THE PANORAMIC PROJECTION OF THE GLOBE FROM SATELLITES

BY

PROFESSOR WAGIH N. HANNA

Faculty of Engineering
Ain Shams University
Abbasis , Cairo , Egypt

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ABSTRACT

A panoramic photograph is a picture of strip of terrain taken transverse to the direction of flight. The exposure is made by a specially designed camera which scans laterally from one side of the flight path to the other. The lateral scan angle maybe as great as 180 , in which the photograph contains a panoramic of the terrain from horizon to horizon. The panoramic photograph is therefore considered a central projection of the globe on the cylindrical film in the satellite , which produces after development what is called "THE PANORAMIC MAP". When the axis of the camera coincides with the geographical axis of the earth joining the two poles N and S ,the map produced is called "VERTICAL PANORAMIC MAP" otherwise it is called "GENERAL PANORAMIC MAP".
This paper deals with the geometry of the vertical panoramic photograph , the derivation of the transformation equations of the points on the globe and its corresponding projections on the cylindrical film and hence with its images on the panoramic map. The second part deals with determination of the panoramic equations of longitudes and latitudes and the illustration of the vertical panoramic map. For the general panoramic projection a simple and short discussion is given to determine the geographic and panoramic coordinates of any given point related to any arbitrary direction of the axis of the camera.

KEY WORDS : Photogrammetry, Panoramic, Cartographic, Mapping, Satellites

1. GEOMETRY OF THE PANORAMIC PHOTOGRAPH

Figure (1) shows an isometric view illustrating the geometry of a vertical panoramic photo taken from exposure station C. The camera focal length is f and the flying height above datum is h. In this system the X axis is taken in the direction of flight passing through the center of the globe O. The Y-axis is taken through O perpendicular to the X axis in the equator horizontal plane.

1.1 Notations and Representation Equations

r= radius of the globe

$P_1(X_1,Y_1,Z_1)$= any point on the globe

U_1 = longitude angle of P_1

V_1 = latitude angle of P_1

P_2 = image of P_1 on the cylindrical surface D

From the figure we have the representation equations;

$$X_1=r\cos V_1\cos U_1$$
$$Y_1=r\cos V_1\sin U_1 \qquad (1)$$
$$Z_1=r\sin V_1$$

To simplify the calculations , the radius r of the globe is assumed to be equal unity,i.e. r=1.

1.2 First Transformation Equations
Figure(2)

This equation expressing the coordinates (X_2,Y_2,Z_2) in terms of either (X_1,Y_1,Z_1) or (U_1,V_1) are called the first transformation equations.The equation of the cylinder film D is

$$Y^2+(z-h)^2=f^2 \qquad (2)$$

The equation of the projection ray P_1CP_2 are

$$X = x_1t$$
$$Y = y_1t \qquad (3)$$
$$Z = h+t(Z_1-h)$$

where t is a parameter.
Since P_2 lies on CP_1 and on the cylinder D then we have

$$X_2 = X_1t$$
$$Y_2 = Y_1t$$
$$Z_2 = h+t(z_1-h)$$
$$Y_2{}^2+(Z_2-h)^2 = f^2$$

which yields to the parameter

$$t=-\frac{f}{\sqrt{Y_1^2+(Z_1-h)^2}}$$

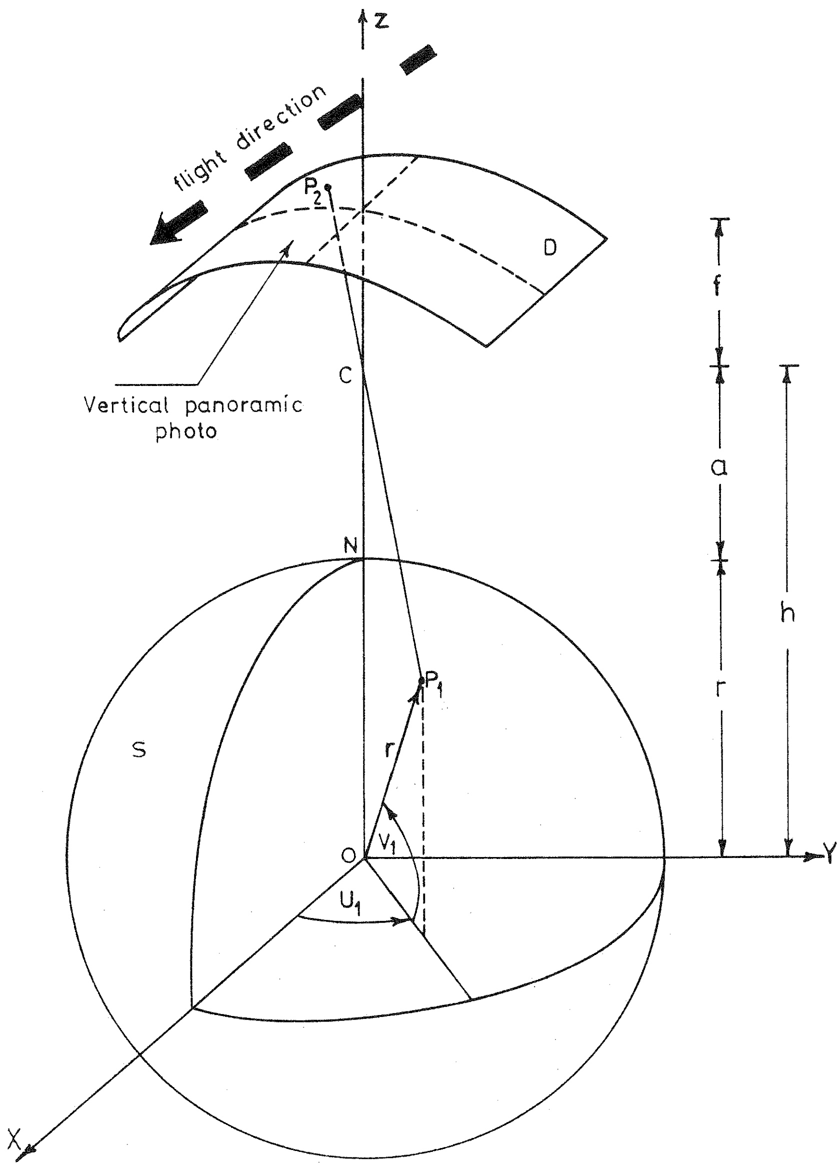


Fig.1. Geometry of a panoramic photograph.

Hence the first transformation equations are :

$$X_2=-\frac{X_1 f}{\sqrt{Y_1^2+(Z_1-h)^2}}$$

$$=-\frac{f\cos V_1\cos U_1}{\sqrt{\cos^2 V_1\sin^2 U_1+(\sin V_1-h)^2}}$$

$$Y_2=-\frac{Y_1 f}{\sqrt{Y_1^2+(Z_1-h)^2}} \tag{4}$$

$$=-\frac{f\cos V_1\cos U_1}{\sqrt{\cos^2 V_1\sin^2 U_1+(\sin V_1-h)^2}}$$

$$Z_2=h+\frac{f(h-Z_1)}{Y_1^2+(Z_1-h)^2}$$

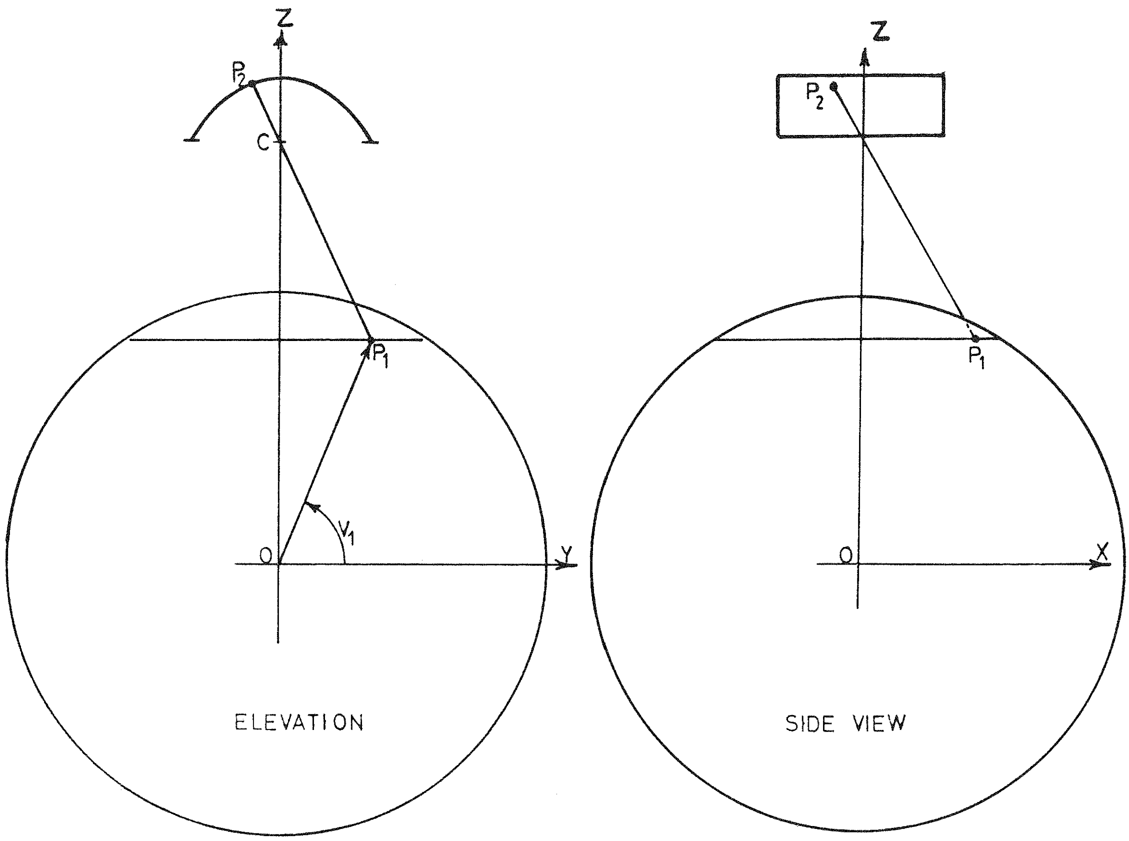
$$=h+\frac{f(h-\sin V_1)}{\sqrt{\cos^2 V_1\sin^2 U_1+(\sin V_1-h)^2}}$$

1.3 Limits of Coordinates

Limits of coordinates are deduced from figure (3):

$$(i)\quad 0\le|X_1,Y_1|\le L$$

$$\frac{1}{h}\le|Z_1|\le 1 \tag{5}$$



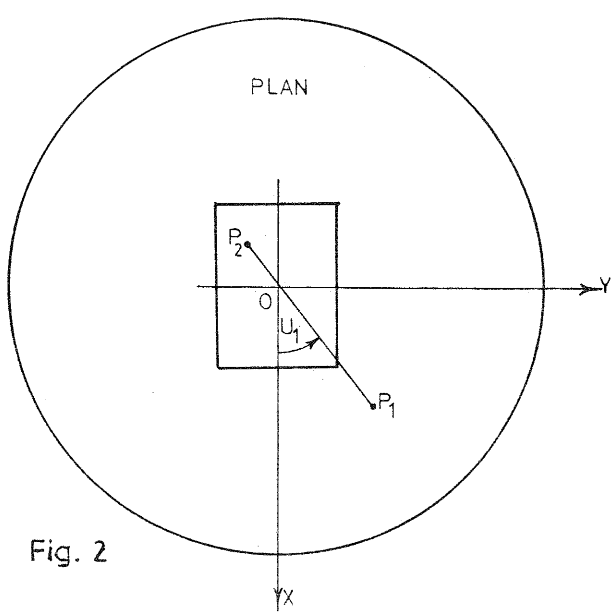


Fig. 2

where

$\sin V_o = \frac{1}{h}$ *and* $\cos V_o = L$

$$(ii) \quad V_o \le V_1 \le \frac{\pi}{2} \qquad (6)$$

$$0 \le U_1 \le 2\pi$$

where

$$V_o = \sin^{-1}\left(\frac{1}{h}\right)$$

$$(iii) \quad 0 \le |X_2| \le f.\tan V_o$$

$$0 \le |Y_2| \le f.\sin V_o \qquad (7)$$

$$h + L.f \le Z_2 \le h + f$$

1.4 The Second Transformation Equations

If we develop the cylindrical film , then we get a panoramic map of the globe. Every point $P_1(X_1, Y_1)$ on the globe will have a corresponding panoramic point $P^*(X^*, Y^*)$ on the map as shown in figures (4) and (5). By transforming the axes from O to C we get:

$$\bar{X}_2 = X_2$$

$$\bar{Y}_2 = Y_2 = f \sin \phi \qquad (8)$$

$$\bar{Z}_2 = (Z_2 - h) = f \cos \phi$$

Hence we get the second transformation equations which express the panoramic coordinates (X^*, Y^*) of ant point P^* in terms of the coordinates (X_2, Y_2, Z_2) :

$$X^* = X_2$$

$$Y^* = f\phi = f\sin^{-1}(Y_2/f)$$

$$= f\cos^{-1}((Z_2 - h)/f) \qquad (9)$$

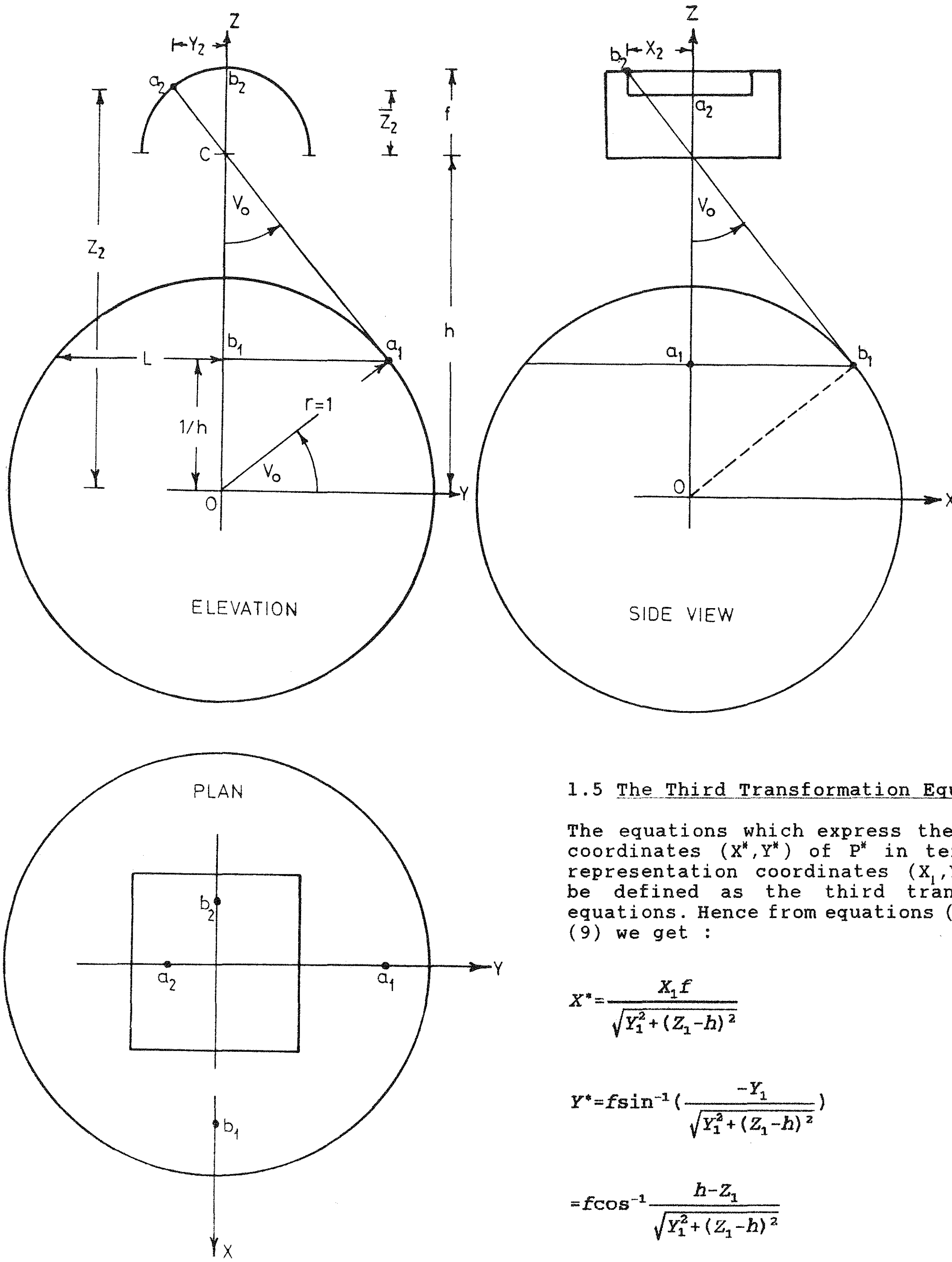


Fig. 3. Limits of coordinats.

1.5 <u>The Third Transformation Equations</u>

The equations which express the panoramic coordinates (X^*, Y^*) of P^* in terms of the representation coordinates (X_1, Y_1, Z_1) will be defined as the third transformation equations. Hence from equations (4),(8),and (9) we get :

$$X^* = \frac{X_1 f}{\sqrt{Y_1^2 + (Z_1 - h)^2}}$$

$$Y^* = f \sin^{-1}\left(\frac{-Y_1}{\sqrt{Y_1^2 + (Z_1 - h)^2}}\right) \tag{10}$$

$$= f \cos^{-1} \frac{h - Z_1}{\sqrt{Y_1^2 + (Z_1 - h)^2}}$$

2. THE PANORAMIC MAP

We deal in this part with determination of the panoramic equations of longitudes and latitudes. Hence it can be easily illustrated to get the required panoramic map.

2.1 <u>Analytical Representation of Longitudes</u>

Longitudes are characterized by constant U_1. Therefore from equations (1) and (10) we deduce the panoramic equation of the longitude :

$$X^* = -\frac{f}{\tan U_1}\sin\left(\frac{Y^*}{f}\right) \tag{11}$$

2.2 Analytical Representation of Latitude

From equation (4) we can get the first transformation equation of the latitude V_1:

$$K^2(X_2^2+Y_2^2) - (Z_2-h)^2 = 0$$

where

$$K = (\sin V_1 - h)/\cos V_1$$

This equation represents the curve of intersection of the circular cone whose vertex is the center of projection C and its base is the latitude V_1 with the cylindrical surface of the film. By substitution in (8) and (9) we deduce the panoramic equation of the latitudes

$$X^* = \pm f\sqrt{\cos^2\left(\frac{Y^*}{Y}\right)(K^{-2}+1)-1} \tag{12}$$

2.3 Representation of The Panoramic Map

Figure(6) shows the panoramic map in which the longitude and latitudes are illustrated according to the equations (11) and (12) respectively. The latitudes which satisfy the inequality:

$$V_0 \le V_1 \le \pi/2, \text{ where } V_0 = \sin^{-1}(1/h)$$

are represented on the map. The latitude which corresponds to V_0 is known as the envelope curve of the map.

2.4 Location of Points From The Panoramic Map

If the panoramic coordinates (X^*, Y^*) of any point P^* are known, then its corresponding point $P_1(U_1, V_1)$ on the globe can be determined from the equations (11) and (12) or approximately directly from the panoramic map.

2.5 The General Panoramic Map

In the general panoramic projection the axis of the camera is in arbitrary position w.r.t the geographical axis of the globe. To get the corresponding transformation equations in this case we have to change the direction of axes of reference without changing the origin O. Let the new system of axes be (X',Y',Z') and the direction cosines of OX',OY', and OZ' referred to the original axes be (L_1, M_1, N_1), (L_2, M_2, N_2), and (L_3, M_3, N_3). Then the coordinates of the point $P_1(X_1', Y_1', Z_1')$ referred to the original system will be;

$$\begin{aligned} X_1 &= L_1 X'_1 + L_2 Y'_1 + L_3 Z'_1 \\ Y_1 &= M_1 X'_1 + M_2 Y'_1 + M_3 Z'_1 \\ Z_1 &= N_1 X'_1 + N_2 Y'_1 + N_3 Z'_1 \end{aligned} \tag{13}$$

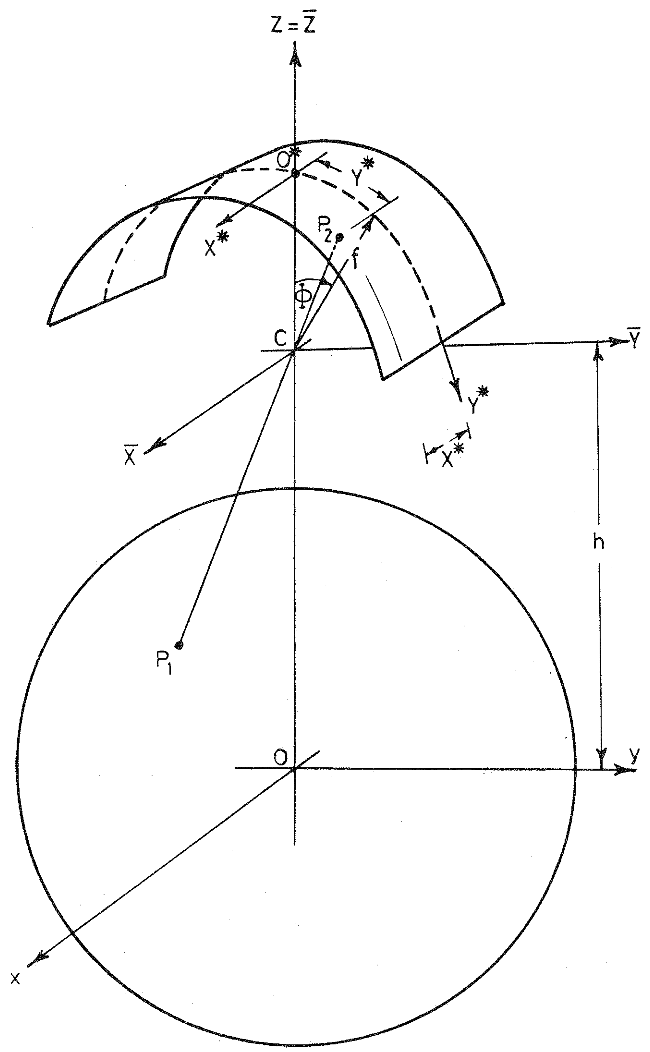


Fig. 4. The panoramic mapping

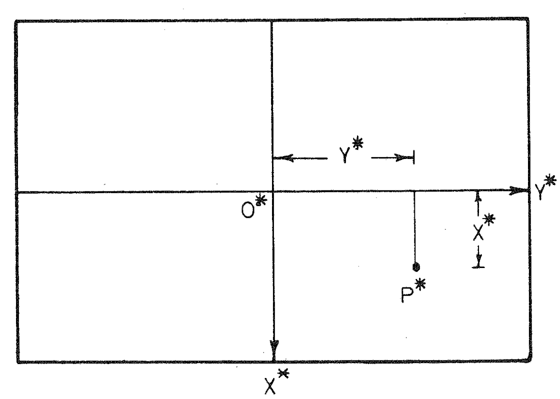


Fig. 5. The panoramic photograph.

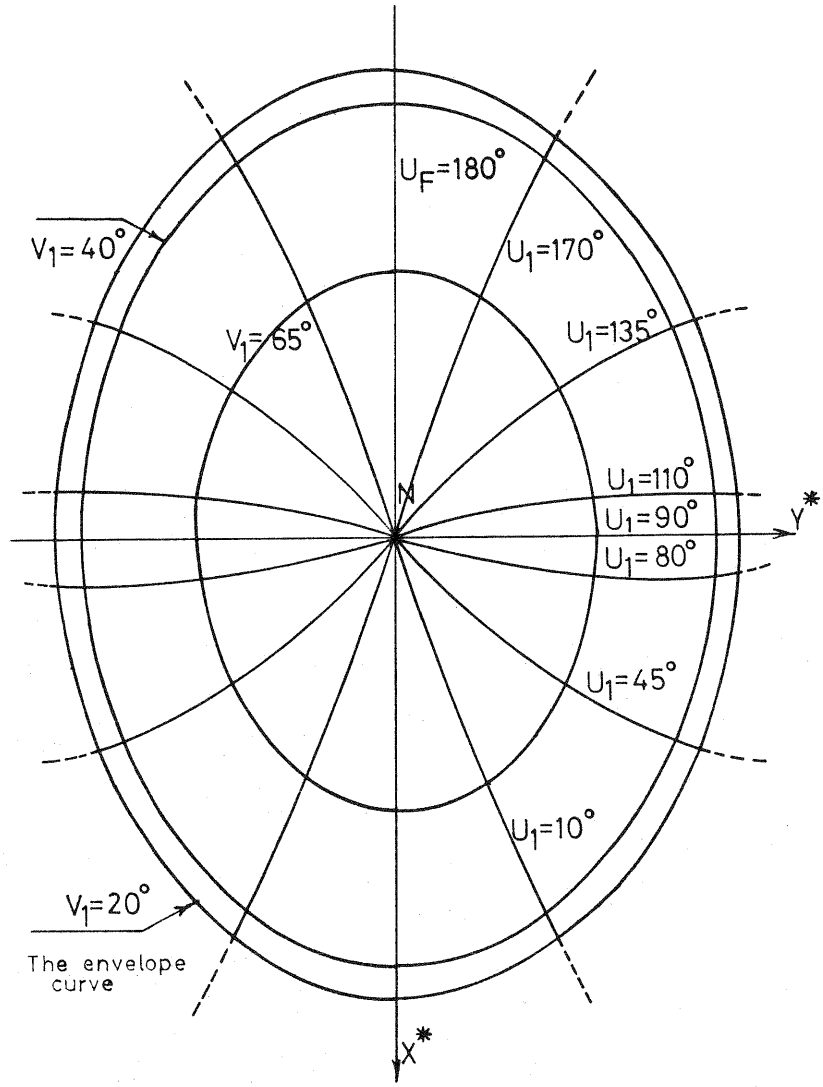


Fig. 6. Panoramic map

By substituting these values in (10) we get its panoramic coordinates X^* and Y^*. Also from (1) we get its geographical coordinates U_1 and V_1.

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