{P}\ ,\quad \dot{N} = A^T\dot{P}A = N + \Delta N\ ,\quad \Delta N = A^T\Delta PA$$

$\dot{N}^{-1}$ can be expanded by using Neumann's series and obtained

$$\Delta G = \sum_{n=1}^{\infty} (-1)^n\ (R \cdot \Delta P)^n \cdot G \tag{4}$$

$$\dot{G} = G + \Delta G \tag{5}$$

1.2 The Increment of Qvv.P as Weight Matrix P only p changs with δp_i

As P_i gets an increment δp_i , the increment of Qvv.P matrix will be

$$\Delta G = \frac{\delta P_i}{P_i}\ [1 + (g_{ii} - 1)\frac{\delta P_i}{P_i} + \cdots + ((g_{ii} - 1)(\frac{\delta P_i}{P_i}))^n + \cdots]\ \Delta R_i \cdot G$$

the equation above can be compressed as

$$\Delta G = S_i \cdot \Delta R_i \cdot G$$

where

$$S_i = \frac{\delta p_i}{p_i}\ (1 - (g_{ii} - 1)\frac{\delta p_i}{p_i})^{-1},$$

$$\Delta R_i = \begin{pmatrix} \varphi & \begin{matrix} g_{1i} \\ \vdots \\ (g_{ii} - 1) \\ \vdots \\ g_{mi} \end{matrix} & \varphi \end{pmatrix}$$

then $\dot{G} = T_i \cdot G \tag{6}$

where
$$T_i = \begin{pmatrix} 1 \cdots 0 \cdots S_i \cdot g_{ii} \cdots 0 \cdots 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 \cdots (1 + S_i(g_{ii} - 1)) \cdots 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 \cdots 0 \cdots S_i \cdot g_{mi} \cdots 0 \cdots 0 \end{pmatrix}$$

Equation(6) is a regorous expression for the changed Qvv.P matrix and provides that the denominator of S_i is not equal to zero, ie.

$$P_i(1 - (g_{ii} - 1)\frac{\delta P_i}{P_i}) \neq 0$$

1.3 The Approximate Expressions of Qvv.P Matrix

Excluding the height order of equation(4), we get

$$\dot{G} = G - R\begin{pmatrix} \delta p_i & & & \varphi \\ & \ddots & & \\ & & \delta p_i & \\ & & & \ddots \\ \varphi & & & & \delta p_m \end{pmatrix} \cdot G \tag{7}$$

using S_i instead of δp_i (i= 1, m) in equation(7), we have

$$\dot{G} = G - R\begin{pmatrix} S_i & & & \varphi \\ & \ddots & & \\ & & S_i & \\ & & & \ddots \\ \varphi & & & & S_m \end{pmatrix} \cdot G \tag{8,a}$$

Equation(8,a) can also be expressed as

$$\dot{G} = G + \begin{pmatrix} (g_{11} - 1)\bar{S}_1 \cdots\cdots g_{1i} \cdot \bar{S}_i \cdots\cdots g_{1m} \cdot \bar{S}_m \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ g_{i1} \cdot \bar{S}_1 \cdots\cdots (g_{ii} - 1) \cdot \bar{S}_i \cdots g_{im} \cdot \bar{S}_m \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ g_{m1} \cdot \bar{S}_1 \cdots\cdots g_{mi} \cdot \bar{S}_i \cdots (g_{mm} - 1) \cdot \bar{S}_m \end{pmatrix} \tag{8,b}$$

where

$$\bar{S}_i = S_i \cdot P_i$$

Equation(8,a) or (8,b) is of more precision than equation(7). However all the approximate equations are only used for analysis and discussions about gross error location rather than to calculation of value of Qvv.p matrix.

Equation(7) and (8) satisfy the condition tr(G)=r, where r is the redundant number because the main diagonal element of equation(8,b) is

$$\Delta g_{ii} = (g_{ii} - 1) \cdot g_{ii} \cdot \bar{S}_i + \sum_{k=1}^{m} g_{ik} \cdot g_{ki} \cdot S_k , \quad (K \neq i)$$

and $tr\,(\Delta G) = \sum_{i=1}^{m} (g_{ii} - 1)\, g_{ii} \cdot \bar{S}_i + \sum_{i=1}^{m}\sum_{k=1}^{m} g_{ik} \cdot g_{ki} \cdot \bar{S}_i , \quad (K \neq i)$

Matrix Qvv.P is a singular idempotent matrix from which we get

$$g_{ii} = \sum_{k=1}^{m} g_{ik} \cdot g_{ki} , \quad g_{ii} - g_{ii}^2 = \sum_{k=1}^{m} g_{ik} \cdot g_{ki} , \quad (K \neq i)$$

$$\sum_{1}^{m} (g_{ii} - 1) g_{ii} \cdot \bar{S}_i = -\sum_{i=1}^{m} \sum_{k=1}^{m} g_{ik}\ g_{ki} , \quad (K \neq j)$$

so tr(G) = 0 and tr(G) = r

The expanded expressions of matrix Qvv.P are very useful tool for the discussion of localizing gross errors. The present author gave the results of capability of localizing gross errors about iterated weight least squares method (Wang Renxiang, 1988b) and gave the conclusion about the power value of negative power function that taken 2.5--4.0 is better then 1.0--2.0 in (Wang Renxiang, 1989c) by using the first order expression of matrix Qvv.P. In this paper, we will further extend the applications about the expanded Qvv.P matrix.

2. CALCULATING Qvv.P MATRIX BY USING FAST RECURSIVE ALGORITHM

As iterated weight leaset squares is performed, Qvv.P matrix will be changed with changing weight matrix P. It is time consuming for caculating Qvv.P matrix according to the eqation(2), when the normal equation with large dimension. If matrix Qvv.P is computed in first iteration, then the sequential iterated Qvv.P matrix can be calculated by using equation(6) and get

$$G^m = \prod_{k=1}^{m} T_i^k \cdot G^0 \tag{9}$$

where

m,k = up foot-not, denoting the number of repeated computation

i = the number of row and column of changing diagonal element of weight matrix P

$G^0 = I - A\,(A^T P A)^{-1} A^T \cdot P$, where P is the initial weight matrix P

G^m= computed Qvv.P according to the fast recursive algorithm with changed weight matrix

$$T_i^K = \begin{pmatrix} 1 \cdots 0 \cdots S_i \cdot g_{ii}^{k-1} \cdots 0 \cdots 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 \cdots\cdots (1 + S_i(g_{ii}^{k-1} - 1) \cdots 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 \cdots 0 \cdots S_i\, g_{mi}^{k-1} \cdots 0 \cdots 1 \end{pmatrix} \tag{10}$$

If all the elements of matrix P have got their own increment, calculate with the fast recursive algorithm according to k=i, otherwisse, if some elements of matrix P nothing changed,then the calculating will have to jump over the order.

Qvv.P matrix calculated from the normal equation with large dimension by equation(9) is more fast then by equation(2), because after the first iteration, there is no inverb matrix in the computation.

3. THE PROPERTIES OF WEIGHT ZERO RESIDUAL

For localizing gross errors, in some robust estimate or iterated least squares method, at least the last iteration is always weighted zero value (or near zero) to the observation of which residual is rather large value in absolute. In this paper,we defined the residual computed with weighted zero to the observation as so-called ' weighted zero residual ' and symbolized $\mathring{v}_i$ (Stefanovic, 1985 called ' swep residual '). It is necessary to investigate the properties of weighted zero residual for further discussion about gross errors localization.

Asumming that P = I. Firstly, we assign zero weight to observation i, ie. $\delta p_i = -1$ and $S_i = -1 / g_{ii}$. According to equation(9) we have

$$G^1 = T_i^1 \cdot G^0$$

Using equatio(9) and (10), we get

$$g_{ij}^1 = \frac{q_{ij}}{q_{ii}} \qquad g_{ii}^1 = 1 \qquad j = 1, m$$

From above, we know that all the elements of i th column are enlarged by a factor of $1/g_{ii}$. If only one observation i is assigned zero weight then its weighted zero residual $\mathring{v}_i$ can be calculated from v_i directly. ie.

$$\mathring{v}_i = v_i / q_{ii} \tag{11}$$

On the otherhand, the elements of k th column will be

$$g_{kj}^1 = q_{kj} - q_{ki} \cdot q_{ij} / q_{ii} , \quad k = 1, m , \quad K \neq i$$

Using

$$\rho_{ik} = q_{ik} / \sqrt{q_{ii} q_{kk}} = q_{ki} / \sqrt{q_{ii}\ q_{kk}}$$

then $g_{kk}^1 = q_{kk} - \rho_{i,k}^2 \cdot q_{kk} = (1 - \rho_{i,k}) q_{kk}$

Let $\rho = 0.7$ $g_{ii} = g_{kk} = 0.5$ we get $g_{ik}^1 = 0.7$ $g_{kk}^1 = 0.25$. In this cse, if gross error ∇_k is included in observation k then to compare the magnitude of residuals, the observation i will be larger then the observation k. So, when correlation coefficient of residuals is big. The gross errors localizing is not reliable. If the absolute value of residual is used as statistical equantity.

Secondly, we further assign zero weight to observation k , ie. $\delta p_k = -1$, $S_k = -1/g_{kk}^1$, then we get

$$\dot{G} = G^2 = T_k^2 \cdot T_i^1 \cdot G^0 = T_k^2 \cdot G^1$$

According to equation(9) and (10), we have

$$\dot{g}_{ij} = (q_{ij} \cdot q_{kk} - q_{kj} \cdot q_{ik}) / (q_{ii} \cdot q_{kk} - q_{ki} \cdot q_{ik})$$

$$\dot{g}_{kj} = (q_{kj} \cdot q_{ii} - q_{ij} \cdot q_{ki}) / (q_{ii} \cdot q_{kk} - q_{ki} \cdot q_{ik})$$

and $\dot{g}_{ii} = 1$, $\dot{g}_{kk} = 1$, $\dot{g}_{jk} = 0$, $\dot{g}_{ki} = 0$

Now there is nolonger correlation between weighted zero residules $\mathring{v}_i$ and $\mathring{v}_k$. In the same way, we obtain

$$\begin{aligned} \dot{g}_{ij} &= (q_{ij} \cdot q_{kk} - q_{kj} \cdot \rho_{i.k} \sqrt{q_{ii} \cdot q_{kk}}) / (1 - \rho_{i,k}^2)\, g_{ii} \cdot q_{kk} \\ \dot{g}_{ij} &= (q_{kj} \cdot q_{ii} - q_{ij} \cdot \rho_{i,k} \sqrt{q_{ii} \cdot q_{kk}}) / (1 - \rho_{i,k}^2)\, g_{ii} \cdot q_{kk} \end{aligned} \tag{12}$$

In the following, we take two conditions for further discussions.

3.1 Let $\rho_{i,k} = 0$

If $\rho_{i,k} = 0$, then

$$\dot{g}_{ij} = q_{ij} / q_{ii} , \qquad \dot{g}_{kj} = q_{kj} / q_{kk}$$

Therefore weihgted zero residuals can be computed by

$$\mathring{V}_i = V_i / q_{ii} \quad , \quad \mathring{V}_k = V_k / q_{kk}$$

Assuming observation i with a gross error ∇_i , then

$$\mathring{V}_i = \nabla_i + \sum_{j=1}^{m} g_{ij} \cdot \varepsilon_j / g_{ii} \quad , \; j=1,m \; , j \neq 1$$

Gross error ∇_i is reverl completely in its weighted zero residual. It must be noted that when q_{ii} is very small , the compoents related no-gross error observations are enlarged evidently in $\dot{g}_{ij}$. It is possible that $\mathring{V}_i$ will have a big magnitude even the observation do not have any gross error. Therefore one using standardized residual (symbolized $\bar{V}_i$) as statistical equantity to do rigorous statistical test for each iteration is quite reasonable.

3.2 Let $\rho_{i,k} \geq 0.7$

We take $\rho \geq 0.7$ as the critical value of correlation of residuals and we symbolized MCCVi as the main component coefficient of standardized residual $\bar{V}_i$. In observation i and observation k of which main component coefficient of standardized residual is as follows :

$$MCC\bar{V}_i = \dot{g}_{ii} / \sqrt{\sum_{j=1}^{m} g_{ij}^2} \qquad MCC\bar{V}_k = \dot{g}_{kk} / \sqrt{\sum_{j=1}^{m} g_{kj}^2} \tag{13}$$

When $\rho > 0.7$, the denominator of equation(12) will be very small and some value of $\dot{g}_{ij}$, $\dot{g}_{kj}$,(j=1,m ,j$\neq$i ,k$\neq$i) would be enlarged evidently. Because of residual i and residual k are strong correlation. Usually, there are several elements of i th and k th column of Qvv.P matrix satisified that $q_{ij} = -q_{kj}$. But $\dot{g}_{ii}$, $\dot{g}_{kk}$, is still equal 1.0. After the residuals have been standardized , one would find that the magnitude of MCCVi or MCCVk will be decreased, as compared with the magnitude computed by using $\rho = 0$, and the capability and the relibility of gross errors localizing would be decreased as well.

Frome the discussions above, we give some conclusions about weighted zero residual as fellows :

3.2.1 Gross error can be revealed in its weighted zero residual completly. Generally speaking, the observation contained gross error its weighted zero residual is of rather large magnitude.

3.2.2 Weighted zero residual is not suitable as statistical quantity for statistical test. It is necessary to be transformed to standardized residual in order to get rigerous statistical test.

3.2.3 The maximum value of main component coefficient of standardized residual MCCVi is $\sqrt{q_{ii}}$ of which magnitude is determined by design matrix. It is impossible to enlarge its value by the way of iteration weighting any small value to the observation.

3.2.4 If any two observations are assigned zero weight, after iteration, the correlation coefficient of the two observational residuals must be zero.

3.2.5 For any two observations in which residuals are strong correlation and weighted zero, the value of main component coefficient of the standardized residuals will be decreased evidently.

The correlation of residuals is an important factor to make mistakes of localizing gross errors. It is difficult to overcome this mistakes by the way of improving the iterational weight function. It is better from the statistical point view to investigate gross errors localization by helping the property of weighted zero residual.

4. CALCULATION EXAMPLES

We take the calculation of photo relative orientation parameters with simulated data for example.

design matrix A

.0	.0	1.0	.0	-1.0
.0	.0	1.0	-1.0	.0
.0	-1.0	2.0	.0	-1.0
-1.0	.0	2.0	-1.0	.0
.0	1.0	2.0	.0	-1.0
1.0	.0	2.0	-1.0	.0
-.8	-.2	2.0	-.8	-.2
.0	.8	1.64	.0	-1.0
-.04	-.16	1.04	-.2	-.8
.16	.04	1.04	-.8	-.2

design matrix B

.0	.0	-1.0	.0	-1.0
.0	.0	-1.0	-1.0	.0
.0	-1.0	-2.0	.0	-1.0
-1.0	.0	-2.0	-1.0	.0
.0	1.0	-2.0	.0	-1.0
1.0	.0	-2.0	-1.0	.0
-.2	-.8	-2.0	-.2	-.8
-.8	-.2	-2.0	-.8	-.2
-.2	-.8	-2.0	-.2	-.8
.8	.2	-2.0	-.8	-.2

The vector of simulated observational errors of vertical parallax is

E =(-.87 -.39 1.5 -.51 .44 .08 -.87 -.75 2.11 -.75)

4.1 Calculating Qvv.P Matrix by Fast Recursive Algorithm

First we take design matrix A and P = I, according to equation(2) to compute G, then using

P=(.9 .2 .7 .4 .5 .5 .6 .3 .8 0)

According to equation(2) and equation(9) to compute G respectively. The discrepancy of the elements of matrix G computed in the two ways is very small and the average of the absolute value of the discrepany is equal to 0.917×10^{-15} . Design matrix B is computed in same way with good results. However there are two cases in which the mastake will be made by using fast recursive algorithm.

4.1.1 Example 1 First we take design matrix B and P = I computed matrix G with equation(2). the elements of 1 th and 2 th column of matrix G is

q_{1i} = (.37 -.37 -.13 .13 -.13 .13 -.08 .08 -.08 .08)

q_{2i} = (-.37 .37 .13 -.13 .13 -.13 .08 -.08 .08 -.08)

Because the correlation coefficient of residual between observation 1 and 2 is equal to 1.0, the donominator of S_2 will be equal to zero and the computed results will be wrong after by the fast recuvsive algorithm with weight matrix $\dot{P}$=diag(0 0 1 1 1 1 1 1 1 1) and equation(9).

4.1.2 Example 2 First We taking matrix B and weight matrix P=diag(1 1 0 1 1 1 1 1 1 1) and computed G with equation(2). Then use weight matrix $\dot{P}$=diag(.9 .2 .7 .4 .5 .5 .6 .3 .8 0) according to fast recursive algorithm , because the denominator of S_3 equal to zero, the computed G is also wrong.

The above results give the examples of limitation by fast recursive algorithm for calculating Qvv.P matrix. The first example can not be overcome by any way except changing the design matrix. However the second one can be treated in an approximate way, for instance, one takes p = 0.01 instead of p = 0, the computed results will be correct.

4.2 Weighted Zero Residual

4.2.1 The Main Component Coefficient of Standardized Residual We take matrix A and P = I, and any two observations with zero weight and calculated MCCVi in Table 1.

TABLE 1 THE MAIN COMPENENT COEFFICIENT OF STANDARDIZED RESIDUAL

Point	P=I	$\rho_{2,3}=.56$ $P_2=P_3=0$	$\rho_{5,8}=.88$ $P_5=P_8=0$	$\rho_{4,7}=.88$ $P_4=P_7=0$	$\rho_{4,10}=-.05$ $P_4=P_{10}=0$	$\rho_{6,9}=.36$ $P_6=P_9=0$
1	.80					
2	.77	.64				
3	.41	.34				
4	.65			.30	.65	
5	.64		.30			
6	.36					.33
7	.78			.35		
8	.79		.30			
9	.84					.78
10	.84				.83	

From Table 1, one may find that when correlation coefficient of residuals is increased the main component coefficient of standardized residual will be decreased.

4.2.2 <u>The Weighted Zero Residual and Standardized Residual When Observations without Gross Error</u> We take design matrix A, error vector E, different weight matrix P and calculated results listed in Table 2.

TABLE 2 WEIGHTED ZERO RESIDUAL AND STANDARDIZED RESIDUAL

	Example 1				Example 2			
Point	$\bar{V}$	$\hat{V}$	ε	p	$\bar{V}$	$\hat{V}$	ε	p
1	1.0	- 1.7	-.87	1	1.7	- 2.7	-.87	1
2	.9	- 1.8	-.39	1	1.6	3.6	-.39	1
3	2.1	-16.9	1.5	0	1.0	-7.8	1.5	0
4	2.1	-17.0	-.51	0	1.0	2.6	-.51	0
5	.2	- .4	-.44	1	.7	1.4	.44	1
6	.7	- 6.6	.08	1	1.0	-9.8	.08	0
7	2.2	-17.1	-.87	0	2.1	-16.4	-.87	1
8	.3	.4	-.75	1	.2	- .4	-.75	1
9	1.0	2.1	2.11	1	2.1	3.1	2.11	1
10	.8	.8	-.75	1	1.9	- 1.9	-.75	1

From Table 2, we find that even observations do not contain any gross error, the magnitude of some weighted zero residual is still large. However the magnitude of standardized residual always is not too large. Therefore using weighted zero residual as statistical quantity may be possible to get wrong decision about localizing gross errors.

5. CONCLUSIONS

The approximate expressions of Qvv.P matrix are power tools for discussion of gross errors localization. The fast recursive algorithm to be calculated Qvv.P matrix would be very helpful using standardized residuals as statistical quantity for statistical test in every iteration. One should be careful about the limitation of the fast recursive algorithm in practical adjustment.

From the analysis of the properties about correlation of residuals and weighted zero residual. We not only have to further improve the weight function but also have to from the strategical point review investigate about gross errors location.

REFERENCES

Shan Jie, 1988. A fast recursive algorithm for repeated computation of the reliability matrix Qvv.P. Acta Geodetica et Cartographica Sinica. vol. 17, No. 4 : 269-227.

Stefanovic, P., 1985. Error treatment in photogrammetric digital techniques. ITC Journal. 1985-2.: 93-95.

Wang Zhizhuo, Theory of gross error detection. Photogrammetric principle (with remote sensing) Publishing House of Surveying and Mapping, Bejing, May 1990. chapter 17 : 196-217.

Wang Renxiang, 1986a. Studies on weight function for robust iterations. Acta Geodetica et Cartographica Sinica. vol. 15, No. 2 : 91-101.

Wang Renxiang, 1988b. Theoretical capacity and limitations of localizing gross errors by robust adjustment. ISPRS, Kyoto, Commission III, July, 1988.

Wang Renxiang, 1989c. Effects of parameters of weihgt function for the iterated weighted least squares adjustment. Acta Geodetica et Cartographica Sinica. 1989/1 (English version). (Special issue of siminar on the Wang Zhizuo's academic thinking) 86-96.