Digital imagery acquired from aircraft contains significant geometric errors resulting from platform instability and variation in viewing geometry. This paper proposes a correction method based on the collinearity equations and using a digital elevation model. Results are presented for a test site in Southern Spain to demonstrate the technique in comparison with traditional methods based on a polynomial fit.

Keywords: Rectification, Registration, Remote Sensing.

1. INTRODUCTION

Digital imagery acquired from aircraft contains significant geometric errors resulting from platform instability and variation in viewing geometry. This paper proposes a correction method based on the collinearity equations and using a digital elevation model. Results are presented for a test site in Southern Spain to demonstrate the technique in comparison with traditional methods based on a polynomial fit.

2. TRADITIONAL POLYNOMIAL TECHNIQUES FOR RECTIFICATION OF IMAGES

2.1 Introduction

The sources of geometric error in satellite imagery are mainly due to instrument error, panoramic distortion, Earth rotation and platform instability. Instrument errors include distortions in the optical system, nonlinearity of the scanning mechanism and non-uniform sampling rates. The panoramic distortion is a function of the angular field of view of the sensor. Earth rotation velocity varies with latitude and has the effect of skewing the image. The platform instability includes variation in altitude and attitude.

Mather (1987), describes two methods for geometric correction of remotely sensed images with a narrow-angular field of view: the orbital geometry method and the map-based method. The first is based on the knowledge of the orbit of the satellite, the Earth's rotation and the along-scan and across-scan sampling rate. It is useful only when the desired accuracy is not high, or where sensor resolution is low, or when suitable maps of the area are not available. Otherwise, the second method, based on ground control points, is preferable. The map-based geometric correction is accomplished by transforming the image point coordinates on to corresponding ground control point coordinates selected from a map or other source.

2.2 Form and Application of the Polynomials

Let \((x,y)\) be the grid coordinates of a point on the map and \((c,r)\) be the row and pixel coordinates of the corresponding point in the image. For the transformation from \((x,y)\) to \((c,r)\) and vice versa, a polynomial relationship is established.

The form of the polynomial should describe the transformation of coordinates including any additional systematic errors present in the resulting image position. Three forms of polynomial were considered, each representing a progressive increase in the order. The same form of polynomial was used in each direction to transform \(c,r\) to \(x\) and then \(c,r\) to \(y\). The reverse transformation of a similar form were also output. \(x,y\) to \(c\) and \(x,y\) to \(r\). The simplest form, a first order polynomial, is given by:

\[

c = a_0 + a_1 x + a_2 y
\]

\[
r = a_0 + a_3 x + a_4 y
\]

For higher order polynomials the following algorithm can be used:

\[
c = \sum_{j=0}^{m} \sum_{k=0}^{n-j} a_{jk} x^j y^k
\]

where: \(m =\) order of polynomial
\(a =\) polynomial coefficient

A similar expression for \(r\) can be written.

The coordinates of the control points are used to determine least squares estimates of the polynomial coefficients.

Once the transformation coefficients have been computed the corrected image can be produced. This involves the transfer of the brightness value from the original image to the corrected image. This resampling process is complicated by the fact that it is unlikely that the pixel centres of the original image will fall at the pixel centres of the correct image. A further requirement might be that the image is resampled to a different resolution from the original, then the area covered by the pixels will be different and thus the
brightness value will again be affected.

Three methods of resampling are commonly used. The first, the nearest neighbour method, takes the value of the pixel in the input image that is closest to the input coordinates. The second, bilinear interpolation, takes the average of the four pixels nearest to the new coordinates. The third method, cubic convolution, computes the new pixel value taking into account the 16 nearest pixels in the input image.

A more complete description of the traditional techniques for geometric rectification as well as for resampling can be found in Jensen (1986) and Mather (1987).

2.3 Application of Traditional Techniques

A geometric rectification was carried out on the ATM image of Antequera using the least squares polynomial fit method described in section 2.2. It was based on the selection of 71 ground control points reasonably well distributed in the image. The UTM geographic coordinates of the points were taken from the 1:10 000 topographic maps and the image coordinates from a display of the image on the Nottingham Image Processing System, NIPS.

The results from the first order polynomial transformation showed some large residuals, of the order of 13 pixels, for both column and row. The Root Mean Square Error (RMSE) for columns was 5.8 and for rows was 5.7 pixels. The results from the second order polynomial showed only a small improvement with RMSE values of 5.6 and 4.5 pixels for column and row respectively. A substantial improvement was achieved with the third order polynomial, the majority of residuals being within ±4 pixels with RMSE values of 2.2 and 1.8 for column and row respectively.

A visual assessment of the effects of the transformations on the image was obtained by producing a corrected image for each order of polynomial and then overlaying the digitized map. The overlay was accomplished by clipping each corrected image by a number of rows and columns in order to get the best match with the map centre. As might be expected, the visual inspection showed increasing displacements of the corresponding features in the two images from the centre to the edges. The displacements were, as expected, bigger in columns than in rows. The majority of residuals being within ±4 pixels with RMSE values of 2.2 and 1.8 for column and row respectively.

A general assessment of the effects of the transformations on the image was obtained by producing a corrected image for each order of polynomial and then overlaying the digitized map. The overlay was accomplished by clipping each corrected image by a number of rows and columns in order to get the best match with the map centre. As might be expected, the visual inspection showed increasing displacements of the corresponding features in the two images from the centre to the edges. The displacements were, as expected, bigger in columns than in rows. The majority of residuals being within ±4 pixels with RMSE values of 2.2 and 1.8 for column and row respectively.

The basic principle of the technique discussed so far is that of modelling systematic errors in the image. The model is defined by a polynomial which should describe the systematic errors present. The polynomial should be based on an analysis of the influences affecting the geometry of the imaging system. Therefore the choice of polynomial is important to obtain the optimum results. An alternative approach is to attempt to model the cause rather than, as in the polynomial case, the effects on the image. The use of the collinearity equations in digital image processing is not new (Konecny, 1979), although the use of time-dependent geometry is not a new concept (El Hassi, 1981; Smith, 1989) although it is becoming more popular with the advent of analytical and digital techniques. For this reason the solution presented here attempts to remain as general, for any similar sensor, as possible. The geometry of the imaging system must be analysed and modelled, which, in the case being considered here, involves the sensor geometry and the aircraft motion during the period of imaging. The principles of the sensor geometry are reasonably well documented nowhere as the aircraft motion is not accurately known and requires certain assumptions to be made.

3. TIME-DEPENDENT GEOMETRY METHOD FOR RECTIFICATION OF IMAGES

3.1 Introduction

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3.2 Analysis of the Imaging System Geometry

The sensing system consists of a rotating mirror that scans a ground swath of a few metres wide on either side of nadir. A scan line of data is collected as a series of 2.5 mrad instantaneous fields of view (IFOV), which therefore defines the resolution. Since the image data is collected at a constant angular measure and constant pixel unit size, the ground area covered by the IFOV will vary depending on the angle from nadir, resulting in compression of the image towards the edges. This is removed from the image by the S-band correction in the digitization process. So the fundamental scanning geometry being described is therefore perspective geometry if the sensor was stationary. Successive scan lines of adjoining ground swath are produced by the forward motion of the aircraft. The putting together of the scan lines is dependent largely on the velocity-to-height ratio of the aircraft, although this is controlled by the allowable scan rates and the required resolution (as mentioned above). Normally these are selected to ensure some overlap, typically 10%. So the aircraft motion has an effect on the scan geometry (perspective geometry) and is used to create the series of scan lines to produce the image. It is therefore necessary to attempt to describe the aircraft motion during the image capture. Considering that each scan line captured, scan rates available are 12.5, 25 or 50 scans/sec. For an image of 1000 scan lines (the approximate image size being considered), the time to capture the image would be 80, 40 or 20 seconds
respectively. It is, therefore, not too unrealistic to assume that the sensor platform (aircraft) is stationary for each scan line and that, for successive scan lines, the aircraft's motion has a constant rate of change.

3.3 Time-Dependent Geometry

If each scan line is considered as a conventional photograph with perspective geometry then the mathematical model for the image can be considered as a series of such photographs of width equal to one scan line. For any of image point coordinates the standard collinearity equations can be written:

\[
\begin{align*}
X_\alpha &= r_{11} (X_\alpha - X_0) + r_{12} (Y_\alpha - Y_0) + r_{13} (Z_\alpha - Z_0) \\
Y_\alpha &= r_{21} (X_\alpha - X_0) + r_{22} (Y_\alpha - Y_0) + r_{23} (Z_\alpha - Z_0) \\
Z_\alpha &= r_{31} (X_\alpha - X_0) + r_{32} (Y_\alpha - Y_0) + r_{33} (Z_\alpha - Z_0)
\end{align*}
\]

where:

- \(X_\alpha, Y_\alpha, Z_\alpha\) = image coordinates of point \(a\)
- \(X_0, Y_0, Z_0\) = ground coordinates of point \(A\)
- \(r_{ij}\) = matrix elements of orthogonal rotation

The determination of the position and orientation of a scan line \(j\), given that \(X_\alpha, Y_\alpha\) are measured and \(X_\alpha, Y_\alpha, Z_\alpha\) and \(f\) are known, involves six unknowns: \(\phi_1, \phi_2, \psi_1, \psi_2, \kappa_1, \kappa_2\). The solution in this form, for all scan lines, results in an impractical situation, with far too many unknowns in the solution. A similar technique to that used in Smith (1989) is adopted, where, considering the assumption made about the aircraft motion above, a relationship exists between all like parameters:

\[
\begin{align*}
\phi_j &= \phi_0 + x_{0\phi} \theta_j \quad (6) \\
\theta_j &= \theta_0 + x_{0\theta} \theta_j \\
\kappa_j &= \kappa_0 + x_{0\kappa} \theta_j \\
X_{0j} &= X_{00} + x_{0X} \theta_j \\
Y_{0j} &= Y_{00} + x_{0Y} \theta_j \\
Z_{0j} &= Z_{00} + x_{0Z} \theta_j
\end{align*}
\]

where:

- \(x_0\) = x coordinate of point \(a\)
- \(X_0\) = scan line number (in direction of flight)
- \(\phi_0, \theta_0, \kappa_0\) = start values of the parameters:
  - \(\phi_0\) = pitch
  - \(\theta_0\) = heading
  - \(X_{00}\) = X plan location
  - \(Y_{00}\) = Y plan location
  - \(Z_{00}\) = flying height
- \(\omega_{0\phi}, \omega_{0\theta}, \omega_{0\kappa}\) = increment coefficients for the parameters

This reduces the number of unknowns to \(\omega_{0\phi}, \omega_{0\theta}, \omega_{0\kappa}, \phi_0, \theta_0, \kappa_0, X_{00}, Y_{00}, Z_{00}\) and a solution can now be obtained for the 12 unknowns (Smith, 1989; El Hassan, 1981; Case, 1961). Once values for \(\phi_0, \theta_0, \kappa_0\) are known, an orientation element for any scan line can be obtained, if \(x\) is known, by substitution into equations (6) to (11).
Table 1. Summary of results

<table>
<thead>
<tr>
<th>Method</th>
<th>Control Points</th>
<th>Check Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No  Pts</td>
<td>x</td>
</tr>
<tr>
<td>TOG</td>
<td>71</td>
<td>5.8</td>
</tr>
<tr>
<td>1st</td>
<td>5.7</td>
<td>5.8</td>
</tr>
<tr>
<td>2nd</td>
<td>4.5</td>
<td>5.6</td>
</tr>
<tr>
<td>3rd</td>
<td>1.6</td>
<td>2.5</td>
</tr>
<tr>
<td>TOG</td>
<td>45</td>
<td>5.4</td>
</tr>
<tr>
<td>1st</td>
<td>5.6</td>
<td>6.2</td>
</tr>
<tr>
<td>2nd</td>
<td>4.2</td>
<td>5.8</td>
</tr>
<tr>
<td>3rd</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>TOG</td>
<td>20</td>
<td>5.9</td>
</tr>
<tr>
<td>1st</td>
<td>6.0</td>
<td>7.5</td>
</tr>
<tr>
<td>2nd</td>
<td>5.2</td>
<td>6.7</td>
</tr>
<tr>
<td>3rd</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Key: TOG = Time Dependent Geometry
1st = 1st Order Polynomial
2nd = 2nd Order Polynomial
3rd = 3rd Order Polynomial

With a small amount of control, the distribution and quality of the control points can significantly affect the results. The values that appear to be most sensitive to this problem are the summation values. Therefore, it was felt that the minimum amount of control that should be considered is 20 control points, which provides a small amount of redundancy.

The time-dependent geometry approach does provide parameters which describe the sensors motion, equations (6) to (11). The parameter values obtained for the 71 control point solution are as follows:
(rotations in radians, coordinates in km)

\[
\begin{align*}
\theta_{\theta} & = -0.063687 \\
\theta_{\phi} & = 0.000000 \\
\theta_{\kappa} & = 0.819822 \\
\phi_{\theta} & = 363.901 \\
\phi_{\phi} & = 95.925 \\
\phi_{\kappa} & = 3.018 \\
\end{align*}
\]

It is interesting to note that the \( \phi_{\kappa} \) value compares very well with the nominal flying height of 3.000 km.

5. CONCLUSIONS

This preliminary study has shown that the time-dependent geometry technique is feasible and, in general, has produced results at least comparable with those obtained from the first and second order polynomial solutions. An improvement in the results from the time-dependent geometry might be produced if the assumption about the aircraft motion is reconsidered with higher order parameters being introduced in equations (6) - (11). Attempting to model aircraft movement during each scan may improve the results, although this might be quite difficult. A more simple solution could be to divide the image into small units. This, however, might lead to inconsistencies at the joining edges.

In summary, results from the time-dependent geometry approach show no significant improvement over the original polynomial solution. The modelling of systematic errors by any method might be extremely difficult if there are irregularities in the data. Such irregularities might be due to short period movement of the sensor. Further work is envisaged, in particular, in the use of the time-dependent geometry approach with other images, especially those with large variations in relief, since the ground height is taken into account by this method.

6. REFERENCES


