Photogrammetric Restitution Based on Linear Features

Kurt Kubik

Professor and Head of School

Queensland University of Technology, Department of Surveying, George Street,

Brisbane 4000. Qld., Australia

ISPRS Commission III

ABSTRACT

In industrial metrology, when measuring the actual shapes of industrial tools and jigs, it is often impossible to define unique measuring points on an object. Straight edges are however, commonly available. In this paper we describe the mensuration procedure using these edges instead of well defined object points. We describe the method of computation and the mensuration procedure, with the minimum number of lines necessary to calculate the problem successfully.

1.   INTRODUCTION

In an industrial metrology, we have to determine with high accuracy the actual shape of tools and to compare it to the desired "blueprint" shape. The problem with this quality control process is that no unique points are defined on the object itself, but linear or circular features abound (Fig. 1).

In order that a simple mensuration process can be used, and to allow online quality control during the manufacturing process, the reconstruction of the 3-D object and its quality control should preferably be done using only the available edges of the object and no auxiliary measuring tools defining virtual points. The problem is thus as follows:

(1) Can we measure, from different measuring stations to arbitrary, non-identical points along the well defined edges of the object and reconstruct its 3-D shape (relative orientation)?

(2) Can we transform the "as is" shape of the object into the "blueprint" shape using common edges instead of common points (absolute orientation)?

This paper discusses the minimum number of measurements to yield a unique solution to the problem. It expands on an earlier paper of the author on this topic (Kubik, 1991).

2. ADVANTAGES IN THE MENSURATION PROCESS

The use of linear features for 3-D reconstruction would greatly simplify the mensuration process, as no homologous points have to be identified. The reconstruction could take place while scanning the image.

In order to explain this idea, let us assume the use of two digital cameras. As each of these cameras scans the object scene, it only needs to register the intersections of the scan line with the linear features. These intersection points are matched in order to find the homologous linear features. Here a learning process can be used to fully utilise the information derived from the previous scan lines.

Also, a quality control on the straightness of the object lines is possible during this process, as straight object lines should map as straight image lines.
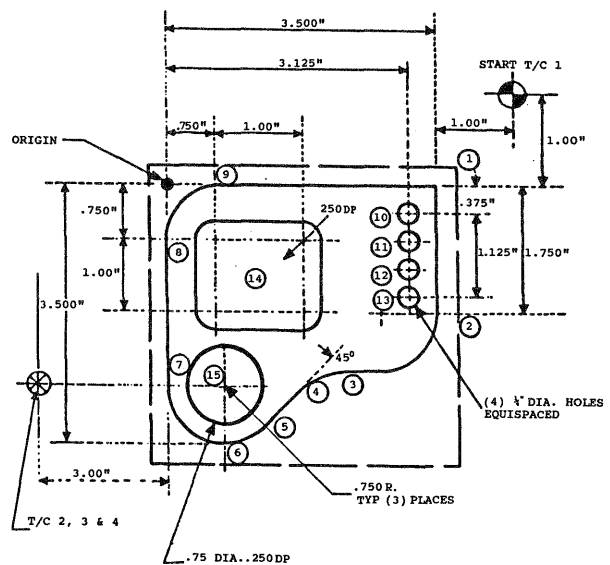

Figure 1: A typical tool shape in industrial metrology

3.   ABSOLUTE ORIENTATION USING EDGES

Let us start with the absolute orientation problem first, as this problem is relatively easy. The formulae for the three dimensional orthogonal transformations are:

$$X_A^{(1)} = \lambda \, R \, X_A^{(2)} + \Delta X \qquad (1)$$

Where:

$X_A^{(2)}$ = the blueprint coordinates (E,N,H) of a point A;

$X_A^{(1)}$ = the "as is" coordinates;

the matrices, $R, \Delta X$ = the rotations and shifts respectively;

λ = the scale factor.

687

We assume a number of points $X_A^{(1)}$ on straight line features of the "as is" object being measured. We only know the "blueprint" expressions for these straight lines:

$$E_A^{(2)} = [E_o^{(2)} + S_A.\text{Cos } \alpha.\text{Cos}\beta]_{line\,p}$$
$$N_A^{(2)} = [N_o^{(2)} + S_A.\text{Sin } \alpha.\text{Cos}\beta]_{line\,p} \qquad (2)$$
$$H_A^{(2)} = [H_o^{(2)} + S_A.\text{Sin } \beta]_{line\,p}$$

Where:

S_A = the distance of point A from reference point θ;
α and β = the azimuth and vertical angles of the straight line feature, respectively (Fig. 2). In matrix notation this becomes:

$$X_A^{(2)} = X_o^{(2)} + S_A.\,TRIG \qquad (3)$$

Where:

$$TRIG = \begin{bmatrix} Cos\,\alpha.Cos\,\beta \\ Sin\,\alpha.Cos\,\beta \\ Sin\,\beta \end{bmatrix}$$

For every measured point A on a line of the "as is" model, three observation equations can be formulated for the seven unknowns, λ, ΔX, R of the transformation (1), and the one additional unknown S_A locating the point A on the line:

$$X_A^{(1)} = \lambda R X_A^{(2)} + \Delta X$$

Where:

$$X_A^{(2)} = X_o^{(2)} + S\,A.TRIG$$

The unknown transformation parameters, and the additional point unknown, S_A, may be determined from these equations by standard least squares adjustment, if sufficient measurements are available.

How many measurements are needed to arrive at a unique solution to the problem? Let us assume we measure N points in the "as is" system, this gives $3N$ observation equations for the $7+N$ unknowns, leading to the inequality: $3N>7+N$. \qquad (4)

At least four points have to be measured in the "as is" system in order to arrive at a unique solution for the system. These four points obviously need to be positioned on at least two linear features in order to avoid singularities. We may, for example, measure two points each on two linear features.
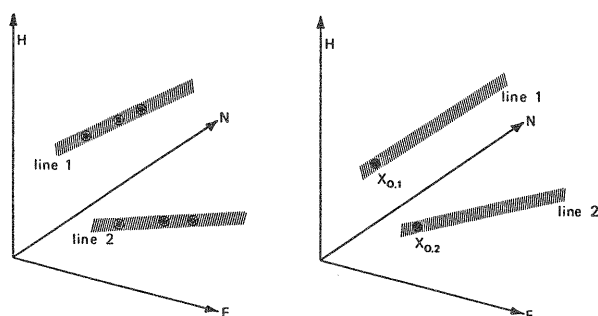
⊙ denote measured points x_A

Figure 2. Absolute orientation:
Determination of the transformation parameters from one coordinate system to another using straight line features only.

We may verify our conclusions with the following example: We wish to find the transformation parameters between the coordinate system of a cube in object space and its blue print shape.
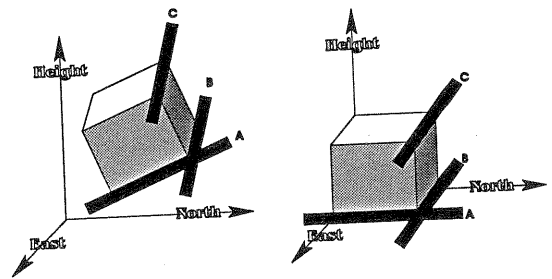

Figure 3: Exemplification of Absolute orientation Problem

Measuring only one line A on both cubes, does allow us to rotate the object cube into the blue print space, but we know neither its position along that line, its orientation with respect to the line nor its scaling. Measuring the two rectangular lines A and B, the object cube can now be positioned and oriented, but we cannot determine the scale factor between object and blue print space. Only when measuring two non-coplanar lines A and C, can the scale factor also be determined and the absolute orientation problem completed.

4. RELATIVE ORIENTATION USING EDGES

We start from the well known projective relationships (American Society of Photogrammetry, 1980) relating the object space to the image space:

$$X_A^{(1)} = P\;(X_A\,;\gamma_{(1)}\,) \qquad (5)$$

Where:

$X_A^{(1)}$ = vector of image coordinates (x,y) of point A in image (1);
X_A = vector of model coordinates (E,N,H) of point A;
$\gamma_{(1)}$ = vector of 6 parameters of image (1); being the tilts, ϕ, ω, κ and projection centre coordinates (E,N,H);
P = projective relationship.

In relative orientation using edges we assume that points on linear features are measured monoscopically both in the left and right image. Non homologous points are observed, that is, points on the linear feature measured in the left image differ from those in the right image. This will usually be the case in automatic industrial metrology, and no online image correlation is required.

The following observation equations can be formulated for two points A and B measured on one and the same linear feature of the object:

Left image:
$$X_A^{(1)} = P(X_A\,;\;\gamma_{(1)}= O) \qquad (6)$$
$$X_A = X_o + S_A.TRIG$$

Right image:
$$X_B^{(2)} = P\;(X_B\,;\;\gamma_{(2)}\,) \qquad (7)$$
$$X_B = X_o + S_B.TRIG$$

The image parameters of the left image are set to zero. Five parameters of the right image are unknown after choosing a suitable model base.

In addition, for every linear feature in model space, there are five unknowns: the three coordinate values X_o of one reference point on the line, and two directions α, β (Fig. 4). Also, for every measured point, the distance S from the reference point X_o is unknown. (For the first point measured on a new line, say point A, we choose S to be zero.)
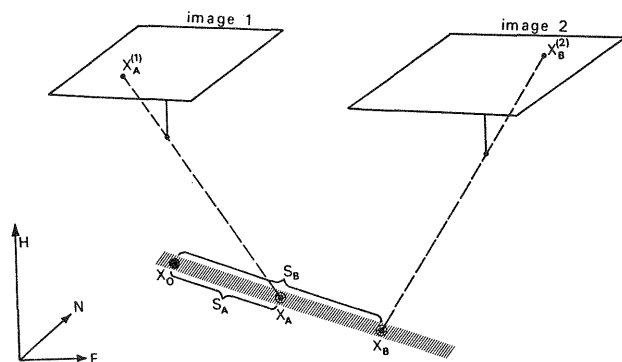
688

Figure 4. Relative orientation using non homologous points
on straight line features.

The unknown parameters may be determined from above equations by standard least squares adjustment, if sufficient measurements are available. How many measurements are needed to make the problem uniquely determined? Let us assume we measure N points in both images for each of the M lines. This gives:

(1) Number of observation equations -
   $4N.M.$

(2) Number of unknowns -
   5 orientation unknowns:
   $(3 + 2)$ M unknowns per linear feature;
   $M(N-1)$ unknown distances S for points measured in left image;
   $M\ N$ unknown distances S for points measured in right image.

(3) Inequality condition -
   $(4\ N\ M)$ equations $\geq 5 + M\ (5 + 2N - 1)$ unknowns; or

$$M \geq \frac{5}{2N - 4}$$

(8)

As two points measurements per line should suffice $(N=2)$ the required number of lines M follows to be infinite. Thus, the relative orientation problem appears to be non-solvable as stated above.

In a variation to the above formulation, we may assume, that pairs of parallel lines are measured. We then ask, how many of such pairs of parallel lines are required for relative orientation. This variation of our argument follows from the requirements of many constructions in projective geometry to know the vanishing points of bundles of parallel lines (Wylie, 1970). Also, in most industrial tools, parallel edges are readily available and recognisable as such.

We now assume measurements of N points made in both images for each of M pairs of parallel lines.

This gives:

(1) Number of observation equations -
   $8\ N,M.$

(2) Number of unknowns -
   5 orientation unknowns;
   $(6 + 2)M$ unknowns per pair of parallel lines;
   $2M(N-1)$ unknown distances S for points measured in left image;
   $2M\ N$ unknown distances S for points measured in right image.

(3) Inequality condition -
   $(8 \geq N\ M)$ equations $\geq 5 + M(8 + 4N - 2)$ unknowns; or

$$M \geq \frac{5}{4N - 6}$$

(9)

For N equal to 2, it results $M \geq 2,5$; that means we need three or more pairs of parallel lines in object space to complete relative orientation.

Again, the above can be verified by a geometric argument. Envisage a cube imaged from two stations.
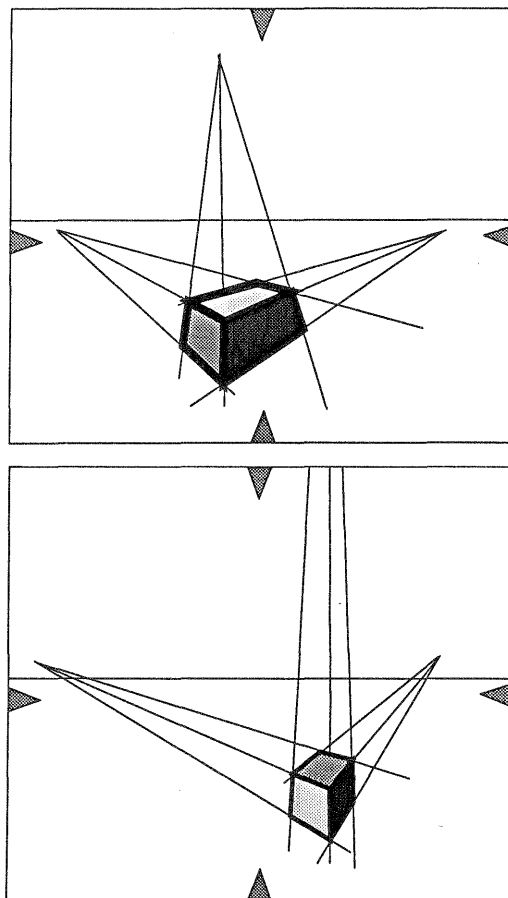

Figure 5. Exemplification of Relative Orientation Process

The attitudes of the left image can be determined from the shape of the triangle formed by the vanishing points, while its relative position can be determined by the position of the focal point with respect to the vanishing points (H. Wylie, 1970). It may however, be difficult to find three pairs of parallel edges in an object. This condition can probably be relaxed and further research is required on suitable minimal measuring arrangements for relative orientation.

689

5.   RESTITUTION

Once relative and absolute orientation is completed, restitution of the 3-D object lines follows. We measure points on a selected linear feature monoscopically both in the left and right image. The observation equations for the 3-D restitution of the linear feature are given in equations (7) and (8), with the image parameters known from the orientation process. Following an argument similar to that of Chapter 4, we find that two points have to be measured monoscopically in both images of the line for its 3-D restitution. Thus, in an automated system, the 3-D restitution of lines can take place as outlined in Chapter 2 of this paper.

6.   CONCLUSION

This paper gives some initial results on the topic of photogrammetric mensuration using homologous lines. This is of very great practical importance in automatic mensuration, as it allows the use of monoscopic measurements with no image correlation required. However, considerable research is still required in this field in order to convert this idea into a proven production method.

REFERENCES

American Society of Photogrammetry, 1980. *'Manual of Photogrammetry'* Am. Soc. Photo-gramm., Falls Church, Va., USA.

Kubik, K, 1991 "*Relative and Absolute Orientation based on linear features*". ISPRS Journal of Photogrammetry and Remote Sensing, 46 (1991).

Wylie, C.R. 1970 "*Introduction to Projective Geometry*", McGraw Hill.