1. INTRODUCTION

In Remote Sensing satellite images, the size of the pixel, in general, may include more than one type of terrain cover. When these sensors observe the earth, the measured radiance is the integration of the radiance of all the objects that are contained in the pixel, implying the existence of the so-called mixture problems.

Two main approaches have been used in the technical literature to solve the mixture problem:

a. To improve the area estimates obtained by conventional classification methods, which are mainly based on the spectral characteristics of the pixels. In general, one class corresponds to a single type of terrain cover. Therefore, the signal obtained by the combination of two or more classes will not be representative of any of these classes and, as a result, an incorrect estimation of the area of each class is obtained. Under this approach, one may mention the works performed by Horwitz et al. (1975) and Ardeo (1983). The main idea that guides this approach is to substitute the conventional classification methods by the estimation of the proportions of classes within a pixel in the computation of the total area of each class in a scene.

b. To develop methods that involve synthetic images derived from proportions of the objects that are contained in the image. The present work is included in this approach, as it is described in the following paragraphs.

Detchmendy and Pace (1972) developed a linear mixing model to explain the variations found on the spectral class signatures. The basic hypothesis of such model is that these variations are mainly caused by structural target characteristics at the sub-pixel scale. These variations can be interpreted as a function of the proportions of the materials (called primary components) that constitute a scene.

As an example, according to Shimabukuro (1987), in forested areas, three main components are found: tree canopy, soil and shadow. Adams et al. (1990) describe the types of land use found in Amazon Forest, in terms of four components: vegetation, soil, shadow and wood. Shimabukuro proposes the use of an image derived from the shadow component in each pixel of the image, called shadow image, as an indication of the structure variations on the forest, that is, the estimated shadow proportions in an image indicate variations in age, type and shape of tree crown cover.

The objective of the present work is to analyze the results obtained in the multispectral image classification (in particular through the Maximum Likelihood criterion under the gaussian hypothesis) with the use of synthetic bands derived from the proportions of primary components within the pixels. Such bands are derived not only from the shadow component, but also from other components of the scene.

The adopted Linear Mixing Model, the problem of estimating the primary components proportions within the pixels and the synthetic bands generation are described in Section 2.

There is a relationship between the number of features used in the classification procedure and the corresponding computational effort. For the Maximum Likelihood classifier, this relationship is quadratic. Therefore, it is important to assure the use of the minimum number of features for an efficient classification (Richards, 1986).

Some of the feature reduction mainly used in Remote Sensing are based on the Jeffries-Matusita (J-M) distance and the Principal Components and Canonical Analysis transformations (Section 3). These methods are also useful for the visualization of color composites of multispectral images (Richards, 1986).

The Mixing Model can be regarded as a tool to reduce the dimensionality of the feature space to the number of mixture components. In the experiment presented in this paper (Section 4), the results obtained through maximum likelihood classification with the use of bands derived from conventional feature reduction procedures, and these synthetic bands derived from the Mixing Model are compared.
Section 5 presents the final conclusions and some important considerations about the use of Mixing Model.

2. ESTIMATION OF THE PROPORTIONS OF COMPONENTS WITHIN A PIXEL

2.1 Linear Mixing Model

By adopting the linear Mixing model, the pixel value in any spectral band is given by the linear combination of the spectral response of each component within the pixel. The model can be expressed as:

\[ r_i = a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n + e_i, \quad i = 1, \ldots, m \]

that is:

\[ r_i = \sum_{j=1}^{n} (a_{ij} x_j) + e_i, \quad i = 1, \ldots, m \]

where,

- \( r_i \) = spectral reflectance of the \( i^{th} \) spectral band of a pixel which contains one or more components;
- \( a_{ij} \) = know spectral reflectance of the \( j^{th} \) component within the pixel on the \( i^{th} \) spectral band;
- \( x_j \) = value of the \( j^{th} \) component proportion within the pixel; and
- \( e_i \) = error for the \( i^{th} \) spectral band.

The estimates for \( x_j \) are subject to the following restrictions:

\[ \sum_{j=1}^{n} x_j = 1 \]  \hspace{1cm} (4.2)

\[ 0 \leq x_j \leq 1 \]  \hspace{1cm} (4.3)

since they represent area proportions within a resolution element.

2.2 Methods for proportions estimations

The proposed methods in the literature to estimate classes proportions within a pixel select those components proportions such that their spectral signature be the best approximation of the observed pixel value (Ranson, 1975).

The methods used in this work are based on the criteria of Least Squares, namely Constrained Least Squares (CLS) and Weighted Least Squares (WLS). The objective is to estimate the proportions \( x_j \) by minimizing the sum of the squares of the errors \( e_i \), subject to the restrictions given by the expressions (4.2) and (4.3). The detailed description of these methods can be found in Shimabukuro et al. (1991) and Aguiar (1991).

Once the proportions \( x_j, j = 1, \ldots, n \) (number of components) are obtained, \( n \) synthetic bands that are linearly related to the estimated proportions \( x_j \) are generated by multiplying the proportions in each pixel by a scale factor 255 (maximum value of the pixel value).

3. DIMENSIONALITY REDUCTION OF THE FEATURE SPACE

This reduction can be obtained through the selection of an appropriate subset of features or through the transformation of the feature space into a smaller dimension space.

The selection of an appropriate feature subset cannot be done in an indiscriminate manner. It is necessary to perform a comparison between the candidate subsets, based on the classification performance. Ideally, the criterion should be based on the error probability, but its computation is usually very difficult. Therefore, indirect criteria that express distances between distribution are used and they provide upper and/or lower bounds on the error probability.

For multiple classes problems, frequently found in Remote Sensing, the Jeffries-Matusita (J-M) distance is one of the most widely used. For a detailed description of this distance, see Swain and Davis (1978) and Richards (1986).

Another method to reduce the dimensionality of the feature space is to transform the data into a new space in which the new features are discarded become evident. Several transformations can be used for this objective. In Remote Sensing one of the most used transformations is the Karhunen-Loève transformation, also known as the Principal Components transformation.

The Principal Components transformation provides a mapping of the original pixel values into a new coordinate system, in which the new random variables are decorrelated. Also, it offers the maximum compression, under the mean square error criterion, for a given dimensionality reduction. It should be observed that the K-L transformation is optimum in the sense of representation of the mixture of classes and does not aim class separability. However, it has been frequently observed that, in Remote Sensing, the data is usually distributed in the direction of the first principal components and, in this case, the K-L transformation is also effective for class separability.

A method that is specifically derived to optimize this last criterion is the so-called Canonical Analysis or Multiple Discriminant Analysis (Duda and Hart, 1973). This transformation rotates the data into a new space, with a maximum dimensionality equal to the number of classes minus one, by maximizing the ratio of the spread of the data between and within classes.

The proposal of this work is to compare the use of these conventional feature reduction methods with the method derived from the Mixing Model, which can be regarded as a way to reduce the dimensionality of the data to the number of primary components within a pixel. In this case, the original bands of the multispectral images are transformed into the synthetic bands, as it was described earlier.
the new coordinate system, the separability of the classes is described in terms of the different proportions of primary components that these classes present.

4. EXPERIMENTAL RESULTS

4.1 Methodology

The experiment was performed over an area ("ITAPEVA") that was reforested with Eucalyptus, with different growing stages, in the state of Mato Grosso do Sul, Brazil.

A Landsat TM image, taken on July 18th, 1984, with bands 1, 2, 3, 4, 5 and 7 was used. The digital numbers in the original bands were converted to reflectance, according to the procedure proposed by Markham and Baker (1986). Figures 4.1 displays the resulting bands, called respectively, 1R, 2R, 3R, 4R, 5R and 7R.

Aerial photographs taken approximately one month before the orbital coverage were available. The experiment was performed over a test area of 161 rows by 161 columns, corresponding to the area covered by the aerial photographs.

The first phase of the experiment was the synthetic bands generation, through the application of the computational methods that were mentioned in Section 2: Constrained Least Squares (CLS) and Weighted Least Squares (WLS).

The choice of the primary components was based on the work of Shimabukuro (1987). For this area, three components were considered: vegetation (eucalyptus), soil and shadow. The reflectance values of the vegetation and soil components were extracted from the image by Shimabukuro (1987), through the sample selection based on the available aerial photographs and reforestation map. The reflectance values of the shadow component were also obtained by Shimabukuro (1987), through the experiments performed by Heimes (1977). Figure 4.2 presents the reflectance curves of the components.

The analysis of the results obtained in this phase was qualitatively performed through the work previously performed by Shimabukuro (1987), since it is very difficult to obtain quantitative information from field work.

The second phase of the work consisted of the comparative analysis of the Maximum Likelihood classifier under the gaussian assumption, through the conventional feature reduction methods described in Section 3 and the Mixing Model.

Classification results were analyzed through the classification matrices generated from training samples. It is well known that the average classification performance estimated over these samples is optimistic but, since the objective of the present analysis is the comparison between different feature reduction methods, this fact was disregarded. Thematic images generated by the classification procedure were analyzed and qualitatively compared by using the available information. Unfortunately, it was not possible to reproduce here the thematic images, so they are not present in this work. They will be displayed in the poster presentation and in a future work in preparation.

4.2 Results

Phase 1: Use of proportion estimators

Synthetic bands derived from the proportions of eucalyptus (Vegetation Band), soil (Soil Band) and shadow (Shadow Band) generated by the CLS method are presented in Figure 4.1. Figure 4.4 presents the synthetic bands generated by the WLS method.

Both methods present very similar results, are compatible with the available ground truth and are similar to the results obtained by Shimabukuro (1987). Therefore, it was decided to follow the experiments using the synthetic bands generated by the CLS method.

Visually, one can notice using the synthetic bands a more clear distinction between two types of eucalyptus. According to Shimabukuro (1987), this difference is due to age variations of the eucalyptus plantation. By analyzing the Shadow Band (Figure 4.3.c), it is possible to notice that one of this areas presents a higher shadow proportion, what means less uniformity and a higher age.

On the basis of this results, the selected classes for the Maximum Likelihood classification were:

New E.: reforestation with eucalyptus, with age between 8 months and 2 years;

Old E.: reforestation with eucalyptus, with age greater than 2 years; and

Soil: exposed soil.

These classes can be discriminated on the basis of the different proportions of primary components, which indicate the structural characteristics of each class. As it can be observed in Figure 4.3, in class New E., the pixels present a greater proportion of the Vegetation component. On the other hand, in class Old E., which is less uniform due to age, one can notice a larger influence of the Soil and Shadow components. Class Soil, as it was expected, is basically due to Soil component.

Phase 2: Comparison between the Mixing Model and the Conventional feature reduction methods.

Tables 4.1 to 4.4 present the classification matrices, when the following bands are used:

a. The first three components (C1, C2 and C3) generated by the Principal Components Transformation;

b. The two components (C1 and C2) generated by the Canonical Analysis procedure;

c. The three original bands (3R, 4R, 5R) selected by J-M Distance; and

d. Synthetic bands derived from the proportions of primary components (Vegetation, Soil and Shadow Bands).

In terms of average performance, the best results were obtained through the Canonical Analysis and the Mixing Model procedures. However, it should be considered that all methods present high values (and optimistic) for estimated average performance (higher than 92% in all cases) and small variations from one case to another. Therefore, it is not possible to present a definite conclusion about the comparison in terms of classification performance.

Through the qualitative analysis of the thematic
Figure 4.1 - Landsat TM image (values converted to reflectance): a) 1R; b) 2R; c) 3R; d) 4R; e) 5R; f) 7R

Figure 4.2 - Spectral signatures of eucalyptus, soil and shadow referring to the study area "ITAPEVA" (TM data).

Figure 4.3 - Synthetic bands generated by the CLS method: a) Vegetation Band; b) Soil Band; c) Shadow Band.

Figure 4.4 - Synthetic bands generated by the WLS method: a) Vegetation Band; b) Soil Band; c) Shadow Band.
images (not present in this work), one concludes that the best results were obtained by Canonical Analysis, followed by the Mixing Model. A clear improvement in terms of non-classified pixels can be observed on these two images.

By analyzing the thematic image that results from the use of synthetic bands, it is possible to notice the highest concentration of non-classified areas, where the mixing between New E. and Old E. classes occurs over a stripe located in the area where the new eucalyptus are planted. As it can be observed in Figure 4.3, these pixels present a greater proportion of the Soil component. This fact makes them similar to the pixels that belong to Old E. class, in terms of proportions. This structural characteristic is not so clearly perceived when other methods are used.

The reason for this structural difference found within the area of New E. class is beyond the scope of the present work. However, a deeper study of forest engineering could be performed, by using additional information about the canopy structure provided by the primary components proportions.

Table 4.1 Classification matrix obtained when the first three Principal Components (C1, C2, C3) were used.

<table>
<thead>
<tr>
<th>Not Classified</th>
<th>New E.</th>
<th>Old E.</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>New E.</td>
<td>3.7</td>
<td>96.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Old E.</td>
<td>2.7</td>
<td>1.0</td>
<td>95.7</td>
</tr>
<tr>
<td>Soil</td>
<td>5.9</td>
<td>0.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Average Performance:</td>
<td>94.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Abstinence:</td>
<td>4.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Confusion:</td>
<td>1.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2 Classification matrix obtained when the Canonical axes (C1, C2) were used.

<table>
<thead>
<tr>
<th>Not Classified</th>
<th>New E.</th>
<th>Old E.</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>New E.</td>
<td>3.3</td>
<td>96.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Old E.</td>
<td>4.7</td>
<td>0.0</td>
<td>95.0</td>
</tr>
<tr>
<td>Soil</td>
<td>2.2</td>
<td>0.0</td>
<td>1.3</td>
</tr>
<tr>
<td>Average Performance:</td>
<td>96.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Abstinence:</td>
<td>3.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Confusion:</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3 Classification matrix obtained when the original bands selected by the J-M distance (3R, 4R, 5R) were used.

<table>
<thead>
<tr>
<th>Not Classified</th>
<th>New E.</th>
<th>Old E.</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>New E.</td>
<td>5.3</td>
<td>94.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Old E.</td>
<td>3.0</td>
<td>0.3</td>
<td>96.3</td>
</tr>
<tr>
<td>Soil</td>
<td>5.6</td>
<td>0.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Average Performance:</td>
<td>94.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Abstinence:</td>
<td>4.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Confusion:</td>
<td>1.09</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4 Classification matrix obtained when the synthetic bands (Vegetation, Soil and Shadow Bands) were used.

<table>
<thead>
<tr>
<th>Not Classified</th>
<th>New E.</th>
<th>Old E.</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>New E.</td>
<td>2.5</td>
<td>95.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Old E.</td>
<td>1.0</td>
<td>0.3</td>
<td>97.0</td>
</tr>
<tr>
<td>Soil</td>
<td>1.0</td>
<td>0.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Average Performance:</td>
<td>96.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Abstinence:</td>
<td>1.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Confusion:</td>
<td>1.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3 Discussion

The results that were obtained with the "ITAPEVA" experiment demonstrate that the Mixing Model can be used as an alternative method for the feature reduction phase of the classification process: its average performance was comparable to conventional methods and, furthermore, additional information about the structural characteristics of the classes was obtained.

It is important to point out that the methods compared in this work aim at different objectives, as it is described in the following.

The most straightforward method is the feature reduction by the J-M Distance. The advantage of this method is to be an indicator of the performance of the Maximum Likelihood classifier, if the gaussian assumption is verified.

The Principal Components transformation aims at the optimum representation of the mixture of the classes in the mean square error sense. The usual choice is for those axes that have the largest variances. However, for classification, the main objective is class discrimination and not the separation of the data (Duda and Hart, 1973). According to Richards (1986) the good classification results obtained with the Principal Components transformation can be credit to the fact that Remote Sensing classes usually are spread over the first principal components axis. This is particularly so for soils and spectrally similar cover types.

On the other hand, Canonical Analysis aims specifically at the discrimination. If the number of classes, k, is less than the original feature dimensionality, p, then the maximum number of features generated by this method is (k-1). The new features can be used as more adequate inputs to the classification process. Another important application is the use of the color composite of the canonical axis, which were found to be very useful for the visual interpretation in heterogeneous areas, since the new features tend to maximize the differences between soil categories (Mather, 1987).

The use of the synthetic bands can also be very useful for the visual interpretation of class distributions. Qualitatively, their color composite (not present in this work) is even better than the color composites obtained by the other methods for visual discrimination between the classes.

Furthermore, with the use of the synthetic bands, classes are not described as a function of their spectral response (spectral signatures) but in...
terms of a physical characteristic (primary components proportions). Therefore, in order to perform visual interpretation, one does not depend on knowledge of the spectral signature of each class, but only on its structural characteristics.

5. CONCLUSIONS

Through the performed experiment, it was found that the Mixing Model can be regarded as a useful alternative for feature reduction both as an input to the Maximum Likelihood process, being comparable to conventional methods, as well as for visual interpretation. In this case, it has the advantage of better visual class discrimination and the fact that the information contained in the synthetic bands represent physical concepts that are more easily assimilated by the analyst than the spectral signatures of the classes.

However, in order that the Mixing Model could provide good results, it is necessary that the primary components be adequately chosen. For forest targets, the components should represent the structural differences of the canopy. The importance of the shadow component for this purpose was confirmed, as well as the positive influence of structural components on the automatic classification process.

Therefore, it is important to develop methods to aid in the selection of primary components and their spectral signatures, in order that the Mixing Model could be effectively applied to larger areas and perhaps other types of classes. Some suggestions of such methods can be found in Detchmendy and Pace (1972), Shimabukuro et al. (1991) and Aguiar (1991).

6. REFERENCES


Aguiar, A.P.D., 1991. Utilização de atributos derivados de proporções de classes dentro de um elemento de resolução de imagem ("pixel") na classificação multispectral de imagens de Sensoriamento Remoto. Dissertação de Mestrado, Instituto Nacional de Pesquisas Espaciais-INPE, São José dos Campos, SP-Brazil.


