BUNDLE ADJUSTMENT IN NO NEED OF APPROXIMATIONS OF PARAMETERS

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ABSTRACT

Bundle adjustment has been widely used in orientation and camera calibration. But since the observation equations are non-linear, approximations of parameters are necessary at the start of adjustment. This paper discusses a method and a procedure to evaluate the approximations automatically associated with any model or object space coordinate system. This method can realize semi-automatic bundle adjustment or photogrammetry without control points. The method is based on relative orientation by the linear coplanarity condition and decomposition of rotation matrices to angular elements. This is validated by experiments of simple orientation of a pair of photographs and camera calibration without control points using a 3-D target field.

Key Word: Bundle Adjustment, Automatic Adjustment, Camera Calibration, Close Range Photogrammetry, Approximations of Parameters

1. INTRODUCTION

Bundle adjustment has been widely used in camera calibration and triangulation. But since observation equations are non-linear, approximations of all parameters are required at the beginning of computation.

In close-range photogrammetry the approximations of exterior orientation parameters are usually recorded at exposing positions. But it is time consuming and sometimes hard, because a convergent or parallel imaging configuration rather than vertical one is often used. For a digital plotter (digital-image-based plotter) which is now being developed in many organizations (Lohmann, 1989, Ohtani, 1989), easy manipulation is substantially required by operators who are not familiar with photogrammetry. Hence an automatic or semi-automatic adjustment procedure is now strongly called for.

This paper shows a method to automatically calculate approximations of exterior orientation parameters and coordinates of object points associated with any model or object space coordinate system. The method is based on relative orientation using the linear coplanarity condition and decomposition of rotation matrices to angular elements (Hattori, 1991).

In practice the purpose of many industrial measurements is focused only on shapes of objects, not absolute coordinates. And camera calibration works also can be executed only by the coplanarity condition in any coordinate system (Fraser,1992). The authors' method solves the problem about the selection of a coordinate system, and realizes photogrammetry without control points. It is very useful in digital plotters, because one can easily define any coordinate system on the screen, observing a model stereo-optically.

2. OUTLINE OF EVALUATION OF INITIAL VALUES OF PARAMETERS

Fig.1 shows an example of an imaging configuration in a camera calibration which will be again referred to in experiments. A three dimensionally allocated targets are imaged convergently at various positions and with various camera rotations. The following is a flow of the procedure to obtain approximations of parameters.

(1) Overlapping photographs are separated to each independent model. Rotation matrices of independent models are evaluated and decomposed to angular elements (see 3.1).
(2) The independent models are linked to make a global model (see 4.1).
(3) If necessary, the global model coordinate system is transformed to the object space coordinate system using more than three control points (see 4.2).
(4) Object space coordinates of target points are calculated. Then the rotation matrix of each photograph in the object space coordinates system (or in the global model coordinate system) is decomposed to angular elements (see 4.3).

3. RELATIVE ORIENTATION BY THE LINEAR COPLANARITY CONDITION

3.1 Coplanarity condition

Let us start with a pair of overlapping photographs. The interior orientation is assumed complete. Model coordinates of two corresponding points are expressed, as shown in Fig.2-1, as

\[
\begin{bmatrix}
X_1 \\
Y_1 \\
Z_1
\end{bmatrix} = \begin{bmatrix}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{bmatrix} \begin{bmatrix}
x_1 \\
y_1 \\
1
\end{bmatrix} + c
\]

\[
\begin{bmatrix}
X_2 \\
Y_2 \\
Z_2
\end{bmatrix} = \begin{bmatrix}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{bmatrix} \begin{bmatrix}
x_2 \\
y_2 \\
1
\end{bmatrix} + c
\]

where \((x_1, y_1, -c)^T, (x_2, y_2, -c)^T\) are photogrammetric coordinates, \((X_1, Y_1, Z_1)^T, (X_2, Y_2, Z_2)^T\) are model coordinates, \(c\) is a camera distance and \(B\) is a base length (unity with an unknown sign). The coplanarity condition:
\[
Y_{p_1}Z_{p_2} - Z_{p_1}Y_{p_2} = 0 \quad (2)
\]
is rewritten to the form;

\[
P_1 x_1^2 + p_2 x_2 + p_3 x_1 (-c) q_1 y_1 y_2 + q_2 y_1 y_2 + q_3 y_1 (-c) + r_1 (-c) x_2 + r_2 (-c) y_2 + r_3 (-c) (-c) = 0 \quad (3)
\]
where

\[
p_1 = m_{21} n_{31} - m_{31} n_{21}, \quad p_2 = m_{21} n_{32} - m_{31} n_{22}, \quad p_3 = m_{22} n_{31} - m_{32} n_{21}, \quad q_1 = m_{22} n_{32} - m_{32} n_{22}, \quad q_2 = m_{23} n_{31} - m_{33} n_{21}, \quad q_3 = m_{23} n_{32} - m_{33} n_{22},
\]

\[
r_1 = m_{23} n_{31} - m_{33} n_{21}, \quad r_2 = m_{23} n_{32} - m_{33} n_{22}, \quad r_3 = m_{23} n_{33} - m_{33} n_{23}. \quad (4)
\]

It is easy to see that a vector has a relation;

\[
a^T a = 2
\]
Expressing eq. (3) in the form of an observation equation

\[
X_3 = Y,
\]
where \(X\) is a design matrix and \(y\) is a residual vector, one can solve \(\beta\) by minimizing \(v^T v\). An objective function for this purpose becomes with a Lagrange multiplier \(u\)

\[
\min v^T v + u (S^T S)^{-2} \quad (6)
\]
By differentiating eq. (5) with \(X\), one gets

\[
(X^T X - u I) S = 0 \quad (7)
\]
Namely \(a\) is an eigen-vector and \(u\) is a variance of residuals; \(|v|^2/2\). If an imaging configuration is good, only one \(u\) that is near zero is obtained. Otherwise multiple candidates of \(u\) may be obtained, out of which the correct one is determined by the following procedure.

3.2 Determination of the rotation matrices and angles

Then the rotation matrices \((n_{ij})\) and \((m_{ij})\) are evaluated from the vector \(a\). Even though Fig. 2-1 is assumed to be correct, Figs.2-2, 2-3, 2-4 as well as 2-1 are included in solutions. Figs.2-1 and 2-2 are equivalent, whereas Figs.2-3 and 2-4 are false, because they are turned over into a negative position.

The rotation matrices must be defined as;

\[
(m_{ij}) = \begin{bmatrix}
\cos \phi_i & \cos \phi_i & -\sin \phi_i \\
\cos \phi_i & \cos \phi_i & -\sin \phi_i \\
\cos \phi_i & \cos \phi_i & -\sin \phi_i
\end{bmatrix} \begin{bmatrix}
\cos \phi_i & \cos \phi_i & -\sin \phi_i \\
\cos \phi_i & \cos \phi_i & -\sin \phi_i \\
\cos \phi_i & \cos \phi_i & -\sin \phi_i
\end{bmatrix}
\]

It should be noted that the rotation order in the definition is unique. For other orders it can be shown that there are some angles at which the rotation matrix becomes singular and fails to be decomposed to angular elements.

3.3 Evaluation of \(\phi_1\)

Since \(m_{23} = 0\), from eqs.(4)

\[
m_{33} n_{21} = -r_1,
\]
\[
m_{33} n_{22} = -r_2,
\]
\[
m_{33} n_{23} = -r_3. \quad (9)
\]
And then

\[
m_{33} \left( n_{21}^2 + n_{22}^2 + n_{23}^2 \right) = r_1^2 + r_2^2 + r_3^2.
\]
Since the photographs are assumed diapositive, \(m_{33} > 0\). From the orthogonality of \((n_{ij})\),

\[
m_{33} = \sqrt{r_1^2 + r_2^2 + r_3^2}. \quad (10)
\]
From eq. (10) two candidates of \(\phi_1\) are obtained. Which is correct is suspended here. Then from eqs.(9)

\[
n_{21} = -r_1/m_{33},
\]
\[
n_{22} = -r_2/m_{33},
\]
\[
n_{23} = -r_3/m_{33}. \quad (11)
\]
Multiplying the first, second and third of eqs.(4) with \(n_{21}, n_{22}\) and \(n_{23}\) respectively and summing them up, one obtains

\[
m_{31} = - (p_1 n_{21} + p_2 n_{22} + p_3 n_{23}). \quad (12-1)
\]
\[
\begin{align*}
\begin{bmatrix} x \\ z \end{bmatrix} &= S \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_m \\ z_m \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},
\end{align*}
\]

is commonly used for 3-D space transformation, where \( S \) is a scale, \( A = (A_{ij}) \) is an orthogonal matrix and \( B = (b_i) \) is a translation vector. \( B \) and \( S \) are evaluated from gravity centers and a scale ratio of two coordinate systems. Thus eq.(18) is reduced to the form:

\[
X_i = A X_{M1}, \quad (i=1,2,n) \quad (19)
\]

where suffix \( i \) means control point No., \( X_i \) and \( X_{M1} \) are coordinate vectors associated with the object space coordinate system and the global model coordinate system respectively. Their origins are assumed already shifted to respective gravity centers and \( X_i \) are assumed to be scaled by \( S \). The matrix \( A \) is determined so as to minimize

\[
E = \sum \left( A X_{M1} - X_i \right)^T \left( A X_{M1} - X_i \right) \quad (20)
\]

This problem was already solved by some researchers (Arun, 1987, Horn, 1988). The authors adopted the Arun's method: By expanding eq.(20) one obtains

\[
E = \sum_{i=1}^{n} \left( X_i^T A X_{M1} - X_i^T X_i \right) - 2X_{M1}^T A T \left( X_i - X_i \right), \quad (21)
\]

\( E \) is minimized when

\[
\text{Trace} \left( \Sigma \left( X_i^T A X_{M1} \right) \right) = \text{Trace} \left( A^T \Sigma \left( X_i^T X_i \right) \right) \quad \forall i = 1 \quad (22)
\]

is maximized. With appropriate orthogonal matrices \( U, V \) which singular-value-decompose \( \Sigma \left( X_i^T X_i \right) \) to

\[
\Sigma \left( X_i^T X_i \right) = W \Lambda U^T, \quad (22)
\]

where \( \Lambda \) is a diagonal matrix, the solution of the matrix \( A \) is given as

\[
A = V U^T, \quad (22)
\]

4.3 Evaluation of angular elements

After all rotation matrices \( (M_{ij}) \) associated with the object space coordinate system (or global model coordinate system) are obtained, they are decomposed to angular elements. Let the matrices related to angular elements \( K, \Theta \) and \( \Phi \) be expressed simply as \( [K], [\Theta] \) and \( [\Phi] \). Here angles are expressed by capital letters. If the rotation order of angles is fixed, the matrix \( (M_{ij}) \) can be singular and unable to be decomposed to unique angular elements. In order to assure unique decomposition, one has to change the order of rotations depending on the values of elements of the rotation matrix; i.e.,

1) If \( M_{ij} = \pm 1 \), \( (M_{ij}) = [Q][\delta][K] \)
2) If \( M_{ij} = \pm 1 \), \( (M_{ij}) = [K][\delta][Q] \)
3) If \( M_{ij} = \pm 1 \) and \( M_{ij} = \pm 1 \), \( (M_{ij}) = [Q][\delta][K] \)

Since the treatments for any cases are similar, here only case 1) is discussed. From equation \( \sin \Theta = -M_{13}, \) one gets two candidates for \( \delta \) for \( -\Theta \leq \Theta \leq \Theta \).

\[
\sin \Theta = \frac{M_{21}}{\cos \Theta}, \quad \cos \Theta = \frac{M_{33}}{\cos \Theta}, \quad \cos \Theta = \frac{M_{11}}{\cos \Theta}, \quad \sin \Theta = \frac{M_{12}}{\cos \Theta} \quad (24)
\]

For each candidate for \( \delta, \Theta \) and \( K \) are determined uniquely. They are tested on whether to satisfy the following equations.

\[
\begin{align*}
\cos Q &+ \sin Q \sin \delta \cos K = M_{21} \\
\cos Q &+ \sin Q \sin \delta \sin K = M_{22} \\
\sin Q &+ \cos Q \sin \delta \cos K = M_{31}(25) \\
-M &+ \cos Q \sin \delta \sin K = M_{32}
\end{align*}
\]

Sets of candidates which do not satisfy all the equations are discarded.

5. EXPERIMENTS

The procedure was applied to two experiments for validity check: A simple relative orientation of a pair of stereo photographs and a camera calibration without control points.

5.1 Relative orientation of a pair of stereo photographs

A target field of 5m x 5m x 0.5m (depth) was imaged by a 35mm metric camera, PENTAX PAMS 645, f = 44.979mm. Two photographs were taken vertically in stereo with a base length of 1.5m, overlapping each other 50%. Common pass-points are 12 in number (minimum requirement is 8). This configuration is not good for the procedure of automatic adjustment but very common in industrial photogrammetry.

In nine eigen-values obtained from eq.(6), three of them were 0.0598,0.146 and 1.02, while others are greater than 100,000. As a result of applying the procedure mentioned in 3., a set of rotation angles with respect to the model coordinate system were obtained only for the third minimal eigen-value. The other eigen-values did not produce misleading false solutions. Residual \( \gamma \)-parallaxes obtained in the ensuing precise orientation were \( \gamma \) um in RMS. Table 2 shows the approximations and precise values of angles.
Likely one gets

\[ m_{32} = -(q_{1}^{2}n_{21} + q_{2}^{2}n_{22} + q_{3}^{2}n_{23}) \]  
\[ m_{33} = -(r_{1}^{2}n_{21} + r_{2}^{2}n_{22} + r_{3}^{2}n_{23}) \]  
(12-2)
(12-3)
where eq.(12-3) is identical to eq.(10).

3.4 Evaluation of \( k_{1} \)

Writing the first six expressions of eqs.(4) in the form of

\[ m_{21}n_{31} = P_{1} + m_{31}n_{21}, \]
\[ m_{21}n_{32} = P_{2} + m_{31}n_{22}, \]
\[ m_{21}n_{33} = P_{3} + m_{31}n_{23}, \]
\[ m_{22}n_{31} = q_{1} + m_{32}n_{21}, \]
\[ m_{22}n_{32} = q_{2} + m_{32}n_{22}, \]
\[ m_{22}n_{33} = q_{3} + m_{32}n_{23}, \]
multiplying the first with the forth, the second with the fifth, and the third with the sixth of each side of the above expressions and summing up them, one can calculate the right side of it. And the left side becomes

\[ m_{21}m_{22}(n_{31}^{2} + n_{32}^{2} + n_{33}^{2}) \]

\[ = -\sin k_{1}\cos k_{1} = -1/2\sin 2k_{1}. \]

This procedure produces four candidates for \( k_{1} \).

Then \( n_{31}, n_{32} \) and \( n_{33} \) are evaluated for each candidate for \( k_{1} \). They are evaluated from following different equations for better precision.

a) for \(-3/4\pi \leq k_{1} \leq \pi/4 \) or \(\pi/4 \leq k_{1} \leq 3/4\pi \)

\[ n_{31} = (P_{1} + m_{31}n_{21})/(-\sin k_{1}), \]
\[ n_{32} = (P_{2} + m_{31}n_{22})/(-\sin k_{1}), \]
\[ n_{33} = (P_{3} + m_{31}n_{23})/\cos k_{1} \]  
(13-1)
b) for \(\pi/4 \leq k_{1} \leq 7/4\pi \) or \(3/4\pi \leq k_{1} \leq 5/4\pi \)

\[ n_{31} = (q_{1} + m_{32}n_{21})/\cos k_{1}, \]
\[ n_{32} = (q_{2} + m_{32}n_{22})/\cos k_{1}, \]
\[ n_{33} = (q_{3} + m_{32}n_{23})/\cos k_{1} \]  
(13-2)

3.5 Evaluation of \( \phi_{2} \) \& \( w_{2} \)

From eqs.(8-2)

\[ \sin w_{2}\cos \phi_{2} = n_{23}, \]
\[ \cos w_{2}\cos \phi_{2} = n_{33}. \]  
(14)
Since \( n_{33} > 0 \), which means \( \cos \phi_{2} \neq 0 \),

\[ \cos \phi_{2} = \frac{1}{\sqrt{n_{33}^{2} + n_{33}^{2}}}. \]  
(15)
There are four candidates for \( \phi_{2} \). And for each candidate for \( \phi_{2} \), angle \( w_{2} \) is evaluated by

\[ \sin w_{2} = n_{23}/\cos \phi_{2}. \]

\[ \cos w_{2} = n_{33}/\cos \phi_{2}. \]  
(16)

3.6 Evaluation of \( k_{2} \)

From eqs.(8-2)

\[ (-\cos w_{2})\sin k_{2} + (\sin w_{2}\sin \phi_{2})\cos k_{2} = n_{21} \]
\[ (\cos w_{2})\cos k_{2} + (\sin w_{2}\sin \phi_{2})\sin k_{2} = n_{22} \]
\[ (\sin w_{2})\sin k_{2} + (\cos w_{2}\sin \phi_{2})\cos k_{2} = n_{31} \]  
(17)
\[ (\sin w_{2})\cos k_{2} + (\cos w_{2}\sin \phi_{2})\sin k_{2} = n_{32}. \]

one solves the first two equations to get \( \sin k_{2} \) and \( \cos k_{2} \). They are always soluble, even if \( \sin \phi_{2} \) is zero. And this \( k_{2} \) is tested by substituting it into the third and fourth equations. Any sets of candidates for \( \phi_{2} \) and \( w_{2} \) that do not satisfy both are abandoned.

3.7 Strict relative orientation and determination of the sign of a base length

Since the precision of approximations evaluated above is usually not sufficient, one should execute relative orientation again using those approximations. An independent model is thus obtained, which is either Fig.1-1 or 1-2.

Next the sign of a base length is determined the way that if \( Zp \) coordinates of objects in the independent model coordinate system are less than 0, it is set plus, and if \( Zp \) coordinates are greater than 0, it is set minus.

4. EVALUATION OF ORIENTATION PARAMETERS IN THE OBJECT SPACE COORDINATE SYSTEM

4.1 Model connection in the global model coordinate system

Independent models thus produced are linked to make a global model by usual successive orientation. Scales of successive models are adjusted by scaling base lengths. As a result exposing positions and rotation matrices associated with the global coordinate system \( XMYMZM \) are determined.

4.2 Transformation from the global model coordinate system to the object space coordinate system

When an object space coordinate system \( XYZ \) is given, global model coordinates \( XMYMZM \) are further transformed to the object coordinates. Here let us consider the case the object space coordinate system is implicitly given in the form of a few of 3-D control points. In most industrial measurements this is common. And in this case one can calculate orientation parameters automatically in the following way.

Similar transformation
A target field shown in Fig.1 was imaged by a metric camera, GEODETIC SERVICE CRC1 f = 240.0mm (changeable), film size 423 cm. The camera is designed to determine precise coordinates of object points by simultaneously adjusting with all other parameters: interior orientation parameters of the camera and exterior orientation parameters of photographs (Fraser, 1982).

The field was 4m(height) x 5m(width) x 2m(depth) in size. 63 target points were allocated three dimensionally. Most of points were imaged in most photographs. Ten photographs were taken, rotating kappa by 90 degrees to each other. The order of linking photographs adopted in the experiment is shown in Fig.3, where the photographs 3 and 8 make a datum model, and others are linked to this model. The base length of the datum model was set to unity (1m).

By applying the procedure mentioned in 3., relative orientation parameters associated with each independent model note system were uniquely determined. All approximations of interior orientation parameters but a camera distance were set to zeros. The camera distance was set initially to 249.5mm, which were read out from a micro-meter-based indicator of the camera. No false solutions did not appear. In additional experiments the authors confirmed that any other combination of photographs from those in Fig.3 could make models, as long as their convergent angles were not near 90 degrees.

In the case of no control points adjustment can be done by the method of free-network or by the method of minimal constraints. The authors adopted the latter. Seven degree of freedom was fixed by giving the infinite precision to ZM of point a and XM, YM, ZM of point b,c in Fig.1. As a result of the procedure in 4. Table 2 was obtained, which includes the approximations and the adjusted values of the interior orientation parameters of the camera (except for ones related to lens distortions) and exterior orientation parameters for photo 1 and 10 as well as a RMS difference between approximations and adjusted values of target point coordinates. The rotation matrices determined by the procedure in 4.3 are both in the form [Q][θ][K].

Table 1 and 2 prove that the algorithm produces approximations of parameters precise enough for ensuing bundle adjustments.

6. CONCLUSION

This paper discusses the algorithm for automatic calculation of approximations of parameters in bundle adjustment. Relative orientation parameters of each pair of photographs are evaluated from the linear coplanarity condition. All models are linked to form a global model. Then their rotation matrices are uniquely decomposed to angular elements. If the object space coordinate system is given, the transformation parameters are also automatically evaluated.

The procedure realizes photogrammetry without control points or easy orientation and camera calibration. It is very useful for digital-image-based plotters (digital plotters), which features easy manipulation for everybody who are not familiar with photogrammetry. Actually this has been already implemented into a digital plotter, TOPCON PI-1000 and now in test use.

The authors would like to acknowledge Mr.H.Ohtani, Mr.M.Chida in TOPCON Corp. and Mr.H.Hasegawa and Mr.K.Uesugi in PASCO Corp. for their cooperation with the experiments.

7. REFERENCES

*References from Journals:

*References from Grey Literature:
Fig. 1 Imaging configuration in camera calibration

Fig. 2 Four solutions retrieved from the coplanarity condition
Table 1 Approximations and the most probable values of relative orientation parameters

<table>
<thead>
<tr>
<th>angles(°)</th>
<th>Approx. M P V</th>
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<tr>
<td>φ₁</td>
<td>349 359</td>
</tr>
<tr>
<td>k₁</td>
<td>345 360</td>
</tr>
<tr>
<td>w₂</td>
<td>-2.43 360</td>
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<tr>
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<tr>
<td>k₂</td>
<td>-1.49 360</td>
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Table 2 Approximations and the most probable values of camera calibration parameters

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<td>camera dist. (mm)</td>
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<td>249.575</td>
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<td>principal xO(mm)</td>
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<td>-0.076</td>
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<td>point YO</td>
<td>0.0</td>
<td>-0.341</td>
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<th>Ext. orientation parameters</th>
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<td>photo 1</td>
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<td></td>
<td>φ(*)</td>
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<td></td>
<td>K(*)</td>
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<td>Z₀(m)</td>
<td>-0.236</td>
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<td>photo 10</td>
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<td>φ(*)</td>
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RMS difference of the approximations and the most probable values of target point coords. in the global model coordinate system 0.918(mm)