HIGH ACCURATE LOCATION ON DIGITAL IMAGE AND APPLICATION IN AUTOMATIC RELATIVE ORIENTATION*

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ABSTRACT

A method, positioning accurately the corner and cross points on digital images, is presented in this paper. Based on the feature extraction by interest operator, the lines, forming the corner and cross point, are located firstly. Then their intersection can be determined. Various simulated images have been used in the test of position accuracy, which is much better than other method's one. The algorithm has been applicated in the relative orientation of real digital image pairs and the results are quite satisfactory.

key words: High accuracy, Location, Orientation, Straight line, Corner, Spread function

1 INTRODUCTION

Location of point and straight line is the basic step of Digital Photogrammetry and Computer Vision. So far, many methods for location of point and straight line have been proposed. Some popular methods are introduced following :

1.1 Moment-preserving method

1.1.1 Gray moment-preserving edge detection (Tabatabai, 1981) If $g(i,j)$ is gray value at pixel (i,j), then the k-order gray moment of digital image is defined as :

$$m_k = 1/N \sum_i \sum_j g^k(i,j) = 1/N \sum_l N_l g_l^k \tag{1}$$

where N is the total number of the pixel in the image, N_l is the total number of the pixel in the image with value g_l. For the one-dimensional case, ideal edge may be expressed as:

$$f(x)=g_1 + (g_2-g_1)\, u(x-x0) \tag{2}$$

where g_1 is the signal value below the edge, g_2 is the signal value above the edge, x_0 is the location of the edge, and $u(x)$ is the unit step function. Since there are three parameters unknown. the first three moments are chosen to solve them:

$$m_j=(x_0-0.5)/N\cdot g_1{}^j + (N-x_0+0.5)/N\cdot g_2{}^j \qquad j=1,2,3.$$

In particular, the solution for x0 is :

$$x_0=N/2\cdot(1-c/\mathrm{sqrt}(4+c^2))+0.5 \tag{3}$$

where

$$c=(3m_1m_2-m_3-2m_1{}^3)/\sigma^3$$
$$\sigma^2=m_2-m_1{}^2$$

1.1.2 Mass moment-preserving corner location (Liu. 1990) If a corner $P_0(x_0,y_0)$ exists in a circular region with radius r. Two intersection points of the boundaries of corner and the circumference of the circle are $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$. The area of the region between angle $p_1p_0p_2$ and arc p_1p_2 is :

$$A=[ar^2+x_2y_1-x_1y_2)+(x_1-x_2)(y_2-y_0)$$
$$-(y_1-y_2)(x_2-x_0)]/2 \tag{4}$$

mass moments:

$$Mx=g2\{(x_1-x_2)(r^2-x_1x_2-y_1y_2)+(y_0+y_1+y_2)$$
$$\cdot[(x_1-x_2)(y_2-y_0)-(y_1-y_2)(x_2-x_0)]\}/6$$
$$My=g2\{(y_1-y_2)(r^2-x_1x_2-y_1y_2)+(x_0+x_1+x_2)$$
$$\cdot[(x_1-x_2)(y_2-y_0)-(y_1-y_2)(x_2-x_0)]\}/6$$
$$Mxy=g2\{r^2(y_2^2-y_1^2-x_2^2+x_1^2)/16+[(x_2y_0-x_0y_2)$$
$$(x_2y_2+x_0y_0)+(x_0y_1-x_1y_0)(x_1y_1+x_0y_0)]/12$$
$$+[x_0^2(y_1^2-y_2^2)+y_0^2(x_2^2-x_1^2)]/24\} \tag{5}$$

and
$$x_1^2+y_1^2=r^2$$
$$x_2^2+y_2^2=r^2 \tag{6}$$

So the point $P_0(x_0,y_0)$, $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ can be determined by solving the six equations. The method is sensitive to noise because noise can notably effect the moments.

1.2 Wong detection method

Wong and Wei-Hsin (Wong. 1986) have developed a method for the location of circular targets on digital images. The first thresholding converts the window area to binary image :

$$\text{Threshold} = (\text{min pixel value} + \text{mean pixel value})/2. \tag{7}$$

Subsequently the position (x,y) and roundness (r) of the target can be computed

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$$x=m_{01}/m_{00}$$
$$y=m_{10}/m_{00}$$
$$r=Mx'/My'$$
$$Mx'=(M_{20}+M_{02})/2+\mathrm{sqrt}[(M_{20}-M_{02})^2/4+M_{11}^2]$$
$$My'=(M_{20}+M_{02})/2-\mathrm{sqrt}[(M_{20}-M_{02})^2/4+M_{11}^2] \quad (8)$$

where $m_{pq}=\Sigma\Sigma\, i^p \cdot j^q \cdot g_{ij}$ and $M_{pq}=\Sigma\Sigma(i-x)^p \cdot (j-y)^q \cdot g_{ij}$, $(p=0,1,\cdots, q=0,1,\cdots)$ are the (p+q) order moment about the origin and moment about the central. If r is smaller than threshold r_0, the target is round, otherwise the target is not round.

Trinder (trinder 1989) found that the result is subject to variations in window size, position and threshold value, and the location error may be up to 0.5 pixel. So he used the gray level value wij as a weight factor for each pixel :

$$X=1/M \cdot \Sigma\Sigma\ j\ g_{ij}\ w_{ij}$$
$$Y=1/M \cdot \Sigma\Sigma\ i\ g_{ij}\ w_{ij}$$
$$M=\Sigma\Sigma\ g_{ij}\ w_{ij} \quad (9)$$

under ideal circumstances, the precision of point with Trinder method can approach 0.02 pixel, but there are few such points in digital image.

1.3 Mikhail method (Mikhail. 1984)

Let f(s,t) represent the output of a perfect imaging system. Considering a linear, spatially-invariant imaging system with a normalized point-spread function p(s,t) assumed known. The sampling value g(s,t) is

$$g(s,t)=f(s,t) * p(s,t) \quad (10)$$

where * denote the convolution operation.

Suppose the distinct target can be characterized with a set of parameters X, then equation 10 may be rewritten as:

$$l(s,t)=f(s,t;X) * p(s,t). \quad (11)$$

Thus least squares method can be used to calculate the set of parameters X.

For one-dimensional edge, if p(x) is Gaussian function, equation 11 may be expressed as:

$$g(x)=f(x;g1,g2,x0) * p(x) \quad (12)$$

Using least squares method, g1, g2 and x0 can be calculated. So the position and shape of edge can be determined. For a cross target, it may be characterized with seven parameters. In ideal condition, the accuracies have reached within 0.03-0.05 pixel. But the point-spread function must be known in the method.

1.4 Hough Transformation (Ballard 1982)

Hough Transformation transforms image space into parameter space. It can detect not only straight line, but also other curves, such as circle, ellipse and parabola. But with the increase of the number of parameters, much computation time and more memory are spent. So Hough Transformation is mainly suitable to detect straight line. A straight line can be represented using two parameters: (1) the angle between the X-axis and the normal of the line (θ), (2) the distance (p) from origin to the line , i.e.

$$p=x\cos\theta+y\sin\theta. \quad (13)$$

Because of the limitation of quantization classes of p and θ, as well as the error of gradient direction and noise, the error of Hough Transformation is large.

1.5 Förstner method (Förstner 1986)

Förstner method is a famous in Photogrammetry. There are the advantages of fast speed and good accuracy in the method. Corner location consists of selecting optimal window and weighting centering. For each image window, the roundness q and weight w can be caclulated:

$$q=\frac{4\,\mathrm{Det}\,N}{(\mathrm{Tr}\,N)^2}$$
$$w=\frac{1}{\mathrm{Tr}\,Q}=\frac{\mathrm{Det}\,N}{\mathrm{Tr}\,N}$$

$$Q=N^{-1}=\begin{bmatrix} g_u^2 & g_u g_v \\ g_u g_v & g_v^2 \end{bmatrix}^{-1} \quad (14)$$

where $g_u=g_{i+1,j+1}-g_{i,j}$
$g_v=g_{i+1,j}-g_{i,j+1}$.

If q and w are larger than their thresholds and if it is extreme maximum, the window is an optimal window. Förstner method is a least squares method. It regards the distance from the origin to the straight line as observed value, and weight of observed value is the square of gradient. There are many advantages with the method. However, its location accuracy is not very good. When window size is 5*5 pixel, The accuracy of corner location is about 0.6 pixel in ideal condition.

Dr. Wu Xiaoliang in Wuhan Technical University of Surveying and Mapping proposed a method which regards the direction of edge as observated value. It seems to be more reasonable in thought, but none of gradient operators can compute accurately the direction of edge.

Most of above methods can be used to location of corners or lines, but the high accuracy can be acquired with only few of them, and it is necesssary in some aspects of Photogrammetry, such as interior orientation and relative orientation. So a high accurate method for the location of point and line has to be developed.

2 HIGH ACCURATE METHOD FOR THE LOCATION OF POINT AND LINE

2.1 The error of gradient operators

If an ideal edge line whose gradient is k passes through the intersection of four pixel. It is easy to compute the gradient k' of line with Roberts operator

$$k'=\begin{cases} 2k-1 & k>1 \\ k/(2-|k|) & -1<k<1 \\ 2k+1 & k<-1. \end{cases} \quad (15)$$

If the edge does not pass the intersection, the error is much larger. The error exists still, with Sobel operator

$$k'=\begin{cases} k & 0<k<1/3 \\ \dfrac{7k^2+6k-1}{-9k^2+22k-1} & 1/3<k<1. \end{cases} \quad (16)$$

That is said, when the line direction is replaced by the gradient direction, there is the model error that can't be neglected. So the methods using gradient direction, such as Hough Transfomation, can not obtain high accuracy .

2.2 The mathematical model of the new method

The corner is the intersection of two straight line. If two edge line forming corner are accurately determined, the corner coordinate can be obtained by solved the cross point. It is well known that the intensity curve of an ideal edge is a knife-edge curve:

$$g(x)=\int_{-\infty}^{x} s(x)\,dx \quad (17)$$

where s(x) is the line-spread function.

So the gradient of image :

$$\Delta g(x)=\frac{dg(x)}{dx}=\frac{d}{dx}\int_{-\infty}^{x} s(x)\,dx = s(x) \quad (18)$$

Considering the different of the intensity of knife-edge curve, a conclusion may be obtained: the gradient of an ideal edge's image is proportion to the line-spread function. An ideal line-spread function is Gauss function:

$$s(x,y)=\frac{1}{\sqrt{2\pi}\,\sigma}\exp[-k(x\cos\theta + y\sin\theta - p)^2] \quad (19)$$

So the gradient of the image can be expressed:

$$\Delta g(x,y)=a\exp[-k(x\cos\theta + y\sin\theta - p)^2] \quad (20)$$

This is the adjustment's function model. Regarding the magnitude of gradient as observed value, we can obtain an error equation

$$v(x,y)=C_0da + C_1dk + C_2dp + C_3d\theta + C_4 \quad (21)$$

where

$C_0=\exp[-k_0(x\cos\theta_0 + y\sin\theta_0 - p_0)^2]$

$C_1=-a_0C_0(x\cos\theta_0 + y\sin\theta_0 - p_0)^2$

$C_2=2a_0k_0C_0\cdot(x\cos\theta_0 + y\sin\theta_0 - p_0)$

$C_3=C_2(x\sin\theta_0 - y\cos\theta_0)$

$C_4=a_0C_0 - g(x,y)$

a_0, k_0, p_0 and θ_0 are the initial parameters approximations.

If Roberts gradient is used, so

$$\Delta g(i,j)=\mathrm{sqrt}[(g_{i+1,j+1}-g_{i,j})^2+(g_{i+1,j}-g_{i,j+1})^2] \quad (22)$$

$$d\Delta g=-\cos\beta\, dg_{i,j} + \sin\beta\, dg_{i+1,j} - \sin\beta\, dg_{i,j+1} + \cos\beta\, dg_{i+1,j+1} \quad (23)$$

where β is the gradient angle

If noise variance is m^2

$$m_{\Delta g}^2=\cos^2\beta\, m^2+\sin^2\beta\, m^2+\sin^2\beta\, m^2+\cos^2\beta\, m^2 =2m^2 \quad (24)$$

It shows that the weights of observed values are equal. After the error equation is normalized and the normal equation is solved iteratively, straight line parameters (p,θ) can be accurately obtained.

2.3 Initial value

The parameters p_0 and θ_0 can be obtained by using Hough Transformation. Because parameter a is the maximal gradient, thus

$$a_0=\max[\Delta g(x,y)] \quad (25)$$

and

$$k_0=\frac{\ln\Delta g(x_0,y_0) - \ln a_0}{(x\cos\theta_0 + y\sin\theta_0 - p_0)^2} \quad (26)$$

where (x_0,y_0) is a point near the line.

2.4 Gross error

In order to reject the gross error, iterative process of weight functions is used. So the gross error can be automatically got away. In our study, weight function is :

$$w_{ij}=\begin{cases} 1 & \sigma_0^2<\sigma_n^2 \text{ or } \sigma_0^2/v_{ij}^2>1 \\ \sigma_0^2/v_{ij}^2 & \text{otherwise} \end{cases} \quad (27)$$

2.5 The window of accurate location

In order to make full use of line message and get away other message, the criterion of window selection is that the window is longer along the line and it is not wider along the normal direction of the line. The points near the corner should be also rejected. Otherwise, they will influence the accuracy of location because of the interference of two line each other (See Fig 1).

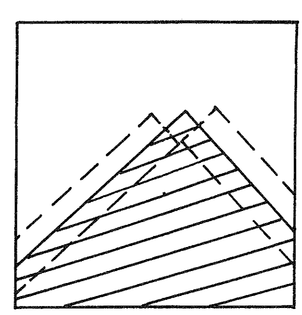

Fig. 1

2.6 Calculated corner coordinates

After two lines forming corner are obtained, corner coordinate (xc,yc) can be computed:

$$xc= (p_1 \sin \theta_2 - p_2 \sin \theta_1) / \sin(\theta_2 - \theta_1)$$
$$yc= (p_2 \cos \theta_1 - p_1 \cos \theta_2) /\sin(\theta_2 - \theta_1) \quad (28)$$

where (p_1,θ_1) and (p_2,θ_2) are the parameters of two straight lines.

3 ACCURACY

3.1 Internal accuracy

Standard error of unit weight:

$$\sigma_0= \text{sqrt}[\ \Sigma VV/(n-4)] \quad (29)$$

where n is the number of observed value.
Inversion of coefficient matrix of normal equation N is covariance matrix $Q=\sigma_0^2N^{-1}$. The covariance matrix of stright line parameters (p,θ) is

$$\begin{bmatrix} \sigma_0^2 q_{pp} & \sigma_0^2 q_{p\theta} \\ \sigma_0^2 q_{p\theta} & \sigma_0^2 q_{\theta\theta} \end{bmatrix} \quad (30)$$

The covariance matrix of two straight lines parameters (p_1, θ_1) and (p_2, θ_2) is

$$\begin{bmatrix} \sigma_{01}^2 q_{p1p1} & \sigma_{01}^2 q_{p1\theta1} & 0 & 0 \\ \sigma_{01}^2 q_{p1\theta1} & \sigma_{01}^2 q_{\theta1\theta1} & 0 & 0 \\ 0 & 0 & \sigma_{02}^2 q_{p2p2} & \sigma_{02}^2 q_{p2\theta2} \\ 0 & 0 & \sigma_{02}^2 q_{p2\theta2} & \sigma_{02}^2 q_{\theta2\theta2} \end{bmatrix} \quad (31)$$

The derivations can be computed by equation 28:

$$dx=Fx^T dL$$
$$dy=Fy^T dL \quad (32)$$

where

$$Fx = \begin{bmatrix} \sin \theta_2/\sin (\theta_2-\theta_1) \\ -p2 \cos \theta1/\sin(\theta_2-\theta_1)+xc\ \text{atan}(\theta_2-\theta_1) \\ -\sin \theta_1/\sin(\theta_2-\theta_1) \\ p_1 \cos \theta_2/\sin(\theta_2-\theta_1)-xc\ \text{atan}(\theta_2-\theta_1) \end{bmatrix}$$

$$Fy= \begin{bmatrix} -\cos \theta_2/\sin(\theta_2-\theta_1) \\ -p_2 \sin \theta_1/\sin(\theta_2-\theta_1)+yc\ \text{atan}(\theta_2-\theta_1) \\ \cos \theta_1/\sin(\theta_2-\theta_1) \\ p^1 \sin \theta_2/\sin(\theta_2-\theta_1)-xc\ \text{atan}(\theta_2-\theta_1) \end{bmatrix}$$

from covariance theorem:

$$\sigma x^2 =Fx^T D_{LL} Fx$$
$$\sigma y^2 =Fy^T D_{LL} Fy \quad (33)$$
$$\sigma xy =Fx^T D_{LL} Fy$$

So internal accuracy and error ellipse can be obtained.

3.2 external accuracy

If there are various errors in mathematical model. internal accuracy is higher than external accuracy. The external accuracy should be used in eveluation of location method. By comparing the coordinates of location (x,y) with real coordinates $(\tilde{x},\tilde{y})$:

$$Dx=x - \tilde{x}$$
$$Dy=y - \tilde{y}$$
$$Mx=\text{sqrt}(\ \Sigma\ DxDx / n)$$
$$My=\text{sqrt}(\ \Sigma\ DyDy / n)$$
$$M = \text{sqrt}(\ Mx^2+My^2) \quad (34)$$

where n is the number of samples, statistic results show that internal accuracy is almost equal to external accuracy in ideal condition (see table 1).

4 EXPEREMENT RESULTS

4.1 Relation between accuracy and the points near the corner

The points near corner are used in method 1 and the points near corner are not used in method 2. From talbe 1, it can be seen that the location precision is higher with method 2 and the external accuracy corresponds with the internal one. So the points near the corner should be rejected.

4.2 Compared with Forstner method

Form table 2, the new method's accuracy is higher than Forstner's. In ideal condition, the accuracy of new method is about 0.02 pixel size.

4.3 Sensibility to noise

From table 3, the accuracy of the new method is smaller than 0.1 pixel size, when there are some noises in image.

4.4 Result on real images

An image whose shape is like chessboard in produced by computer. So all intersection points' coordinates are known. After taken the photograph, it is digitised with 25*25 pixel size in scanner. Because scanner coordinates do not correspond with photo coordinates, we used the affine transformation for orientation and distortion correction. The new method is used for location, and the results see table 4 .

The table 4 shows that the real accuracy is 0.091 pixel, the line's direction is obtained at the same time.

4.5 Relative orientation

The new location method and the method of high precision least squares matching are used in relative orientation of a urban image (table 5). Because of the influence of urban buildings, the surface is no smooth, and only affine transformation is used in the method of least squares matching for geometric correction. It can not completely compensate the errors. But the problem does not exist in the new method, so its accuracy is higher than that in the method of least squares matching.

5 CONCLUSIONS

In this paper some popular location method have been analysed simply, and a new location method for straight line and corner has been proposed. By means of analyses and experements, a few conclusions have been obtained. (1) The new method can locate accurately the straight line. Its accuracy is much higher than that of Hough Transformation. (2) The high accuracy in the location of corner point can also be acquired with the new method. In the ideal condition, the accuracy is about 0.02 pixel size. (3) The new method can provide not also the position of the corner point, but also the directions of two lines forming the corner, which is useful to matching and recognition of image. (4) On the urban image with large scale, the result of the method of least squares matching may not be very good because of the influence of projective error. But it is satisfactory to use the new method. So the new method can be used in relative orientation, absolute orientation and other aspects which high accuracy is requared.

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unit: pixel

window size	method 1	method 2	method 3
9*9	0.248	0.039	0.043
11*11	0.185	0.033	0.035
13*13	0.125	0.025	0.027
15*15	0.092	0.027	0.022
17*17	0.046	0.028	0.018
19*19	0.036	0.025	0.016
21*21	0.028	0.023	0.015

*method 3: internal accuracy

Table 1

unit: pixel

window size	Forstner Algorithm	new Algorithm
9*9	0.216	0.039
11*11	0:177	0.033
13*13	0.153	0.025
15*15	0.140	0.027
17*17	0.132	0.028
19*19	0.124	0.025
21*21	0.122	0.023

Table 2

noise (grey level)	standard error (pixel)
0	0.023
2	0.029
4	0.037
6	0.050
8	0.067
10	0.092

(pixel size 21 * 21)

Table 3

unit: pixel

0.008, -0.015	-0.081, 0.007	0.040, -0.059	0.036, -0.118
0.028, 0.100	-0.006, 0.010	0.052, -0.037	0.076, 0.018
-0.102, 0.036	0.039, -0.008	-0.020, 0.055	-0.098, 0.066
-0.005, -0.011	-0.105, -0.020	-0.039, 0.066	0.049, 0.092
-0.081, -0.044	0.160, -0.095	-0.074, -0.078	-0.040, -0.002

mm = 0.091 pixel

Table 4

parameters (radian)	κ	ϕ	κ'	ϕ'	ω'	max dq (pixel)
New Algorithm	-0.019	-0.021	0.007	-0.009	0.002	0.02
L.S. Matching	-0.019	-0.014	0.008	-0.002	0.004	0.04

Table 5