

COMPARISON OF PRECISION AND RELIABILITY OF POINT COORDINATES USING DLT AND BUNDLE APPROACH

By: M.Sc. Lars-Åke Edgardh
Department of Photogrammetry
The Royal Institute of Technology
S-10044 Stockholm
SWEDEN
Commission number: III

ABSTRACT

The direct linear transform is a popular alternative to the Bundle adjustment method as it does not require calibrated cameras and the transform parameters can be computed directly from a linear function. However, the method has some drawbacks in precision and reliability compared to Bundle adjustment.

This paper reports a comparative evaluation of the two methods. The methods are evaluated using a theoretical test where control points, check points and control points have been simulated. Precision of the two methods have been calculated using data from control point calculations. Internal reliability have been estimated using data from the calculation of parameters.

DLT has been evaluated using both an iterative and a linear approach of the parameter calculations. Bundle adjustment has been evaluated in two ways: internal and external orientation parameters as unknowns, and only external orientation parameters as unknown.

The evaluation shows that Bundle adjustment gives a better precision and internal reliability compared to DLT when 6 control points are used. When the number of control points is increased, the difference decreases in both precision and internal reliability.

Key words: Photogrammetry, non-metric, accuracy

1. INTRODUCTION

Bundle adjustment and the Direct Linear Transform (DLT) are the most commonly used methods when calculating point coordinates from image data. This paper reports a theoretical study of the two methods focused on precision and estimation of internal reliability.

Bundle adjustment is based on the collinearity equations, where the physical reality is modelled in a straightforward way. The collinearity equations form a perspective transform which mathematically describes that the object point, the perspective centre of the camera and the measured image point ideally lie on a straight line. The transform includes parameters for interior and exterior orientation. Exterior orientation parameters determine the position of the camera in terms of position coordinates and rotation of the image relative to an object space coordinate system.

DLT is a projective transform, where the transform parameters are not directly interpretable in terms of interior or exterior orientation. One of the advantages of DLT is that the parameters can be calculated without any initial approximations. This has made the method popular to use in cases where only non-metric cameras are available, i.e. cameras without calibration or without fiducial marks.

1.1 Bundle adjustment

The collinearity equations are defined by:

$$x = x_p - c \frac{T_x}{N} \quad (1)$$

$$y = y_p - c \frac{T_y}{N} \quad (2)$$

and

$$T_x = r_{11}(X - X_c) + r_{21}(Y - Y_c) + r_{31}(Z - Z_c) \quad (3)$$

$$T_y = r_{12}(X - X_c) + r_{22}(Y - Y_c) + r_{32}(Z - Z_c) \quad (4)$$

$$N = r_{13}(X - X_c) + r_{23}(Y - Y_c) + r_{33}(Z - Z_c) \quad (5)$$

where x and y are the measured image coordinates in the comparator coordinate system and X, Y, Z are the object coordinates in an object space coordinate system. The collinearity equation contains in total nine unknown parameters where: $X_c, Y_c,$ and Z_c are the coordinates of the perspective centre in the object space coordinate system; x_p and y_p are the coordinates of the principal point in the image system; c is the principal distance of the camera; r_{11}, \dots, r_{33} are elements of the rotation matrix describing the rotation of the film plane into the object space. The nine rotation matrix elements are functions of three independent angles Ω, Φ and K , which describe the rotations around the $X, Y,$ and Z axes.

The collinearity equations are non-linear and must be linearized. Initial approximations are needed for both the resection problem, i.e. estimation of inner and outer orientation elements, and the intersection problem, i. e. calculation of object coordinates of new points.

1.2 DLT

In DLT, comparator coordinates are expressed in terms of object coordinates and eleven transform parameters.

DLT is defined by [McGlone et al 1989]:

$$x = \frac{L_1X + L_2Y + L_3Z + L_4}{L_9X + L_{10}Y + L_{11}Z + 1} \quad (6)$$

$$y = \frac{L_5X + L_6Y + L_7Z + L_8}{L_9X + L_{10}Y + L_{11}Z + 1} \quad (7)$$

where L_1, \dots, L_{11} are the transform parameters.

The DLT transform is non-linear but by making the assumption that corrections to the observed image points are small and negligible, a linear equation system results. This linear equation system is used to determine the unknown parameters. A more rigorous method is to treat the equations as non-linear and compute the parameters iteratively. Point coordinates in object space are calculated by treating the equations as non-linear.

2. METHOD

The basic idea is to simulate an ideal test where control points, check points and cameras are simulated, and to use a unit camera, i. e. a camera with a unit principle distance c and a unit reference variance σ_0^2 .

Control points have been defined in two different configurations with 6 and 13 points. Check points are defined in an 11 by 11 by 11 grid. The coordinates of both the control and check points are defined to be without any error. Cameras have been defined in two configurations with two and four cameras, see table 1. Comparator readings of the image points have been simulated using the collinearity equation without any distortion component.

test	number of cameras	control points
1	2	6
2	4	6
3	2	13
4	4	13

Table 1: The two methods have been compared using four different tests with two or four cameras and 6 or 13 control points

The comparison of DLT and Bundle adjustment has been done using linear and iterative solutions of DLT-parameters. The collinearity equations have been used in two ways: interior and exterior orientation parameters unknown, and interior orientation parameters known and exterior orientation parameters unknown. In total, four cases have been tested and compared, see table 2.

case	method	solution of parameters	parameters and number of parameters
A	DLT	linear	L_1, \dots, L_{11} (11)
B	DLT	iterative	L_1, \dots, L_{11} (11)
C	Bundle	iterative	$X_0, Y_0, Z_0, \Omega, \Phi, K, x_p, y_p, c$ (9)
D	Bundle	iterative	$X_0, Y_0, Z_0, \Omega, \Phi, K$ (6)

Table 2: The four tested methods and their parameters.

2.1 Control points, check points and cameras

The control point grids have been configured according to figure 1.

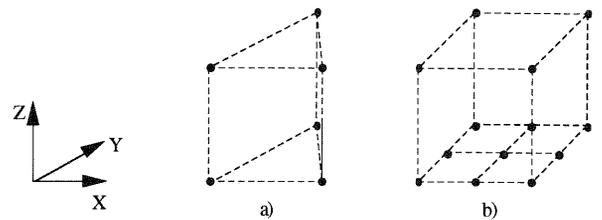


Fig 1 Control points used for the calculation of parameters. Configuration a) has 6 points and b) has 13 points.

The checkpoints have been arranged into an equal spaced grid of size 11 by 11 by 11, see figure 2. Side length of the control point grids and the check point grid have been selected to one.

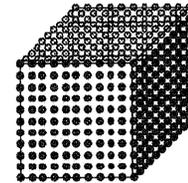


Fig 2 Check points arranged in an 11 by 11 by 11 grid.

The simulated cameras have been set to a height one unit above the control or check point grids, pointing to the centre of the grids, see figure 3.

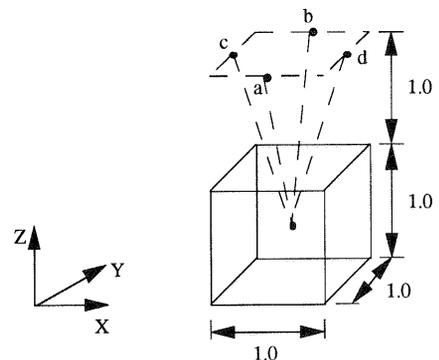


Fig 3 Configuration of the simulated cameras. Test 1 and 3 uses cameras a and b while test 2 and 4 uses all four cameras. The cameras are pointing to the middle of each grid configuration.

2.2 Solution of DLT parameters

As mentioned earlier, the DLT parameters can be calculated either in a direct way where the corrections to the observations are assumed to be small and can be ignored or by an iterative non-linear solution. The first type with a direct solution will be called linear DLT and the second type will be called iterative DLT.

Direct solution: The equations for the direct solution can be written as:

$$F_1 = L_1X + L_2Y + L_3Z + L_4 - L_9xX - L_{10}xY - L_{11}xZ - (x - v_x) = 0 \quad (8)$$

$$F_2 = L_5X + L_6Y + L_7Z + L_8 - L_9yX - L_{10}yY - L_{11}yZ - (y - v_y) = 0 \quad (9)$$

where v_x, v_y are residuals of the measured image points.

Iterative solution In the iterative solution, all observations have corrections except the control points, which are assumed to be without error. The equations for the iterative solution of the parameters can be written as:

$$F_3 = L_1X + L_2Y + L_3Z + L_4 - L_9(x - v_x)X - L_{10}(x - v_x)Y - L_{11}(x - v_x)Z - (x - v_x) = 0 \quad (10)$$

$$F_4 = L_5X + L_6Y + L_7Z + L_8 - L_9(y - v_y)X - L_{10}(y - v_y)Y - L_{11}(y - v_y)Z - (y - v_y) = 0 \quad (11)$$

Since equations (10) and (11) are non-linear they must be linearized before they can be used in a least squares solution.

2.3 Solution of parameters in the collinearity equations

As in the case with the iterative DLT, all observations have corrections. The collinearity equations for the iterative solution can be written as:

$$F_5 = cT_x - N x_p + N(x - v_x) = 0 \quad (12)$$

$$F_6 = cT_y - N y_p + N(y - v_y) = 0 \quad (13)$$

In case D, see table 2, equation (12) and (13) are linearized with respect to the unknown parameters $X_0, Y_0, Z_0, \Omega, \Phi, K$. For case C they are also linearized with respect to parameters x_p, y_p , and c .

2.4 Solution of object coordinates

Both the DLT and the collinearity equations are treated

as non-linear. Both the comparator readings and the calculated parameters are treated as observations.

2.5 Evaluation

The methods have been evaluated with respect to precision and internal reliability. Precision refers to the statistical variability of the result and the internal reliability indicates to what extent gross errors in an observation can be detected.

Precision values have been calculated for all control points in the 11 by 11 by 11 grid. An average value is calculated for the X, Y and Z component of the check point coordinates.

Internal reliability is evaluated by using the cofactor matrix Q_{vv}^P and the correlation matrix of the residuals resulting from the parameter calculations. An average of the off-diagonal elements in the correlation matrix has been calculated to get a value for comparison of how correlated the parameters are in the four cases.

Each diagonal element of the covariance matrix Q_w^P gives an indication of to what extent a gross error in the observations can be detected into a specific observation. A value of above 0.5 has been selected to indicate that an error can be detected in that position with a high degree of confidence.

3. ADJUSTMENT PROCESS

In a first step, the transform parameters for each of the four cases A, B, C, and D in table 2, are calculated using a least square adjustment method. In the same step, cofactor matrix Q_{vv}^P and the corresponding correlation matrix of the measurement residuals are computed. In a second step the calculated parameters and their calculated cofactor matrix Q_{xx}^P is used to calculate the standard deviation σ of the check points in order to estimate the precision of the coordinates of each point.

For all test set-ups the adjustment is overdetermined, i.e. the number of observations exceeds the number of parameters carried in the adjustment process, see table 3.

case	test	parameters (u)	observations (n)	redundancy (r)
A	1,2	11	12	1
B	1,2	11	12	1
B	1,2	9	12	3
C	1,2	6	12	6
A	3,4	11	26	15
B	3,4	11	26	15
C	3,4	9	26	17
D	3,4	6	26	20

Table 3: The four tested methods A, B, C, and D and redundancy in the adjustment process. See table 1 and 2 for cases and tests.

The simulated cameras are defined to be independent of each other, i.e. the parameters for each camera can be calculated independently.

3.1 Adjustment of the parameters

DLT Parameters: Parameters of the linear DLT, case A equation (8) and (9), are calculated using adjustment of indirect observations of the form [Mikhail 1976]:

$$\mathbf{v} + \mathbf{B} \Delta = \mathbf{f} \quad (14)$$

where \mathbf{v} is the vector of residuals of the measurements, \mathbf{B} is the matrix of parameter coefficients, Δ is the vector of parameter estimates, and \mathbf{f} is the vector of observations.

In order to calculate the parameters of the iterative DLT, case B, equations (10) and (11) are linearised by a Taylor series using only the first order terms. The linearized DLT equations look like:

$$F_3^0 + \frac{\delta F_3}{\delta L_1} dL_1 + \frac{\delta F_3}{\delta L_2} dL_2 + \dots + \frac{\delta F_3}{\delta L_{11}} dL_{11} + \frac{\delta F_3}{\delta x} dx + \frac{\delta F_3}{\delta y} dy = 0 \quad (15)$$

$$F_4^0 + \frac{\delta F_4}{\delta L_1} dL_1 + \frac{\delta F_4}{\delta L_2} dL_2 + \dots + \frac{\delta F_4}{\delta L_{11}} dL_{11} + \frac{\delta F_4}{\delta x} dx + \frac{\delta F_4}{\delta y} dy = 0 \quad (16)$$

The parameters are calculated using general least square adjustment. Expressing the coefficients of the measured image points by matrix \mathbf{A} , and the parameter corrections by vector Δ results in:

$$\mathbf{A} \mathbf{v} + \mathbf{B} \Delta = \mathbf{f} \quad (17)$$

Matrix \mathbf{A} and \mathbf{B} are based on equations (15) and (16). \mathbf{A} have the structure:

$$\mathbf{A} = \begin{pmatrix} J_{A1} & & & \\ & J_{A2} & & \\ & & \dots & \\ & & & J_{Ak} \end{pmatrix} \quad (18)$$

where sub-matrix J_{Ai} is the jacobian for image measurement of control point i :

$$J_{Ai} = \begin{pmatrix} \frac{\delta F_3}{\delta x} & \frac{\delta F_3}{\delta y} \\ \frac{\delta F_4}{\delta x} & \frac{\delta F_4}{\delta y} \end{pmatrix} \quad (19)$$

The number of control points is k . Matrix \mathbf{B} have the structure:

$$\mathbf{B} = \begin{pmatrix} J_{B1} \\ J_{B2} \\ \dots \\ J_{Bk} \end{pmatrix} \quad (20)$$

where the sub-matrix J_{Bi} is the jacobian of parameters for control point i :

$$J_{Bi} = \begin{pmatrix} \frac{\delta F_3}{\delta L_1} & \frac{\delta F_3}{\delta L_2} & \dots & \frac{\delta F_3}{\delta L_{11}} \\ \frac{\delta F_4}{\delta L_1} & \frac{\delta F_4}{\delta L_2} & \dots & \frac{\delta F_4}{\delta L_{11}} \end{pmatrix} \quad (21)$$

Collinearity equation parameters: The unknown parameters are calculated by linearization of equations (12) and (13) in the same way as for the DLT equations. Case C uses nine and case D six unknown parameters, see table 2. The resulting equations for nine parameters looks:

$$F_5^0 + \frac{\delta F_5}{\delta \Omega} d\Omega + \frac{\delta F_5}{\delta \Phi} d\Phi + \frac{\delta F_5}{\delta K} dK + \frac{\delta F_5}{\delta X_c} dX_c + \frac{\delta F_5}{\delta Y_c} dY_c + \frac{\delta F_5}{\delta Z_c} dZ_c + \frac{\delta F_5}{\delta x_p} dx_p + \frac{\delta F_5}{\delta y_p} dy_p + \frac{\delta F_5}{\delta c} dc + \frac{\delta F_5}{\delta x} dx + \frac{\delta F_5}{\delta y} dy = 0 \quad (22)$$

$$F_6^0 + \frac{\delta F_6}{\delta \Omega} d\Omega + \frac{\delta F_6}{\delta \Phi} d\Phi + \frac{\delta F_6}{\delta K} dK + \frac{\delta F_6}{\delta X_c} dX_c + \frac{\delta F_6}{\delta Y_c} dY_c + \frac{\delta F_6}{\delta Z_c} dZ_c + \frac{\delta F_6}{\delta x_p} dx_p + \frac{\delta F_6}{\delta y_p} dy_p + \frac{\delta F_6}{\delta c} dc + \frac{\delta F_6}{\delta x} dx + \frac{\delta F_6}{\delta y} dy = 0 \quad (23)$$

The system of equations is solved using the general case least square in equation (17) where \mathbf{A} is identical to the jacobian in equation (18) and (19) and \mathbf{B} is identical to equation (20) but J_{Bi} is substituted by:

$$J_{Bi} = \begin{pmatrix} \frac{F_5}{\delta \Omega} & \frac{\delta F_5}{\delta \Phi} & \dots & \frac{\delta F_5}{\delta y_p} & \frac{\delta F_5}{\delta c} \\ \frac{F_6}{\delta \Omega} & \frac{\delta F_6}{\delta \Phi} & \dots & \frac{\delta F_6}{\delta y_p} & \frac{\delta F_6}{\delta c} \end{pmatrix} \quad (24)$$

Solution of parameters: The parameters for each camera are calculated separately for each of the four cases A, B, C, and D. The parameters in the linear DLT equation are calculated by [Mikhail 1976]:

$$\Delta = \mathbf{N}^{-1} \mathbf{t} \quad (25)$$

where

$$\mathbf{N} = \mathbf{B}^t \mathbf{W} \mathbf{B} \quad (26)$$

$$\mathbf{t} = \mathbf{B}^t \mathbf{W} \mathbf{f} \quad (27)$$

$$\mathbf{W} = \mathbf{Q}^{-1} = \mathbf{I} \quad (28)$$

The image measurements are defined to be independent and with unit weight, thus $\mathbf{Q} = \mathbf{I}$ in equation (28).

The cofactor matrix of the residuals used in the estimation of internal reliability is :

$$\mathbf{Q}_{vv}^p = \mathbf{Q} - \mathbf{B} \mathbf{N}^{-1} \mathbf{B}^t = \mathbf{I} - \mathbf{B} \mathbf{N}^{-1} \mathbf{B}^t \quad (29)$$

The non-linear DLT parameters and the Bundle adjustment parameters are calculated by:

$$\mathbf{N} = \mathbf{J}_B^t \mathbf{W}_e \mathbf{J}_B \quad (30)$$

$$\mathbf{t} = \mathbf{J}_B^t \mathbf{W}_e \mathbf{f} \quad (31)$$

$$\mathbf{W}_e = (\mathbf{J}_A \mathbf{Q} \mathbf{J}_A^t)^{-1} = (\mathbf{J}_A \mathbf{J}_A^t)^{-1} \quad (32)$$

The cofactor matrix of the residuals \mathbf{Q}_{vv}^p is:

$$\mathbf{Q}_{vv}^p = \mathbf{J}_A^t \mathbf{W}_e (\mathbf{I} - \mathbf{J}_B \mathbf{N}^{-1} \mathbf{J}_B^t \mathbf{W}_e) \mathbf{J}_A \quad (33)$$

The cofactor matrix of the computed parameters for both the direct and the iterative methods is:

$$\mathbf{Q}_{xx}^p = \mathbf{N}^{-1} \quad (34)$$

The cofactor matrix \mathbf{Q}_{xx}^p is important as it is used in the following step of check point calculation together with the calculated parameters.

3.2 Adjustment of object point coordinates

Object points from the DLT: Object coordinates are calculated for case A and B in table 2 by using non-linear functions similar to equations (10) and (11), in the following denoted F'_3 and F'_4 . The equation is different in that all eleven parameters are treated as observations together with the comparator readings, i. e. L_1, \dots, L_{11} have residuals added to them in the same way as the comparator readings x and y . The linearized DLT equation is the same as in equations (15) and (16),

with partial derivatives for the object coordinates X , Y , and Z added. The unknown object coordinates are calculated using the general least square adjustment in equation (17). The check points are calculated separately but with data from all cameras. The \mathbf{A} and \mathbf{B} matrix has the same structure as in equation (18) and (20) but the jacobian \mathbf{J}_{Ai} for camera i is:

$$\mathbf{J}_{Ai} = \begin{pmatrix} \frac{\delta F'_3}{\delta L_{1i}} & \frac{\delta F'_3}{\delta L_{2i}} & \dots & \frac{\delta F'_3}{\delta L_{11i}} & \frac{\delta F'_3}{\delta x_i} & \frac{\delta F'_3}{\delta y_i} \\ \frac{\delta F'_4}{\delta L_{1i}} & \frac{\delta F'_4}{\delta L_{2i}} & \dots & \frac{\delta F'_4}{\delta L_{11i}} & \frac{\delta F'_4}{\delta x_i} & \frac{\delta F'_4}{\delta y_i} \end{pmatrix} \quad (35)$$

and the jacobian \mathbf{J}_{Bi} for camera i is:

$$\mathbf{J}_{Bi} = \begin{pmatrix} \frac{\delta F'_3}{\delta X} & \frac{\delta F'_3}{\delta Y} & \frac{\delta F'_3}{\delta Z} \\ \frac{\delta F'_4}{\delta X} & \frac{\delta F'_4}{\delta Y} & \frac{\delta F'_4}{\delta Z} \end{pmatrix} \quad (36)$$

Object points from the collinearity equation:

Analogous to DLT, equations (12) and (13) are used with the modifications that all parameters calculated in the previous step have residuals added to them in the same way as the measured image points. The modified functions are denoted F'_5 and F'_6 . Also the linearization is analogue, i.e. partial derivatives for the object coordinates X , Y , and Z are added to equations (22) and (23). The unknown object coordinates are calculated in the same way as with DLT but the jacobian of the observation coefficients \mathbf{J}_{Ai} is:

$$\mathbf{J}_{Ai} = \begin{pmatrix} \frac{F'_5}{\delta \Omega} & \frac{\delta F'_5}{\delta \Phi} & \dots & \frac{\delta F'_5}{\delta y_p} & \frac{\delta F'_5}{\delta c} & \frac{\delta F'_5}{\delta x} & \frac{\delta F'_5}{\delta y} \\ \frac{F'_6}{\delta \Omega} & \frac{\delta F'_6}{\delta \Phi} & \dots & \frac{\delta F'_6}{\delta y_p} & \frac{\delta F'_6}{\delta c} & \frac{\delta F'_6}{\delta x} & \frac{\delta F'_6}{\delta y} \end{pmatrix} \quad (37)$$

The jacobian \mathbf{J}_{Bi} is the same as in equation (36) except that functions F'_5 and F'_6 are used instead of F'_3 and F'_4 .

Solution of cofactor matrix: The basis for estimation of precision is the cofactor matrix \mathbf{Q}_{xx}^c of the computed check points. In case A, B, C, and D the cofactor matrix is given by:

$$\mathbf{Q}_{xx}^c = \mathbf{N}^{-1} = (\mathbf{J}_B^t \mathbf{W}_e \mathbf{J}_B)^{-1} \quad (38)$$

$$\mathbf{W}_e = (\mathbf{J}_A \mathbf{Q}_{11} \mathbf{J}_A^t)^{-1} \quad (39)$$

Matrix \mathbf{Q}_{11} is the a priori cofactor matrix of the observations, i.e. the calculated parameters from the first step and the measured image coordinates of the check

points. For each camera used in the calculation of object points, a sub-matrix Q_i is defined as:

$$Q_i = \begin{pmatrix} Q_{xx,i}^p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (40)$$

where $Q_{xx,i}^p$ is the cofactor matrix from equation (34) of the computed parameters for camera with index i . The ones in the diagonal is due to the fact that the measured image coordinates of the used check point have unit weight. The resulting Q_{11} matrix for k cameras is:

$$Q_{11} = \begin{pmatrix} Q_1 & & & \\ & Q_2 & & \\ & & \dots & \\ & & & Q_k \end{pmatrix} \quad (41)$$

4. RESULTS

In order to estimate precision, the standard deviation σ is calculated for each of the coordinate components of the check points. The diagonal elements q_{ii} of the cofactor matrix Q_{xx}^c in equation (38) have been used to calculate σ :

$$\sigma = \sigma_0 \sqrt{q_{ii}} = \sqrt{q_{ii}} \quad (42)$$

where σ_0 is set to 1.

The standard deviation have been calculated for each check point in the grid of figure 2. In order to get a value for comparison of case A, B, C and D in table 2, the mean deviation has been computed for each of the coordinate components, i.e. $\sigma_x, \sigma_y, \sigma_z$ see table 4 and 5.

test	component	A	B	C	D
1	σ_x	3.50	2.47	1.83	1.30
	σ_y	3.78	2.66	1.88	1.30
	σ_z	9.74	7.27	5.29	3.70
2	σ_x	2.32	1.70	1.40	1.01
	σ_y	2.47	1.77	1.48	1.01
	σ_z	6.49	5.08	4.38	3.07

Table 4: Mean deviation σ of the coordinate components X, Y and Z using 6 control points. Test 1 uses two cameras and test 2 uses four cameras. See table 2 for case A, B, C, and D.

Figure 4 Shows the mean deviation in the X-component, σ_x , for case A, B, C, and D. The mean deviations of the Y-component gives the same relative difference for case A, B, C, and D.

test	component	A	B	C	D
3	σ_x	1.57	1.12	1.39	1.01
	σ_y	1.66	1.13	1.47	1.01
	σ_z	4.84	3.41	4.35	3.05
4	σ_x	1.46	1.05	1.37	0.98
	σ_y	1.53	1.05	1.43	0.98
	σ_z	4.51	3.16	4.25	2.98

Table 5: Mean deviation σ of the coordinate components X, Y and Z using 13 control points. Test 3 uses two cameras and test 4 uses four cameras.

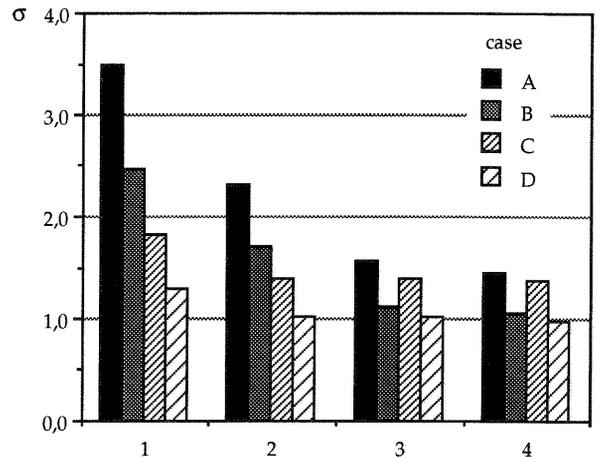


Fig 4 Mean deviation in the X-component, σ_x , from the check point calculation in test 1, 2, 3, and 4, see table 1.

Figure 5 shows the Z-component of the same check point grid as in figure 4.

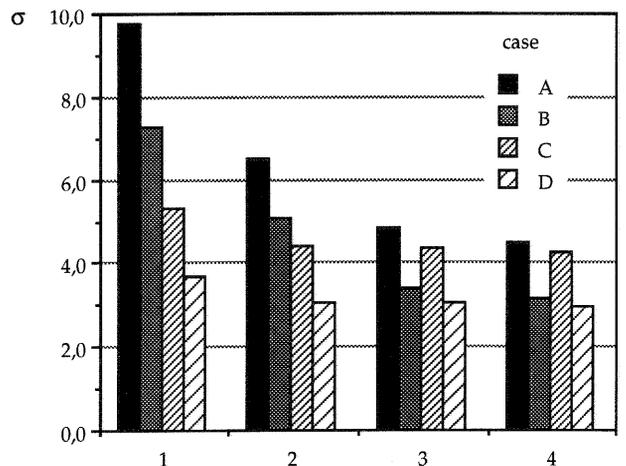


Fig 5 Mean deviation in the Z component, σ_z , from the check point calculation in test 1, 2, 3, and 4.

To get an idea of the internal reliability the correlation of the residuals resulting from the calculation of transform parameters can be studied. Figure 6 shows an average of the correlation values of the residuals for the four cases A, B, C, and D. Correlation is calculated from Q_{vv}^p in equation (29) and (33) by:

$$\rho_{ij} = \frac{q_{ij}}{q_{ii} q_{jj}} \quad (43)$$

Another way of getting an idea of the internal reliabi-

lity is to see how many of the observations have an diagonal element value above 0.5 in Q_{vv}^p as shown in figure 7.

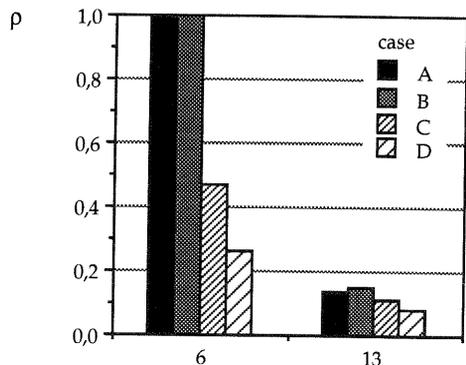


Fig 6 Average values of the residual correlations ρ_{ij} from the parameter calculation using 6 or 13 control points.

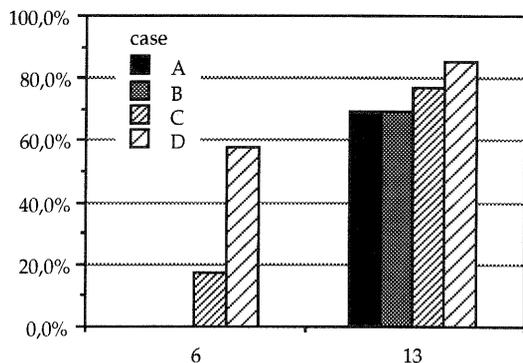


Fig 7 Estimation of internal reliability. The staples indicates the percent of parameters in case A, B, C, and D with a value above 0.5 in the diagonal of Q_{vv}^p using 6 or 13 control points. The values for case A and B using 6 control points are zero.

5. DISCUSSION

The evaluation is intended to give an estimate of the precision and an idea of the internal reliability, using different number of control points and cameras. The evaluation is theoretical in that it is based on an ideal simulated camera and comparator measurements. The distance between camera and control/check points have been short.

The test results of precision is shown in tables 4, 5 and figures 5, 6. The values of mean deviation in table 4, 5 does not give anything on the distribution of precision in the check point grid. However, the values are intended for a comparison of case A, B, C and D as shown in figure 4 and 5.

Figure 4 and 5 confirms the intuitive idea that iterative DLT should give a better precision compared to linear DLT and that bundle adjustment should give a better precision with 6 compared to 9 unknown parameters. It is also shown that the difference in precision decreases between DLT and bundle adjustment when the number of control points is increased. An interesting result is that the difference

between DLT, case A and B, and Bundle adjustment with 9 unknown parameters, case C, is rather small when 13 control points are used. The iterative DLT in case B is even better than Bundle adjustment in case C.

In general, the mean deviation in the Z-component is about 2.5 to 3 times higher compared with the X and Y component. The relative difference of the Z-component between case A, B, C and D is approximately the same as for the X and Y components.

Correlation of the comparator measurement residuals resulting from the calculation of transform parameters are shown as average values in figure 6. It indicates that DLT needs many control points to have an acceptable internal reliability. In case A and B, with only 6 control points, the residuals are fully correlated, i. e. the residuals of the comparator measurements are dependent on each other. A gross errors in one of the comparator measurements is not possible to locate when DLT is used. Bundle adjustment has lower correlation compared to DLT when 6 control points are used. When 13 control points are used the difference between DLT and Bundle adjustment becomes small, but still Bundle adjustment is better.

Figure 7 gives an idea of in how many of the comparator measurements a gross error can be detected. Using DLT and 6 control points, i. e. case A and B, a gross error can not be detected in any of the comparator measurements. For Bundle adjustment the situation is better. With 13 control points the difference between DLT and Bundle adjustment becomes smaller. A gross error can be detected in 70 - 85% of the measurements in case A, B, C, and D.

An indication of future work in evaluating the methods is to make a more extensive study of internal and also external reliability which are important aspects of the methods.

6. CONCLUSIONS

Bundle adjustment gives a better precision and internal reliability compared to DLT when the control points are few. When more control points are used the difference decreases in both precision and internal reliability.

In a case where a non-metric camera is used, i. e. the internal orientation of the camera is unknown, and many control points are available and well distributed in object space, the iterative DLT is preferable when precision is important. The linear DLT gives almost the same precision as Bundle adjustment but may also be preferable because no estimates of the parameters are needed.

The results from the estimates of the internal reliability indicates that Bundle adjustment in general has a better internal reliability than DLT but the difference decreases with the number of control points.

7. ACKNOWLEDGEMENTS

The ideas for this investigation originates from Professor Kennert Torlegård, Department of Photogrammetry, Royal Institute of Technology, Stockholm, to whom I would like to express my gratitude for many fruitful discussions and for his guidance. I would also like to thank my college Holger Zielinski for his useful cooperation and support.

REFERENCES

MCGLONE, J., AND MIKHAIL E. M., AND PADERES, F. C. 1989. Analytic Data-Reduction Schemes in Non-Topographic Photogrammetry. In *Non-Topographic Photogrammetry*. 2nd edition. Karara, H. M., Ed. American Society for Photogrammetry and Remote Sensing. Ch 4, pp 37-57.

MIKHAIL, E. M. 1976. *Observations and Least Squares*. IEP-A Dun-Donnelley Publisher. New York.