# ON THE PRINCIPLES AND THE APPROACHES OF IMPLEMENTING THE STRICT DIGITAL GEOMETRIC RECTIFICATION FOR SPOT IMAGERY

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## ABSTRACT

This paper presents the theories and the approaches of carrying out the digital geometric rectification for the SPOT imagery by means of collinearity equations. It focuses on the mathematical model of calculating the exterior orientation elements according to the image-forming mechanism of the SPOT imagery, the principles of realizing the indirect method for the anchor points' coordinates using the strict formula, and specially, the first employment of direct method in digital geometric rectification and its advantages in use. The experiments using SPOT image window in Yiyang area and the precision analysis show us that the proposed approaches and key techniques using collinearity equation are correct and reasonable.

KEY WORDS: Linear array scanning imagery, Collinearity equations, Rectification for anchor points, Indirect and direct methods.

SPOT images are of the central projection only at lines. Each scan line has its own projective center. The exterior orientation elements vary with the change of scan line number. Therefore, SPOT images are different from aerophotographs in imaging geometry. The collinearity equations based on the special geometric nature must be taken when using SPOT image for strict rectifications.

### 1. The projective deformation due to relief

The frame size of SPOT image is of generally 150mm ×150mm which can cover the ground area of 60km×60km. If taking into account a point with the distance to the nadir r=75mm, and hight difference h=800m, the estimated projective deformation will be Dh=0.0723 at that point. The displacement will be 1.112mm when creating a photomap of the scale 1:50000 without geometric processing. It is almost 3 times of allowable mapping error. So the projective deformation is very distinct, even in the general case. The necessity of employing collinearity equations can be seen according to the estimation above.

### 2. The solution for the exterior orientation elements of central line

#### 2.1 Mathematic model

One can define the collinearity equations for SPOT image as the same formula as that for aerial photographs. But the position and the attitude parameters of CCD sensor in those equations are not for whole image, but only for one line, and x=0, because of the image geometry. ( here, x represents the flight direction ) Further more, each exterior element must be expressed as that at central line added with its change rate per line multiplicated by line change number to central line. Thus the unknowns in the equations are the exterior orientation elements for central line and the change rate of those elements. As CCD oblique scanning can be performed by SPOT system, the y direction expression in the equations should be written as the following formula while considering scanning angle:

$$y_i = f\times(y\times\cos\varphi + f\times\sin\varphi)/(y\times\sin\varphi - f\times\cos\varphi) \qquad (1)$$

where $y$ is the raw image coordinate.

Usually, the SPOT 1A level image, not other level, is used in rectification. But the image coordinates (sequence number of line and column ) should be transformed to those of a new coordinate system which origin is the center of image. Since earth curveture effects are very significant, one has to chose tangent plane coordinate system as ground coordinate system. The corresponding round point to the center of image must be determined as the origin of tangent plane system.

In order to avoid the strong correlation between the linear and angular elements ( i.e. the position and attitude of sensor ), one must calculate them separately during the iteration. While computing one type of exterior orientation element (the position, for example), the other type could be considered as constant(the attitude remained unchanged, in this case). This has been proved effective in the experiments.

#### 2.2 Preparation for basic data

For the solution of exterior orientation elements and the estimation of precision, it is needed to select sufficient number of control points and check points. The importance is the determination of image coordinates with subpixel accuracy as much as possible.

As to the origin of tangent plane system, one can firstly fit up a second order of polinomial by means of control points, then calculate the ground coordinates (Xc, Yc ) for the corresponding point of image center. The elevation value of that point could be read from topomap according to Xc and Yc. Because the attitude of satellite can be read from " Leader File " of SPOT CCT tape, the equivalent focal length therefore should be evaluated for the image of interest.

The preliminary values of exterior orientation elements must be assigned since it is meaningful for the rapid convergence of solutions. Generally, the angle elements and all the change rate parameters should be zero when starting iteration, but the position of sensor must be determined using satellite altitude in "Leader File" as the preliminary value of Zs for central line, and assigning the zero-order terms as Xs and Ys after fitting up first order polinomial by four control points at the corners of image frame.

### 3. Indirect approach for anchor point grid

As each scan line of SPOT image has its own orientation elements, the line sequence number has to be estimated firstly for a ground point for which the corresponding image coordinates should be calculated by means of collinearity equations. Therefore, one must fit up second order polinomials using some control points. With those polinomials, the approximate value of line number x can be computed for a ground grid point ( i.e. anchor point ) and the estimated orientation elements could be given out. Then, $x_K$ ( here k is the number of iteration ) can be calculated using the first collinearity equation for getting the coordinate in line number. That value should be zero according to the SPOT imaging geometry. In practical calculations, one should set up a threshold, which is generally of 0.1 pixel. If $x_K$ is less than that, it converges to the solution, and

$$x' = x + x_K \qquad (2)$$

where x' could be considered as the line coordinate for the corresponding image anchor point. If x is not less than the threshold, we have

$$x_{K+1} = x_K + x_{K-1} \qquad (3)$$

then compute x' using formula (2), and estimate the orientation elements again.

When the iteration converges to the solution, the coordinate y can be calculated by means of the second collinearity equation. As the ground coordinates are those of tangent plane system ( when using collinearity equations), the image coordinates x', y' must be transformed to the raw image coordinates (line and pixel sequence number) for grey level interpolations.

Because the iterative calculations are time consumed for the rectification at each point, it is needed to determine a regular grid ( i.e. anchor point grid ) in ground coordinate system. The interval of that grid could be defined as of 200--500m along both x and y direction. The calculations mentioned above should be performed only for the grid points.

As the three dimensional coordinate for ground system are needed for the resections, one has to prepare the digital elevation model (DEM) in advance. The DEM grid could be defined as same as anchor point grid so that two grids are overlayed with same interval distance.

4. The employment of direct approach

Generally, the direct approach is never used for digital image rectification, because its result is not a regular grid and must be further interpolated so that the algorithm is more complicated and more time-consuming.

But for SPOT image, it will have more advantages, since it needs no iterations for computing the orientation elements. If the line number is known, one can calculate directly the orientation elements with higher accuracy. In this case, the direct approach is available.

Firstly, one must determine a grid in the image window to be rectified. The interval of this grid could be same as that of DEM, but the range should be smaller than that of DEM. Then one can calculate the ground coordinates X, Y for each image grid point according to the following formula:

$$X = (Z - Zs)\frac{a_1 y - a_3 f}{c_1 y - c_3 f} + Xs \qquad (4)$$

$$Y = (Z - Zs)\frac{b_1 y - b_3 f}{c_1 y - c_3 f} + Ys$$

where $a_i$ $b_i$ $c_i$ (i=1, 2, 3) are the functions of attitude parameters and Xs, Ys, Zs are the position parameters for the line concerned. The value Z can be obtained from DEM and f is the equivalent focal length. Here, the key is the Z value which should be interpolated from DEM. Fitting up the second order polinomials is necessary for calculating ground coordinates X', Y' using image grid point coordinates, and then one can perform the interpolation for Z'. Z' should be transformed to the Z value in tangent plane system. Here, Z', as a approximate value, must be checked. Thus, X', Y', Z' should be transformed to tangent plane coordinates and then be used for resection to get corresponding image coordinates which will be compared with the image anchor point coordinates. If the deviation of image point is less than 0.1 pixel, Z value can be accepted. Otherwise, one has to take in use the values Z'±5, Z'±10 (unit in meter) to perform the resection and comparison in order to chose Z' value which should meet the need of precision.

After the calculation using formula (4), X , Y must be transformed to Gause-Kruger coordinates in order to form a coordinate set for a grid which is irregular. Furthermore, the interpolation is needed for a regular grid using that coordinate set as the control by means of first order polinomials only.

5. Rectification for getting orthophoto

After having the pair of coordinate sets of anchor point grid, one can complete the rectification for whole image window. Since there are four points in each case within the grid, the first order polinomials are sufficient for the interpolations of image coordinates one point by one point so that one can perform the grey level interpolations from raw image window. Each pixel of rectified image has the same size as that of raw image so that the resolutions are identical.

Because the interval of anchor point grid is of 200-500m, each grid case could be considered as a tilt plane on ground surface, so that there is only the displacement and rotation of image distortions and the first order polinomials are sufficient to express this kind of distortions.

6. Results of experiments and analysis of precision

6.1 The solutions of orientation elements and their precision

The SPOT image window selected as test area is of good quality, so that one can easily choose the control points and check points. Since the preliminary values of exterior orientation elements and the equivalent focal length have been reasonably determined, and the approach of alternate iteration for the solution of linear orientation elements and angular elements has been taken in use, the convergence to the solutions is very quick and generally three or four times of iterations are enough to converge the solutions with higher precision. The Mean Square Error (MSE) of 14 control points is of only 0.56 pixel. The MSE of 37 check points is of 0.9 pixel. Thus, it makes up the good basis for the geometric rectification to reach to the higher precision.

6.2 Coordinate calculations for anchor points and the accuracy

The coordinate set of anchor points is one of the keys in rectifications. The precision of those coordinates is very important to predict if the process will be successful or failure. When taking the indirect approach, 50 check points have been selected to examine the accuracy of anchor point calculations, The mean absolute errors (M.A.E) for those points are dx= 0.61(pixel) dy= 0.56(pixel) and 94% of those points can reach to the precision of subpixel.

One the other hand, the traces in iterations show us that one or two times of iteration can converge to the solutions during the indirect process, so the calculating rate is very high and it reduces effectively the time of rectification.

Since there is no iteration during the procedure of direct approach, the calculation is relatively quicker. As to the precision of anchor point coordinates, the checks in 4087 anchor points by resections show us that the error of point is very small. The MAE for those points is of only 0.000006 pixel. The maximum is 0.0000236 pixel. The minimum is 0.0000038 pixel. Therefore, the precision of anchor points is very high, and this shows us that the calculations of X Y and the interpolation of Z can perfectly assure the accuracy of coordinate Z for anchor point.

Two kinds of approaches have created two files of anchor point coordinates. In order to estimate the precision of further interpolation within the cases of anchor point grid, we have computed the MSE of point after fitting up the first order polinomials by four anchor points of each case, and the further statistics have been performed. The following Table 1. is the result of statistics (unit is pixel).

Table 1.

| | Indirect Approach | Direct Approach |
|---|---|---|
| Average MSE | Mx=0.0150 My=0.0169 | Mx=0.0133 My=0.0079 |
| Maximum Error | Ex=0.2486 Ey=0.6426 | Ex=0.1679 Ey=0.4340 |
| Minimum Error | Ex=0.0011 Ey=0.0003 | Ex=0.0015 Ey=0.0002 |

As there is one value of MSE of point for each case. So we have the average MSE of point. From Table 1, we can find out that the error of point for direct approach is less than that for indirect approach.

6.3 The precision of orthophotomap

The final results of two approaches in rectification are two orthophotomaps, on which the check points have been selected to fit up the first order polinomials for examining the precision. Table 2 is the fitting error of point on those check points (unit is meter).

Table 2.

| Point No. | Indirect Approach | | Direct Approach | |
|---|---|---|---|---|
| | line | pixel | line | pixel |
| 1 | -4.4 | 2.3 | 1.9 | 6.1 |
| 2 | 7.0 | 2.3 | -5.7 | 8.4 |
| 3 | -10.8 | -3.0 | 4.4 | 3.3 |
| 4 | -6.8 | -10.6 | 6.5 | -9.9 |
| 5 | 1.9 | -7.6 | 5.5 | -4.6 |
| 6 | 8.9 | -3.8 | -2.5 | -9.1 |
| 7 | -6.5 | 6.1 | -2.5 | 3.8 |
| 8 | 4.6 | 14.4 | -8.2 | 2.3 |
| MSE | 6.08 | 6.27 | 4.40 | 5.66 |

Table 2 shows us that the rectification results have both the precision of subpixel.That is about one half of a pixel. Furthermore the precision of direct approach is higher than that of indirect approach. So taking into account the imaging geometry of SPOT image and employing the corresponding flexible algorithm for the rectification are very important to assure the quality of geometric processing. As the last results , Table 2 has proved the correctness, reasonableness, and flexibility of the principles and methods suggested above.

7 Conclusion

When performing digital geometric corrections for SPOT images, it is not only necessary, but also flexible to rectify the prejective distortion due to relief. The suggested principles and methods based upon the imaging geometry of SPOT have been successful in making first use of collinearity equations for linear-array scanned images. According to the characteristies of SPOT images, The employment of direct approach in rectifications has shown us for the first time that it has more advantages to this kind of images.

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