SPACE TRIANGULATION USING AFFINE TRANSFORMATION

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ABSTRACT

If the DLT method is applied for space triangulation with satellite photographs, high correlations arise among the orientation unknowns due to very small height differences in the terrain for the flying height of the platform. This difficulty can however be overcome by using ground point coordinates calculated by means of affine transformation as the initial values for the iterative solution. This approach is tested with simulated examples and is revealed to have a fairly good accuracy.

INTRODUCTION

The DLT method(Abdel-Aziz and Karara(1971)) is a general orientation approach of photographs. This method is usually applied for the analysis of non-metric photographs. Employing the DLT approach for the analysis of satellite photographs, the attained accuracy may not be so high due to very high correlations among the orientation parameters, because height differences in the photographed terrain are very small in comparison with the flying height of the platform. In order to overcome this problem, this paper proposes an orientation method of adopting ground point coordinates as the initial values for the iterative solution, which were calculated by means of an orientation theory of photographs based on affine transformation(Okamoto, et al(1989,1991,1992)). Also, the proposed method is applied for space triangulation with simulated satellite photographs taken consecutively in a convergent manner. The camera is assumed to be a non-metric one. Further, the analysis of satellite CCD camera imageries with a very narrow field angle is performed using the orientation theory based on affine transformation.

OUTLINE OF THE PROPOSED ORIENTATION METHOD

The basic theory for analyzing affine imageries has first been constructed by Okamoto(1989) and in detail explained in the paper by Okamoto, et al(1991,1992). However, this theory cannot be applied for the analysis of conventional photographs without transforming the central-perspective imageries into affine ones. Thus, this transformation will first be discussed.

Let the ground surface be flat and a central-perspective photograph be taken with the rotation angles ω and φ. The reference coordinate system (X,Y,Z) is selected as a right-handed, rectangular Cartesian system with its origin at the projection center of the photograph and with its X-Y plane parallel to the scaled ground surface, as is demonstrated in Figure-1. Further, the photograph is considered to intersect the scaled ground surface in such a way that its principal point H lies on the surface. The three-dimensional coordinates (X_p, Y_p, Z_p) of an image point p(x,y) of the central-perspective photograph are expressed with respect to the reference coordinate system in the form

$$\begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 1 & \cos\varphi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} x \\ y \\ -c \end{bmatrix} \quad (1)$$
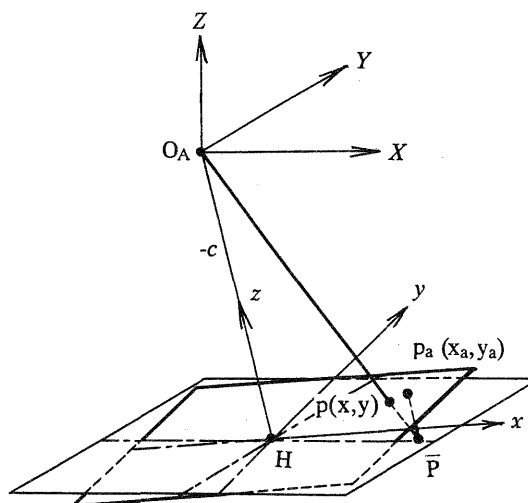

Figure-1 : transformation of a central-perspective photograph into an affine one

in which c denotes the principal distance of the picture. Also, the principal point H of the central-perspective picture is given with respect to the reference coordinate system as

$$
\begin{bmatrix} X_H \\ Y_H \\ Z_H \end{bmatrix} = \begin{bmatrix} -c \cdot \sin \varphi \cos \omega \\ -c \cdot \sin \omega \\ -c \cdot \cos \varphi \cos \omega \end{bmatrix} \tag{2}
$$

Further, Let $\overline{P}(\overline{X},\overline{Y},\overline{Z})$ denote the point at which the ray $\overrightarrow{O_A p}$ intersects the scaled ground surface. The three-dimensional coordinates of this point can be described as

$$
\begin{aligned}
\overline{X} &= \frac{X_p}{Z_p} Z_H \\
\overline{Y} &= \frac{Y_p}{Z_p} Z_H \\
\overline{Z} &= Z_H
\end{aligned} \tag{3}
$$

Observing the point $\overline{P}(\overline{X},\overline{Y},\overline{Z})$ on the scaled ground surface with respect to the picture coordinate system (x,y,z) we can find the corresponding affine image point $p_a(x_a,y_a)$ in the form

$$
\begin{aligned}
x_a &= (\overline{X} - X_H) \cos \varphi \\
y_a &= (\overline{X} - y_H) \sin \omega \sin \varphi + (\overline{Y} - Y_H) \cos \omega
\end{aligned} \tag{4}
$$

Next, we will consider image transformation errors which are produced by the errors of the given orientation parameters and by height differences in the photographed terrain. The exterior orientation parameters (ω, φ) and the interior ones (x_H, y_H, c) of satellite photographs are usually given with very accurate approximations. Thus, the image transformation errors due to those of the orientation parameters are considered to be small, if the ground surface is flat. However, when the photographed terrain has height differences, the image transformation errors remarkably increase unlike in the case of transforming a central-perspective line image into an affine one, because the field angle of a camera used for taking satellite photographs is much wider than that of a satellite CCD line-scanner. Therefore, the orientation theory of two-dimensional affine imageries cannot be employed for the rigorous analysis of satellite photographs taken with conventional cameras.

On the other hand, the general orientation approach (the DLT method) of central-perspective photographs may not be effectively applied for the analysis of satellite photographs, because height differences in the photographed terrain are rather small in comparison with the flying height of the platform. In order to overcome this problem, the following method can be developed for the analysis of satellite photographs taken under bad photogrammetric conditions:

1) The satellite photographs are first analyzed using the orientation theory based on affine transformation and coordinates of unknown ground points are calculated.

2) The DLT method is adopted for the rigorous analysis of the photographs by using the ground point coordinates above as the initial values for the iterative solutions. The solutions obtained by means of affine transformation are little correlated with those based on projective transformation, because the two transformations are mathematically quite different.

TESTS WITH SIMULATED EXAMPLES

SPACE TRIANGULATION

The proposed orientation method of satellite photographs was tested with simulated examples. For the construction of the simulation models ten satellite photographs taken consecutively in a convergent manner were assumed to be employed (See Figure-2.). The image coordinates of 39 object points were calculated by means of the collinearity equations under the following conditions:

flying height of the satellite : $H = 250$ km
focal length of the used camera : $c = 300$ mm
picture format : 230×230 mm
picture scale : 1/800,000
maximum height differences in
the photographed terrains : 100 m, 500 m, 3,000 m
convergent angles : ± 30 deg.

The perturbed image coordinates were provided in which the perturbation consisted of random normal deviates having a standard deviation of 2.5 micrometers.

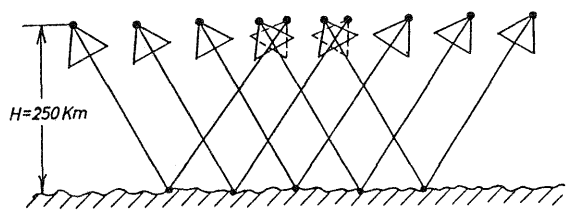In the orientation calculation, maximum errors of the orientation parameters were assumed to be 15

Figure-2 : ten satellite photographs taken consecutively in a convergent manner

minutes for the rotation parameters, 1000 m for the translation parameters, and 1 mm for the interior orientation elements. Also, the arrangement of ground control points are shown in Figures-3a, -3b, and -3c. The obtained results regarding the standard error of unit weight, the average external error at the check points calculated by means of
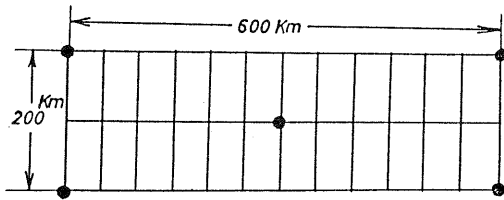
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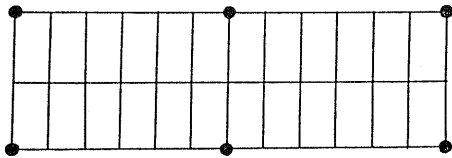Figure-3a : configuration of 34 check and 5 control points

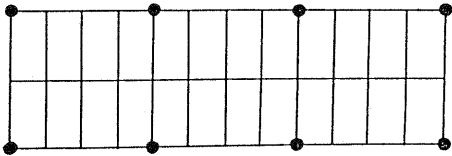Figure-3b : configuration of 33 check and 6 control points

Figure-3c : configuration of 31 check and 8 control points

the affine transformation, and the average internal and external errors based on the proposed method are given in Tables-1a, 1b, and 1c. From these re-

	A	B	C
$\hat{\sigma}_A$ (m)	6 4 8. 8	3 4 7. 6	2 3 3. 9
$\hat{\sigma}_0$ (μm)	2. 7	2. 6	2. 6
$\hat{\sigma}_I$ (m)	5. 3	3. 6	2. 9
$\hat{\sigma}_E$ (m)	4. 4	2. 6	2. 3

Table-1a : for the case of maximum height difference in the terrain : 100 m

A : the case of five ground control points

B : the case of six ground control points

C : the case of eight ground control points

$\hat{\sigma}_A$: the average external error at check points using affine transformation

$\hat{\sigma}_0$: the standard error of unit weight

$\hat{\sigma}_I$: the average internal error at check points using projective transformation

$\hat{\sigma}_E$: the average external error at check points using projective transformation

	A	B	C
$\hat{\sigma}_A$ (m)	2 9 5. 6	1 0 4. 1	8 1. 3
$\hat{\sigma}_0$ (μm)	2. 5	2. 5	2. 5
$\hat{\sigma}_I$ (m)	5. 9	4. 3	3. 3
$\hat{\sigma}_E$ (m)	5. 5	3. 4	2. 9

Table-1b : for the case of maximum height difference in the terrain : 500 m

	A	B	C
$\hat{\sigma}_A$ (m)	2 6 5. 1	7 9. 1	6 7. 0
$\hat{\sigma}_0$ (μm)	2. 5	2. 5	2. 4
$\hat{\sigma}_I$ (m)	8. 5	5. 6	3. 9
$\hat{\sigma}_E$ (m)	7. 9	4. 3	3. 5

Table-1c : for the case of maximum height difference in the terrain : 3,000 m

sults we can discuss the characteristics of the proposed orientation approach of satellite photographs as follows:

1) Five control points are mathematically required(Okamoto(1981)) for the analysis of overlapped central-perspective photographs by means of the general approach (the DLT method). The results in Tables-1a, 1b, and 1c show that the general orientation theory is mathematically sound and useful for the analysis of photographs.

2) The orientation results of the method based on affine transformation are worse than the expected ones. This means that the wide field angle of the used camera plays a significant role for the image transformation errors.

3) In case of five ground control points the method using affine transformation has great external errors. From this fact we can also see that the image transformation errors cause a large model deformation.

4) The orientation accuracies obtained by means of the proposed method are rather high. In particular, when the maximum height difference in the terrain is 100m and eight ground control points are available, the external error is almost identical to the theoretical one.

5) The external errors increase with the height differences in the terrains.

ANALYSIS OF SATELLITE CCD CAMERA IMAGERIES USING AFFINE TRANSFORMATION

The orientation method based on affine transformation was also checked with simulated satellite

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CCD camera imageries taken in a convergent manner under the following conditions:

flying height of the satellite : H = 450 km
focal length of the used CCD camera :
    c = 150 mm
picture format : 10×10 mm
pixel size : 10×10 μm
picture scale : 1/3,000,000
number of satellite photographs : 2(a stereopair)
convergent angles : 20 deg.
number of control points : 5
number of check points : 20

As the pixel size is 10×10 micrometers, the average image error is considered to be 2.5 micrometers and the theoretical error at the ground scale is 7.5 meters. In order to employ the orientation theory based on affine transformation, the central-perspective image coordinates of the CCD camera imageries were first transformed into affine ones by means of the transformation method discussed in the previous section. In the process of the image transformation, errors of the rotation parameters (ω, φ) were assumed to be 15 minutes and errors of the interior orientation parameters were 0.5 mm for the principal point coordinates and 1.0 mm for the principal distance. The maximum height differences in the terrain was assumed to be 100 meters and 500 meters.

Then, the orientation method based on affine transformation was applied for the analysis of the stereopair of transformed affine images. The orientation calculation was performed as follows. The ground point coordinates obtained in the first orientation calculation give us knowledge of the height differences in the terrain. Thus, the image transformation errors due to the height differences can be corrected with this knowledge. Using the corrected affine image coordinates the orientation calculation is repeated.

	the first orientation calculation	the second orientation calculation
A	1 2. 5 m	9. 1 m
B	1 6. 5 m	1 0. 5 m

Table-2 : the obtained results by means of affine transformation

A : the case of maximum height difference : 100 m
B : the case of maximum height difference : 500 m

The obtained external errors are given in Table-2. From these results we can see that the orientation theory using affine transformation is applicable for the analysis of satellite CCD camera imageries, if the height differences in the terrain are not so large.

CONCLUDING DISCUSSIONS

In this paper an orientation method of satellite photographs has been developed in which the DLT approach is employed by using the ground point coordinates obtained by means of affine transformation as the initial values for the iterative solutions. Further, the orientation theory based on affine transformation has been applied for the analysis of satellite CCD camera imageries having a very narrow field angle. Through the tests with simulated examples these methods have proved to have a fairly good accuracy.

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